Semimagic generating functions and constituents

(general form, with cubic data)

Notation:
S: semimagic squares (all positive values).
s: normalized squares (symmetry types).
R: reduced squares (least element is 0).
r: reduced normalized squares (reduced symmetry types).
gf: generating function in some form.
gfsum: generating function as a sum of simple terms.
c: Cubic (fixed strict upper bound; weak upper bound for reduced).
a: Affine (fixed magic sum).

We start by recomputing rs from the semimagic count. From the Latte results we get the closed Ehrhart g.f. of each flat, which depends on whether we're doing cubic or affine. We also need
p = period (or truncated period in affine),
d = degree/dimension,
RtoSfactor = the rational function that multiplies Rgf to Sgf and rgf to sgf.
enddegree: The number of terms of the sequences to print out.

```
> enddegree:=20;
```
$$enddegree := 20$$

This is for cubic: set up main constants.
```
> d:=5; p:=60;
  RtoSfactor:=x^2/(1-x)^2;
```
$$d := 5$$
$$p := 60$$
$$RtoSfactor := \frac{x^2}{(1-x)^2}$$

This is also for cubic: set up simplex data.
```
> simplexname[1]:="OABC": ehrgf[1]:= 1/((1-x)^3*(1-x^2)) : dimen
  [1]:=3:
  simplexname[2]:="OEE2": ehrgf[2]:= 1/((1-x)*(1-x^2)*(1-x^3)) :
  dimen[2]:=2:
  simplexname[3]:="OAE2": ehrgf[3]:= 1/((1-x)*(1-x^2)^2) : dimen
  [3]:=2:
  simplexname[4]:="ADE2": ehrgf[4]:= 1/((1-x^2)^3) : dimen[4]:=2:
  simplexname[5]:="DE1E2": ehrgf[5]:= 1/((1-x^2)^2*(1-x^3)) :
  dimen[5]:=2:
  simplexname[6]:="OCE": ehrgf[6]:= 1/((1-x)^2*(1-x^3)) : dimen
  [6]:=2:
  simplexname[7]:="BDE1": ehrgf[7]:= 1/((1-x)*(1-x^2)*(1-x^3)) :
  dimen[7]:=2:
```

```
simplexname[8]:="ABD": ehrgf[8]:= 1/((1-x)*(1-x^2)^2) : dimen
[8]:=2:
simplexname[9]:="FG1": ehrgf[9]:= 1/((1-x^3)*(1-x^5)) : dimen
[9]:=1:
simplexname[10]:="EF": ehrgf[10]:= 1/((1-x^3)^2) : dimen[10]:=
1:
simplexname[11]:="OG": ehrgf[11]:= 1/((1-x)*(1-x^4)) : dimen
[11]:=1:
simplexname[12]:="FG": ehrgf[12]:= 1/((1-x^3)*(1-x^4)) : dimen
[12]:=1:
simplexname[13]:="AF": ehrgf[13]:= 1/((1-x^2)*(1-x^3)) : dimen
[13]:=1:
simplexname[14]:="DG": ehrgf[14]:= 1/((1-x^2)*(1-x^4)) : dimen
[14]:=1:
simplexname[15]:="DG2": ehrgf[15]:= 1/((1-x^2)*(1-x^5)) : dimen
[15]:=1:
simplexname[16]:="DE": ehrgf[16]:= 1/((1-x^2)*(1-x^3)) : dimen
[16]:=1:
simplexname[17]:="H": ehrgf[17] := 1/(1-x^5) : dimen[17]:=0:
for n from 1 to 17 do print(simplexname[n], dimen[n], ehrgf[n])
; od;
```

$$\text{"OABC"}, 3, \frac{1}{(1-x)^3 \, (1-x^2)}$$

$$\text{"OEE2"}, 2, \frac{1}{(1-x) \, (1-x^2) \, (1-x^3)}$$

$$\text{"OAE2"}, 2, \frac{1}{(1-x) \, (1-x^2)^2}$$

$$\text{"ADE2"}, 2, \frac{1}{(1-x^2)^3}$$

$$\text{"DE1E2"}, 2, \frac{1}{(1-x^2)^2 \, (1-x^3)}$$

$$\text{"OCE"}, 2, \frac{1}{(1-x)^2 \, (1-x^3)}$$

$$\text{"BDE1"}, 2, \frac{1}{(1-x) \, (1-x^2) \, (1-x^3)}$$

$$\text{"ABD"}, 2, \frac{1}{(1-x) \, (1-x^2)^2}$$

$$\text{"FG1"}, 1, \frac{1}{(1-x^3) \, (1-x^5)}$$

$$\text{"EF"}, 1, \frac{1}{\left(1-x^3\right)^2}$$

$$\text{"OG"}, 1, \frac{1}{\left(1-x\right)\left(1-x^4\right)}$$

$$\text{"FG"}, 1, \frac{1}{\left(1-x^3\right)\left(1-x^4\right)}$$

$$\text{"AF"}, 1, \frac{1}{\left(1-x^2\right)\left(1-x^3\right)}$$

$$\text{"DG"}, 1, \frac{1}{\left(1-x^2\right)\left(1-x^4\right)}$$

$$\text{"DG2"}, 1, \frac{1}{\left(1-x^2\right)\left(1-x^5\right)}$$

$$\text{"DE"}, 1, \frac{1}{\left(1-x^2\right)\left(1-x^3\right)}$$

$$\text{"H"}, 0, \frac{1}{1-x^5}$$

The closed E.g.f. is converted to the open E.g.f.:

```
> for n from 1 to 17 do
    mu[n]:=(-1)^(dimen[1]-dimen[n]):
  od:
  mu[14]:=2*mu[14]:
  for n from 1 to 17 do
    openehrgf[n]:=simplify(-(-1)^dimen[n]*subs(x=1/x,ehrgf[n])):
  od:
  for n from 1 to 17 do
    rgfterm[n]:=openehrgf[n]:
  od:
```

Now compute the number of reduced symmetry types:

```
> rgfsum:=sum(mu[nn]*rgfterm[nn],nn=1..17);
  rgf:=simplify(rgfsum):
```

$$rgfsum := \frac{x^5}{(x-1)^3\left(x^2-1\right)} + \frac{2\,x^6}{(x-1)\left(x^2-1\right)\left(x^3-1\right)} + \frac{2\,x^5}{(x-1)\left(x^2-1\right)^2}$$

$$+ \frac{x^6}{\left(x^2-1\right)^3} + \frac{x^7}{\left(x^2-1\right)^2\left(x^3-1\right)} + \frac{x^5}{(x-1)^2\left(x^3-1\right)} + \frac{x^8}{\left(x^3-1\right)\left(x^5-1\right)}$$

$$+ \frac{x^6}{\left(x^3-1\right)^2} + \frac{x^5}{(x-1)\left(x^4-1\right)} + \frac{x^7}{\left(x^3-1\right)\left(x^4-1\right)} + \frac{2\,x^5}{\left(x^2-1\right)\left(x^3-1\right)}$$

$$+ \frac{2\,x^6}{\left(x^2-1\right)\left(x^4-1\right)} + \frac{x^7}{\left(x^2-1\right)\left(x^5-1\right)} + \frac{x^5}{x^5-1}$$

We need the total number of reduced squares, Rgf:

```
> Rgfsum:=72*rgfsum;
  Rgf:=simplify(Rgfsum):
```

$$Rgfsum := \frac{72\,x^5}{(x-1)^3\,(x^2-1)} + \frac{144\,x^6}{(x-1)\,(x^2-1)\,(x^3-1)} + \frac{144\,x^5}{(x-1)\,(x^2-1)^2}$$

$$+ \frac{72\,x^6}{(x^2-1)^3} + \frac{72\,x^7}{(x^2-1)^2\,(x^3-1)} + \frac{72\,x^5}{(x-1)^2\,(x^3-1)} + \frac{72\,x^8}{(x^3-1)\,(x^5-1)}$$

$$+ \frac{72\,x^6}{(x^3-1)^2} + \frac{72\,x^5}{(x-1)\,(x^4-1)} + \frac{72\,x^7}{(x^3-1)\,(x^4-1)} + \frac{144\,x^5}{(x^2-1)\,(x^3-1)}$$

$$+ \frac{144\,x^6}{(x^2-1)\,(x^4-1)} + \frac{72\,x^7}{(x^2-1)\,(x^5-1)} + \frac{72\,x^5}{x^5-1}$$

Hence Sgf, the g.f. of the number of semimagic squares, equals

```
> Sgfsum:=RtoSfactor*Rgfsum;
  Sgf:=simplify(Sgfsum):
```

$$Sgfsum := \frac{1}{(1-x)^2}\left(x^2\left(\frac{72\,x^5}{(x-1)^3\,(x^2-1)} + \frac{144\,x^6}{(x-1)\,(x^2-1)\,(x^3-1)} \right.\right.$$

$$+ \frac{144\,x^5}{(x-1)\,(x^2-1)^2} + \frac{72\,x^6}{(x^2-1)^3} + \frac{72\,x^7}{(x^2-1)^2\,(x^3-1)} + \frac{72\,x^5}{(x-1)^2\,(x^3-1)}$$

$$+ \frac{72\,x^8}{(x^3-1)\,(x^5-1)} + \frac{72\,x^6}{(x^3-1)^2} + \frac{72\,x^5}{(x-1)\,(x^4-1)} + \frac{72\,x^7}{(x^3-1)\,(x^4-1)}$$

$$\left.\left. + \frac{144\,x^5}{(x^2-1)\,(x^3-1)} + \frac{144\,x^6}{(x^2-1)\,(x^4-1)} + \frac{72\,x^7}{(x^2-1)\,(x^5-1)} + \frac{72\,x^5}{x^5-1} \right) \right)$$

The g.f. of the total number of symmetry types, sgf:

```
> sgfsum:=RtoSfactor*rgfsum;
  sgf:=simplify(sgfsum):
```

$$sgfsum := \frac{1}{(1-x)^2}\left(x^2\left(\frac{x^5}{(x-1)^3\,(x^2-1)} + \frac{2\,x^6}{(x-1)\,(x^2-1)\,(x^3-1)} \right.\right.$$

$$+ \frac{2\,x^5}{(x-1)\,(x^2-1)^2} + \frac{x^6}{(x^2-1)^3} + \frac{x^7}{(x^2-1)^2\,(x^3-1)} + \frac{x^5}{(x-1)^2\,(x^3-1)}$$

$$+ \frac{x^8}{(x^3-1)\,(x^5-1)} + \frac{x^6}{(x^3-1)^2} + \frac{x^5}{(x-1)\,(x^4-1)} + \frac{x^7}{(x^3-1)\,(x^4-1)}$$

$$\left.\left. + \frac{2\,x^5}{(x^2-1)\,(x^3-1)} + \frac{2\,x^6}{(x^2-1)\,(x^4-1)} + \frac{x^7}{(x^2-1)\,(x^5-1)} + \frac{x^5}{x^5-1} \right) \right)$$

Generate the series expansions of the g.f.'s.

Expressing the rational function with standard denominator gives an orders-of-magnitude speedup in the series expansion.

Standard denominator $(1-x^p)^{\{d+1\}}$.

```
> pdenom:=(1-x^p):
  standenom:=pdenom^(d+1);
```
$$standenom := \left(1 - x^{60}\right)^6$$

G.f. as rational function with standard denominator.

```
> Sgfstandnum:=simplify(numer(Sgf)*simplify(standenom/denom(Sgf))
  ):
  Sgf:=Sgfstandnum/standenom;
```

$$Sgf := \frac{1}{\left(1 - x^{60}\right)^6}\left(72\,x^{10}\,(18\,x^9 + 46\,x^8 + 69\,x^7 + 74\,x^6 + 65\,x^5 + 46\,x^4 + 26\,x^3\right.$$

$$+\,11\,x^2 + 4\,x + 1)\,(x^{55} + x^{50} + x^{45} + x^{40} + x^{35} + x^{30} + x^{25} + x^{20} + x^{15} + x^{10} + x^5$$

$$+\,1)\,(1 + x + x^2 + x^3 + x^5 + x^4 + x^6 + x^7 + x^8 + x^{10} + x^{15} + x^{12} + x^9 + x^{13}$$

$$+\,x^{11} + x^{19} + x^{14} + x^{16} + x^{17} + x^{18} + x^{58} + x^{54} + x^{56} + x^{52} + x^{50} + x^{48} + x^{46} + x^{44}$$

$$+\,x^{42} + x^{40} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} + x^{28} + x^{26} + x^{24} + x^{22} + x^{20} + x^{31} + x^{43}$$

$$+\,x^{55} + x^{37} + x^{49} + x^{25} + x^{59} + x^{57} + x^{53} + x^{51} + x^{47} + x^{45} + x^{41} + x^{39} + x^{35} + x^{33}$$

$$+\,x^{29} + x^{27} + x^{23} + x^{21})^2\,(1 + x^2 + x^4 + x^6 + x^8 + x^{10} + x^{12} + x^{14} + x^{16} + x^{18}$$

$$+\,x^{58} + x^{54} + x^{56} + x^{52} + x^{50} + x^{48} + x^{46} + x^{44} + x^{42} + x^{40} + x^{38} + x^{36} + x^{34} + x^{32}$$

$$+\,x^{30} + x^{28} + x^{26} + x^{24} + x^{22} + x^{20})\,(1 - x + x^3 - x^5 + x^6 + x^{15} + x^{12} - x^{13} - x^{17}$$

$$+\,x^{18} + x^{54} + x^{48} + x^{42} + x^{36} + x^{30} + x^{24} - x^{37} - x^{49} - x^{25} - x^{53} + x^{51} - x^{41} + x^{39}$$

$$-\,x^{29} + x^{27})\,(1 - x + x^2 + x^6 - x^7 + x^8 + x^{12} - x^{13} - x^{19} + x^{14} + x^{18} + x^{54} + x^{56}$$

$$+\,x^{50} + x^{48} + x^{44} + x^{42} + x^{38} + x^{36} + x^{32} + x^{30} + x^{26} + x^{24} + x^{20} - x^{31} - x^{43} - x^{55}$$

$$\left.-\,x^{37} - x^{49} - x^{25})\right)$$

G.f. as rational function with standard denominator.

```
> Rgfstandnum:=simplify(numer(Rgf)*standenom/denom(Rgf)):
  Rgf:=Rgfstandnum/standenom;
```

$$Rgf := \frac{1}{\left(1 - x^{60}\right)^6}\left(72\,(18\,x^9 + 46\,x^8 + 69\,x^7 + 74\,x^6 + 65\,x^5 + 46\,x^4 + 26\,x^3 + 11\,x^2\right.$$

$$+\,4\,x + 1)\,x^8\,(x^{57} + x^{54} + x^{51} + x^{48} + x^{45} + x^{42} + x^{39} + x^{36} + x^{33} + x^{30} + x^{27}$$

$$+\,x^{24} + x^{21} + x^{18} + x^{15} + x^{12} + x^9 + x^6 + x^3 + 1)^2\,(-1 + x - x^2 + x^3 + x^5 - x^4$$

$$-\,x^6 + x^7 - x^8 - x^{10} + x^{15} - x^{12} + x^9 + x^{13} + x^{11} + x^{19} - x^{14} - x^{16} + x^{17} - x^{18}$$

$$-\,x^{58} - x^{54} - x^{56} - x^{52} - x^{50} - x^{48} - x^{46} - x^{44} - x^{42} - x^{40} - x^{38} - x^{36} - x^{34} - x^{32}$$

$$-\,x^{30} - x^{28} - x^{26} - x^{24} - x^{22} - x^{20} + x^{31} + x^{43} + x^{55} + x^{37} + x^{49} + x^{25} + x^{59} + x^{57}$$

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$$+ x^{53} + x^{51} + x^{47} + x^{45} + x^{41} + x^{39} + x^{35} + x^{33} + x^{29} + x^{27} + x^{23} + x^{21}) (-1$$

$$+ x^{60}) (1 + x^2 + x^4 + x^6 + x^8 + x^{10} + x^{12} + x^{14} + x^{16} + x^{18} + x^{58} + x^{54} + x^{56} + x^{52}$$

$$+ x^{50} + x^{48} + x^{46} + x^{44} + x^{42} + x^{40} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} + x^{28} + x^{26} + x^{24}$$

$$+ x^{22} + x^{20}) (x^{52} - x^{51} + x^{48} - x^{46} + x^{44} - x^{41} + x^{40} + x^{32} - x^{31} + x^{28} - x^{26} + x^{24}$$

$$- x^{21} + x^{20} + x^{12} - x^{11} + x^8 - x^6 + x^4 - x + 1))$$

G.f. as rational function with standard denominator.

```
> sgfstandnum:=simplify(numer(sgf)*simplify(standenom/denom(sgf))
  ):
  sgf:=sgfstandnum/standenom;
```

$$sgf := \frac{1}{(1 - x^{60})^6} \left(x^{10} (18 x^9 + 46 x^8 + 69 x^7 + 74 x^6 + 65 x^5 + 46 x^4 + 26 x^3 + 11 x^2 \right.$$

$$+ 4 x + 1) (x^{55} + x^{50} + x^{45} + x^{40} + x^{35} + x^{30} + x^{25} + x^{20} + x^{15} + x^{10} + x^5$$

$$+ 1) (1 + x + x^2 + x^3 + x^5 + x^4 + x^6 + x^7 + x^8 + x^{10} + x^{15} + x^{12} + x^9 + x^{13} + x^{11}$$

$$+ x^{19} + x^{14} + x^{16} + x^{17} + x^{18} + x^{58} + x^{54} + x^{56} + x^{52} + x^{50} + x^{48} + x^{46} + x^{44} + x^{42}$$

$$+ x^{40} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} + x^{28} + x^{26} + x^{24} + x^{22} + x^{20} + x^{31} + x^{43} + x^{55}$$

$$+ x^{37} + x^{49} + x^{25} + x^{59} + x^{57} + x^{53} + x^{51} + x^{47} + x^{45} + x^{41} + x^{39} + x^{35} + x^{33} + x^{29}$$

$$+ x^{27} + x^{23} + x^{21})^2 (1 + x^2 + x^4 + x^6 + x^8 + x^{10} + x^{12} + x^{14} + x^{16} + x^{18} + x^{58}$$

$$+ x^{54} + x^{56} + x^{52} + x^{50} + x^{48} + x^{46} + x^{44} + x^{42} + x^{40} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30}$$

$$+ x^{28} + x^{26} + x^{24} + x^{22} + x^{20}) (1 - x + x^3 - x^5 + x^6 + x^{15} + x^{12} - x^{13} - x^{17} + x^{18}$$

$$+ x^{54} + x^{48} + x^{42} + x^{36} + x^{30} + x^{24} - x^{37} - x^{49} - x^{25} - x^{53} + x^{51} - x^{41} + x^{39} - x^{29}$$

$$+ x^{27}) (1 - x + x^2 + x^6 - x^7 + x^8 + x^{12} - x^{13} - x^{19} + x^{14} + x^{18} + x^{54} + x^{56} + x^{50}$$

$$+ x^{48} + x^{44} + x^{42} + x^{38} + x^{36} + x^{32} + x^{30} + x^{26} + x^{24} + x^{20} - x^{31} - x^{43} - x^{55} - x^{37}$$

$$- x^{49} - x^{25}))$$

G.f. as rational function with standard denominator.

```
> rgfstandnum:=simplify(numer(rgf)*standenom/denom(rgf)):
  rgf:=rgfstandnum/standenom;
```

$$rgf := \frac{1}{(1 - x^{60})^6} \left((18 x^9 + 46 x^8 + 69 x^7 + 74 x^6 + 65 x^5 + 46 x^4 + 26 x^3 + 11 x^2 \right.$$

$$+ 4 x + 1) x^8 (x^{57} + x^{54} + x^{51} + x^{48} + x^{45} + x^{42} + x^{39} + x^{36} + x^{33} + x^{30} + x^{27}$$

$$+ x^{24} + x^{21} + x^{18} + x^{15} + x^{12} + x^9 + x^6 + x^3 + 1)^2 (-1 + x - x^2 + x^3 + x^5 - x^4$$

$$- x^6 + x^7 - x^8 - x^{10} + x^{15} - x^{12} + x^9 + x^{13} + x^{11} + x^{19} - x^{14} - x^{16} + x^{17} - x^{18}$$

$$- x^{58} - x^{54} - x^{56} - x^{52} - x^{50} - x^{48} - x^{46} - x^{44} - x^{42} - x^{40} - x^{38} - x^{36} - x^{34} - x^{32}$$

$$-x^{30} - x^{28} - x^{26} - x^{24} - x^{22} - x^{20} + x^{31} + x^{43} + x^{55} + x^{37} + x^{49} + x^{25} + x^{59} + x^{57}$$
$$+ x^{53} + x^{51} + x^{47} + x^{45} + x^{41} + x^{39} + x^{35} + x^{33} + x^{29} + x^{27} + x^{23} + x^{21}) \, (-1$$
$$+ x^{60}) \, (1 + x^2 + x^4 + x^6 + x^8 + x^{10} + x^{12} + x^{14} + x^{16} + x^{18} + x^{58} + x^{54} + x^{56} + x^{52}$$
$$+ x^{50} + x^{48} + x^{46} + x^{44} + x^{42} + x^{40} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} + x^{28} + x^{26} + x^{24}$$
$$+ x^{22} + x^{20}) \, (x^{52} - x^{51} + x^{48} - x^{46} + x^{44} - x^{41} + x^{40} + x^{32} - x^{31} + x^{28} - x^{26} + x^{24}$$
$$- x^{21} + x^{20} + x^{12} - x^{11} + x^8 - x^6 + x^4 - x + 1))$$

Expand the series to find the first few values of the number of squares.
```
> Sseries:=series(Sgf,x=0,enddegree+1);
```
$$Sseries := 72\,x^{10} + 288\,x^{11} + 936\,x^{12} + 2592\,x^{13} + 5760\,x^{14} + 11520\,x^{15}$$
$$+ 20952\,x^{16} + 35712\,x^{17} + 57168\,x^{18} + 88272\,x^{19} + 131112\,x^{20} + O(x^{21})$$

Expand the series to find the first few values of the number of reduced squares.
```
> Rseries:=series(Rgf,x=0,enddegree+1);
```
$$Rseries := 72\,x^8 + 144\,x^9 + 432\,x^{10} + 1008\,x^{11} + 1512\,x^{12} + 2592\,x^{13} + 3672\,x^{14}$$
$$+ 5328\,x^{15} + 6696\,x^{16} + 9648\,x^{17} + 11736\,x^{18} + 15552\,x^{19} + 17856\,x^{20}$$
$$+ O(x^{21})$$

Expand the series to find the first few values of the number of symmetry types.
```
> sseries:=series(sgf,x=0,enddegree+1);
```
$$sseries := x^{10} + 4\,x^{11} + 13\,x^{12} + 36\,x^{13} + 80\,x^{14} + 160\,x^{15} + 291\,x^{16} + 496\,x^{17}$$
$$+ 794\,x^{18} + 1226\,x^{19} + 1821\,x^{20} + O(x^{21})$$

Expand the series to find the first few values of the number of reduced symmetry types.
```
> rseries:=series(rgf,x=0,enddegree+1);
```
$$rseries := x^8 + 2\,x^9 + 6\,x^{10} + 14\,x^{11} + 21\,x^{12} + 36\,x^{13} + 51\,x^{14} + 74\,x^{15} + 93\,x^{16}$$
$$+ 134\,x^{17} + 163\,x^{18} + 216\,x^{19} + 248\,x^{20} + O(x^{21})$$

Find the constituents

Casculate the zeroth constituent of the **total semimagic counting function**. Find its constant term.
```
> Szeroth:=expand(
  sum(coeff(Sgfstandnum,x,p*jj)*binomial(d+t/p-jj,d),jj=0..d+1) )
  ;
  print(subs(t=0,Szeroth)):
```
$$Szeroth := -1296 + 1464\,t - \frac{5989}{10}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5$$
$$-1296$$

Extract the constituents of the total semimagic counting function.

```
> Sconstituent[0]:=Szeroth:
  for r from 1 to p do
     Sconstituent[r]:=expand(sum( coeff(Sgfstandnum,x,p*jj+r)*
  binomial(d+(t-r)/p-jj,d), jj=0..d)):
  #  print(r):
  #  print( Sconstituent[r] ):
     print( factor(Sconstituent[r]) ):
  od;
```

$$Sconstituent_1 := \frac{2831}{2}\, t - \frac{11933}{20}\, t^2 - \frac{110413}{120} + \frac{331}{3}\, t^3 - \frac{75}{8}\, t^4 + \frac{3}{10}\, t^5$$

$$\frac{1}{120}\, (t-1)\, (36\, t^4 - 1089\, t^3 + 12151\, t^2 - 59447\, t + 110413)$$

$$Sconstituent_2 := 1464\, t - \frac{5989}{10}\, t^2 + \frac{331}{3}\, t^3 - \frac{75}{8}\, t^4 + \frac{3}{10}\, t^5 - \frac{3824}{3}$$

$$\frac{1}{120}\, (t-2)\, (36\, t^4 - 1053\, t^3 + 11134\, t^2 - 49600\, t + 76480)$$

$$Sconstituent_3 := \frac{2847}{2}\, t - \frac{11933}{20}\, t^2 + \frac{331}{3}\, t^3 - \frac{75}{8}\, t^4 + \frac{3}{10}\, t^5 - \frac{47727}{40}$$

$$\frac{1}{120}\, (t-3)\, (36\, t^4 - 1017\, t^3 + 10189\, t^2 - 41031\, t + 47727)$$

$$Sconstituent_4 := 1456\, t - \frac{5989}{10}\, t^2 + \frac{331}{3}\, t^3 - \frac{75}{8}\, t^4 + \frac{3}{10}\, t^5 - \frac{18152}{15}$$

$$\frac{1}{120}\, (t-4)\, (36\, t^4 - 981\, t^3 + 9316\, t^2 - 34604\, t + 36304)$$

$$Sconstituent_5 := \frac{2847}{2}\, t - \frac{11933}{20}\, t^2 + \frac{331}{3}\, t^3 - \frac{75}{8}\, t^4 + \frac{3}{10}\, t^5 - \frac{25705}{24}$$

$$\frac{1}{120}\, (t-5)\, (36\, t^4 - 945\, t^3 + 8515\, t^2 - 29023\, t + 25705)$$

$$Sconstituent_6 := 1464\, t - \frac{6192}{5} - \frac{5989}{10}\, t^2 + \frac{331}{3}\, t^3 - \frac{75}{8}\, t^4 + \frac{3}{10}\, t^5$$

$$\frac{1}{120}\, (t-6)\, (36\, t^4 - 909\, t^3 + 7786\, t^2 - 25152\, t + 24768)$$

$$Sconstituent_7 := \frac{2831}{2}\, t - \frac{11933}{20}\, t^2 + \frac{331}{3}\, t^3 - \frac{75}{8}\, t^4 + \frac{3}{10}\, t^5 - \frac{25193}{24}$$

$$\frac{1}{120}\, (t-7)\, (36\, t^4 - 873\, t^3 + 7129\, t^2 - 21695\, t + 17995)$$

$$Sconstituent_8 := 1464\, t - \frac{5989}{10}\, t^2 + \frac{331}{3}\, t^3 - \frac{75}{8}\, t^4 + \frac{3}{10}\, t^5 - \frac{19552}{15}$$

$$\frac{1}{120}\,(t-8)\,(36\,t^4-837\,t^3+6544\,t^2-19516\,t+19552)$$

$$Sconstituent_9 := \frac{2847}{2}\,t - \frac{11933}{20}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 - \frac{44847}{40} + \frac{3}{10}\,t^5$$

$$\frac{1}{120}\,(t-9)\,(36\,t^4-801\,t^3+6031\,t^2-17319\,t+14949)$$

$$Sconstituent_{10} := 1456\,t - \frac{5989}{10}\,t^2 - \frac{3544}{3} + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5$$

$$1456\,t - \frac{5989}{10}\,t^2 - \frac{3544}{3} + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5$$

$$Sconstituent_{11} := \frac{2847}{2}\,t - \frac{11933}{20}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5 - \frac{130253}{120}$$

$$\frac{2847}{2}\,t - \frac{11933}{20}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5 - \frac{130253}{120}$$

$$Sconstituent_{12} := -1296 + 1464\,t - \frac{5989}{10}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5$$

$$-1296 + 1464\,t - \frac{5989}{10}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5$$

$$Sconstituent_{13} := \frac{2831}{2}\,t - \frac{11933}{20}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5 - \frac{120781}{120}$$

$$\frac{2831}{2}\,t - \frac{11933}{20}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5 - \frac{120781}{120}$$

$$Sconstituent_{14} := 1464\,t - \frac{5989}{10}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5 - \frac{19552}{15}$$

$$\frac{1}{120}\,(t-8)\,(36\,t^4-837\,t^3+6544\,t^2-19516\,t+19552)$$

$$Sconstituent_{15} := \frac{2847}{2}\,t - \frac{11933}{20}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5 - \frac{9315}{8}$$

$$\frac{2847}{2}\,t - \frac{11933}{20}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5 - \frac{9315}{8}$$

$$Sconstituent_{16} := 1456\,t - \frac{5989}{10}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5 - \frac{16856}{15}$$

$$1456\,t - \frac{5989}{10}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5 - \frac{16856}{15}$$

$$Sconstituent_{17} := \frac{2847}{2}\,t - \frac{11933}{20}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5 - \frac{25705}{24}$$

$$\frac{1}{120}\,(t-5)\,(36\,t^4-945\,t^3+8515\,t^2-29023\,t+25705)$$

$$Sconstituent_{18}:=1464\,t-\frac{5989}{10}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5-\frac{6624}{5}$$

$$1464\,t-\frac{5989}{10}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5-\frac{6624}{5}$$

$$Sconstituent_{19}:=-\frac{129421}{120}+\frac{2831}{2}\,t-\frac{11933}{20}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5$$

$$-\frac{129421}{120}+\frac{2831}{2}\,t-\frac{11933}{20}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5$$

$$Sconstituent_{20}:=1464\,t-\frac{5989}{10}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5-\frac{3824}{3}$$

$$\frac{1}{120}\,(t-2)\,(36\,t^4-1053\,t^3+11134\,t^2-49600\,t+76480)$$

$$Sconstituent_{21}:=\frac{2847}{2}\,t-\frac{11933}{20}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5-\frac{41391}{40}$$

$$\frac{2847}{2}\,t-\frac{11933}{20}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5-\frac{41391}{40}$$

$$Sconstituent_{22}:=1456\,t-\frac{5989}{10}\,t^2-\frac{3544}{3}+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5$$

$$1456\,t-\frac{5989}{10}\,t^2-\frac{3544}{3}+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5$$

$$Sconstituent_{23}:=\frac{2847}{2}\,t-\frac{140621}{120}-\frac{11933}{20}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5$$

$$\frac{2847}{2}\,t-\frac{140621}{120}-\frac{11933}{20}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5$$

$$Sconstituent_{24}:=1464\,t-\frac{5989}{10}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5-\frac{6624}{5}$$

$$1464\,t-\frac{5989}{10}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5-\frac{6624}{5}$$

$$Sconstituent_{25}:=\frac{2831}{2}\,t-\frac{11933}{20}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5-\frac{23465}{24}$$

$$\frac{2831}{2}\,t-\frac{11933}{20}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5-\frac{23465}{24}$$

$$Sconstituent_{26}:=-\frac{18256}{15}+1464\,t-\frac{5989}{10}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5$$

$$-\frac{18256}{15} + 1464\,t - \frac{5989}{10}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5$$

$$Sconstituent_{27} := \frac{2847}{2}\,t - \frac{11933}{20}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5 - \frac{9315}{8}$$

$$\frac{2847}{2}\,t - \frac{11933}{20}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5 - \frac{9315}{8}$$

$$Sconstituent_{28} := 1456\,t - \frac{5989}{10}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5 - \frac{18152}{15}$$

$$\frac{1}{120}\,(t-4)\,(36\,t^4 - 981\,t^3 + 9316\,t^2 - 34604\,t + 36304)$$

$$Sconstituent_{29} := \frac{2847}{2}\,t - \frac{131981}{120} - \frac{11933}{20}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5$$

$$\frac{2847}{2}\,t - \frac{131981}{120} - \frac{11933}{20}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5$$

$$Sconstituent_{30} := -1296 + 1464\,t - \frac{5989}{10}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5$$

$$-1296 + 1464\,t - \frac{5989}{10}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5$$

$$Sconstituent_{31} := \frac{2831}{2}\,t - \frac{119053}{120} - \frac{11933}{20}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5$$

$$\frac{2831}{2}\,t - \frac{119053}{120} - \frac{11933}{20}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5$$

$$Sconstituent_{32} := 1464\,t - \frac{5989}{10}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5 - \frac{3824}{3}$$

$$\frac{1}{120}\,(t-2)\,(36\,t^4 - 1053\,t^3 + 11134\,t^2 - 49600\,t + 76480)$$

$$Sconstituent_{33} := \frac{2847}{2}\,t - \frac{11933}{20}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 - \frac{44847}{40} + \frac{3}{10}\,t^5$$

$$\frac{1}{120}\,(t-9)\,(36\,t^4 - 801\,t^3 + 6031\,t^2 - 17319\,t + 14949)$$

$$Sconstituent_{34} := 1456\,t - \frac{5989}{10}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5 - \frac{18152}{15}$$

$$\frac{1}{120}\,(t-4)\,(36\,t^4 - 981\,t^3 + 9316\,t^2 - 34604\,t + 36304)$$

$$Sconstituent_{35} := \frac{2847}{2}\,t - \frac{11933}{20}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5 - \frac{27433}{24}$$

$$\frac{2847}{2}\,t - \frac{11933}{20}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5 - \frac{27433}{24}$$

$$Sconstituent_{36} := 1464\,t - \frac{6192}{5} - \frac{5989}{10}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5$$

$$\frac{1}{120}\,(t-6)\,(36\,t^4 - 909\,t^3 + 7786\,t^2 - 25152\,t + 24768)$$

$$Sconstituent_{37} := \frac{2831}{2}\,t - \frac{11933}{20}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5 - \frac{23465}{24}$$

$$\frac{2831}{2}\,t - \frac{11933}{20}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5 - \frac{23465}{24}$$

$$Sconstituent_{38} := 1464\,t - \frac{5989}{10}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5 - \frac{19552}{15}$$

$$\frac{1}{120}\,(t-8)\,(36\,t^4 - 837\,t^3 + 6544\,t^2 - 19516\,t + 19552)$$

$$Sconstituent_{39} := \frac{2847}{2}\,t - \frac{11933}{20}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5 - \frac{47727}{40}$$

$$\frac{1}{120}\,(t-3)\,(36\,t^4 - 1017\,t^3 + 10189\,t^2 - 41031\,t + 47727)$$

$$Sconstituent_{40} := 1456\,t - \frac{5989}{10}\,t^2 - \frac{3544}{3} + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5$$

$$1456\,t - \frac{5989}{10}\,t^2 - \frac{3544}{3} + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5$$

$$Sconstituent_{41} := \frac{2847}{2}\,t - \frac{11933}{20}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5 - \frac{121613}{120}$$

$$\frac{2847}{2}\,t - \frac{11933}{20}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5 - \frac{121613}{120}$$

$$Sconstituent_{42} := -1296 + 1464\,t - \frac{5989}{10}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5$$

$$-1296 + 1464\,t - \frac{5989}{10}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5$$

$$Sconstituent_{43} := -\frac{129421}{120} + \frac{2831}{2}\,t - \frac{11933}{20}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5$$

$$-\frac{129421}{120} + \frac{2831}{2}\,t - \frac{11933}{20}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5$$

$$Sconstituent_{44} := 1464\,t - \frac{5989}{10}\,t^2 + \frac{331}{3}\,t^3 - \frac{75}{8}\,t^4 + \frac{3}{10}\,t^5 - \frac{19552}{15}$$

$$\frac{1}{120}\,(t-8)\,(36\,t^4-837\,t^3+6544\,t^2-19516\,t+19552)$$

$$Sconstituent_{45}:=\frac{2847}{2}\,t-\frac{11933}{20}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5-\frac{8739}{8}$$

$$\frac{2847}{2}\,t-\frac{11933}{20}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5-\frac{8739}{8}$$

$$Sconstituent_{46}:=1456\,t-\frac{5989}{10}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5-\frac{16856}{15}$$

$$1456\,t-\frac{5989}{10}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5-\frac{16856}{15}$$

$$Sconstituent_{47}:=\frac{2847}{2}\,t-\frac{11933}{20}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5-\frac{27433}{24}$$

$$\frac{2847}{2}\,t-\frac{11933}{20}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5-\frac{27433}{24}$$

$$Sconstituent_{48}:=1464\,t-\frac{5989}{10}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5-\frac{6624}{5}$$

$$1464\,t-\frac{5989}{10}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5-\frac{6624}{5}$$

$$Sconstituent_{49}:=\frac{2831}{2}\,t-\frac{11933}{20}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5-\frac{120781}{120}$$

$$\frac{2831}{2}\,t-\frac{11933}{20}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5-\frac{120781}{120}$$

$$Sconstituent_{50}:=1464\,t-\frac{5989}{10}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5-\frac{3824}{3}$$

$$\frac{1}{120}\,(t-2)\,(36\,t^4-1053\,t^3+11134\,t^2-49600\,t+76480)$$

$$Sconstituent_{51}:=\frac{2847}{2}\,t-\frac{44271}{40}-\frac{11933}{20}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5$$

$$\frac{2847}{2}\,t-\frac{44271}{40}-\frac{11933}{20}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5$$

$$Sconstituent_{52}:=1456\,t-\frac{5989}{10}\,t^2-\frac{3544}{3}+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5$$

$$1456\,t-\frac{5989}{10}\,t^2-\frac{3544}{3}+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5$$

$$Sconstituent_{53}:=\frac{2847}{2}\,t-\frac{131981}{120}-\frac{11933}{20}\,t^2+\frac{331}{3}\,t^3-\frac{75}{8}\,t^4+\frac{3}{10}\,t^5$$

$$\frac{2847}{2} t - \frac{131981}{120} - \frac{11933}{20} t^2 + \frac{331}{3} t^3 - \frac{75}{8} t^4 + \frac{3}{10} t^5$$

$$Sconstituent_{54} := 1464\, t - \frac{5989}{10} t^2 + \frac{331}{3} t^3 - \frac{75}{8} t^4 + \frac{3}{10} t^5 - \frac{6624}{5}$$

$$1464\, t - \frac{5989}{10} t^2 + \frac{331}{3} t^3 - \frac{75}{8} t^4 + \frac{3}{10} t^5 - \frac{6624}{5}$$

$$Sconstituent_{55} := \frac{2831}{2} t - \frac{11933}{20} t^2 + \frac{331}{3} t^3 - \frac{75}{8} t^4 + \frac{3}{10} t^5 - \frac{25193}{24}$$

$$\frac{1}{120} (t - 7) (36\, t^4 - 873\, t^3 + 7129\, t^2 - 21695\, t + 17995)$$

$$Sconstituent_{56} := -\frac{18256}{15} + 1464\, t - \frac{5989}{10} t^2 + \frac{331}{3} t^3 - \frac{75}{8} t^4 + \frac{3}{10} t^5$$

$$-\frac{18256}{15} + 1464\, t - \frac{5989}{10} t^2 + \frac{331}{3} t^3 - \frac{75}{8} t^4 + \frac{3}{10} t^5$$

$$Sconstituent_{57} := \frac{2847}{2} t - \frac{11933}{20} t^2 + \frac{331}{3} t^3 - \frac{75}{8} t^4 + \frac{3}{10} t^5 - \frac{8739}{8}$$

$$\frac{2847}{2} t - \frac{11933}{20} t^2 + \frac{331}{3} t^3 - \frac{75}{8} t^4 + \frac{3}{10} t^5 - \frac{8739}{8}$$

$$Sconstituent_{58} := 1456\, t - \frac{5989}{10} t^2 + \frac{331}{3} t^3 - \frac{75}{8} t^4 + \frac{3}{10} t^5 - \frac{18152}{15}$$

$$\frac{1}{120} (t - 4) (36\, t^4 - 981\, t^3 + 9316\, t^2 - 34604\, t + 36304)$$

$$Sconstituent_{59} := \frac{2847}{2} t - \frac{140621}{120} - \frac{11933}{20} t^2 + \frac{331}{3} t^3 - \frac{75}{8} t^4 + \frac{3}{10} t^5$$

$$\frac{2847}{2} t - \frac{140621}{120} - \frac{11933}{20} t^2 + \frac{331}{3} t^3 - \frac{75}{8} t^4 + \frac{3}{10} t^5$$

$$Sconstituent_{60} := -1296 + 1464\, t - \frac{5989}{10} t^2 + \frac{331}{3} t^3 - \frac{75}{8} t^4 + \frac{3}{10} t^5$$

$$-1296 + 1464\, t - \frac{5989}{10} t^2 + \frac{331}{3} t^3 - \frac{75}{8} t^4 + \frac{3}{10} t^5$$

Extract the coefficients of the constituents.

```
> for r from 1 to p do
    for coeffdeg from 0 to d do
     Sc[coeffdeg,r]:=coeff(Sconstituent[r],t,coeffdeg):
     #print( r, Sc[coeffdeg,r] ):
    od:
   od:
```

Print and analyze the constituent coefficients for periods. First the higher

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coefficients, which ought to be constant. Print the first coefficient, then any that don't repeat the preceding value.

```
> for coeffdeg from 3 to d do
    print("degree", coeffdeg, "coeff", Sc[coeffdeg,1]):
    print(1,Sc[coeffdeg,1]);
    for r from 2 to p do
      stepdifference:=Sc[coeffdeg,r]-Sc[coeffdeg,r-1]:
      if( stepdifference<>0 ) then
      print(r,Sc[coeffdeg,r],stepdifference):
      fi:
    od:
    print("Compared all coefficients of degree", coeffdeg);
  od:
```

$$\text{"degree", 3, "coeff", } \frac{331}{3}$$

$$1, \frac{331}{3}$$

$$\text{"Compared all coefficients of degree", 3}$$

$$\text{"degree", 4, "coeff", } -\frac{75}{8}$$

$$1, -\frac{75}{8}$$

$$\text{"Compared all coefficients of degree", 4}$$

$$\text{"degree", 5, "coeff", } \frac{3}{10}$$

$$1, \frac{3}{10}$$

$$\text{"Compared all coefficients of degree", 5}$$

Next, the constant terms, whose period is expected to be p. Print all constant terms up to the presumed period "stepsize". Print the difference (at step "stepsize") if they are not repeating.
Note that the even terms repeat at step 30 (a period of 15, half the expected period).

```
> stepsize:=30;
  for r from 1 to stepsize do
    print(r, Sc[0,r]);
  od:
  for r from stepsize+1 to p do
    stepdifference:=Sc[0,r]-Sc[0,r-stepsize]:
    if( stepdifference<>0 ) then print(r,Sc[0,r],stepdifference):
    fi:
    #print(r,Sc[0,r],stepdifference);
  od:
  print("Constant terms completed.");
```

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$$stepsize := 30$$

$$1, \ -\frac{110413}{120}$$

$$2, \ -\frac{3824}{3}$$

$$3, \ -\frac{47727}{40}$$

$$4, \ -\frac{18152}{15}$$

$$5, \ -\frac{25705}{24}$$

$$6, \ -\frac{6192}{5}$$

$$7, \ -\frac{25193}{24}$$

$$8, \ -\frac{19552}{15}$$

$$9, \ -\frac{44847}{40}$$

$$10, \ -\frac{3544}{3}$$

$$11, \ -\frac{130253}{120}$$

$$12, \ -1296$$

$$13, \ -\frac{120781}{120}$$

$$14, \ -\frac{19552}{15}$$

$$15, \ -\frac{9315}{8}$$

$$16, \ -\frac{16856}{15}$$

$$17, \ -\frac{25705}{24}$$

$$18, \ -\frac{6624}{5}$$

$$19, -\frac{129421}{120}$$

$$20, -\frac{3824}{3}$$

$$21, -\frac{41391}{40}$$

$$22, -\frac{3544}{3}$$

$$23, -\frac{140621}{120}$$

$$24, -\frac{6624}{5}$$

$$25, -\frac{23465}{24}$$

$$26, -\frac{18256}{15}$$

$$27, -\frac{9315}{8}$$

$$28, -\frac{18152}{15}$$

$$29, -\frac{131981}{120}$$

$$30, -1296$$

$$31, -\frac{119053}{120}, -72$$

$$33, -\frac{44847}{40}, 72$$

$$35, -\frac{27433}{24}, -72$$

$$37, -\frac{23465}{24}, 72$$

$$39, -\frac{47727}{40}, -72$$

$$41, -\frac{121613}{120}, 72$$

$$43, -\frac{129421}{120}, -72$$

$$45, -\frac{8739}{8}, 72$$

$$47, -\frac{27433}{24}, -72$$

$$49, -\frac{120781}{120}, 72$$

$$51, -\frac{44271}{40}, -72$$

$$53, -\frac{131981}{120}, 72$$

$$55, -\frac{25193}{24}, -72$$

$$57, -\frac{8739}{8}, 72$$

$$59, -\frac{140621}{120}, -72$$

$$\text{"Constant terms completed."}$$

Now, the linear terms. First print all linear coefficients up to the presumed period "stepsize".. Then analyze for period and print the difference (at step "stepsize") if they are not repeating.

```
> stepsize:=6;
  for r from 1 to stepsize do
    print(r, Sc[1,r]);
  od:
  for r from stepsize+1 to p do
    stepdifference:=Sc[1,r]-Sc[1,r-stepsize]:
    if( stepdifference<>0 ) then print(r,Sc[1,r],stepdifference):
  fi:
  od:
  print("Linear coefficients completed.");
```

$$stepsize := 6$$

$$1, \frac{2831}{2}$$

$$2, 1464$$

$$3, \frac{2847}{2}$$

$$4, 1456$$

$$5, \frac{2847}{2}$$

18

6, 1464

The quadratic terms. First print all quadratic coefficients up to the presumed period "stepsize".. Then analyze for period and print the difference (at step "stepsize") if they are not repeating.

```
> stepsize:=2;
  for r from 1 to stepsize do
    print(r, Sc[2,r]);
  od:
  for r from stepsize+1 to p do
    stepdifference:=Sc[2,r]-Sc[2,r-stepsize]:
    if( stepdifference<>0 ) then print(r,Sc[2,r],stepdifference):
  fi:
  od:
  print("Quadratic coefficients completed.");
```

$$stepsize := 2$$

$$1, -\frac{11933}{20}$$

$$2, -\frac{5989}{10}$$

"Quadratic coefficients completed."

Casculate the zeroth constituent of the **semimagic symmetry-type counting function**. Find its constant term.

```
> szeroth:=expand(
  sum(coeff(sgfstandnum,x,p*jj)*binomial(d+t/p-jj,d),jj=0..d+1) )
  ;
  print(subs(t=0,szeroth)):
```

$$szeroth := -18 + \frac{61}{3}\,t - \frac{5989}{720}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5$$

$$-18$$

Extract the constituents of the semimagic symmetry-type counting function.

```
> sconstituent[0]:=szeroth:
  for r from 1 to p do
    sconstituent[r]:=expand(sum( coeff(sgfstandnum,x,p*jj+r)*
  binomial(d+(t-r)/p-jj,d), jj=0..d)):
  #  print(r):
  #  print( sconstituent[r] ):
    print( factor(sconstituent[r]) ):
  od;
```

$$sconstituent_1 := \frac{2831}{144}\,t - \frac{11933}{1440}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5 - \frac{110413}{8640}$$

$$\frac{1}{8640}\,(t-1)\,(36\,t^4 - 1089\,t^3 + 12151\,t^2 - 59447\,t + 110413)$$

$$sconstituent_2 := \frac{61}{3}\,t - \frac{5989}{720}\,t^2 - \frac{478}{27} + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5$$

$$\frac{1}{8640}\,(t-2)\,(36\,t^4 - 1053\,t^3 + 11134\,t^2 - 49600\,t + 76480)$$

$$sconstituent_3 := \frac{949}{48}\,t - \frac{11933}{1440}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5 - \frac{5303}{320}$$

$$\frac{1}{8640}\,(t-3)\,(36\,t^4 - 1017\,t^3 + 10189\,t^2 - 41031\,t + 47727)$$

$$sconstituent_4 := \frac{182}{9}\,t - \frac{2269}{135} - \frac{5989}{720}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5$$

$$\frac{1}{8640}\,(t-4)\,(36\,t^4 - 981\,t^3 + 9316\,t^2 - 34604\,t + 36304)$$

$$sconstituent_5 := \frac{949}{48}\,t - \frac{11933}{1440}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5 - \frac{25705}{1728}$$

$$\frac{1}{8640}\,(t-5)\,(36\,t^4 - 945\,t^3 + 8515\,t^2 - 29023\,t + 25705)$$

$$sconstituent_6 := \frac{61}{3}\,t - \frac{5989}{720}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5 - \frac{86}{5}$$

$$\frac{1}{8640}\,(t-6)\,(36\,t^4 - 909\,t^3 + 7786\,t^2 - 25152\,t + 24768)$$

$$sconstituent_7 := \frac{2831}{144}\,t - \frac{11933}{1440}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5 - \frac{25193}{1728}$$

$$\frac{1}{8640}\,(t-7)\,(36\,t^4 - 873\,t^3 + 7129\,t^2 - 21695\,t + 17995)$$

$$sconstituent_8 := \frac{61}{3}\,t - \frac{5989}{720}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5 - \frac{2444}{135}$$

$$\frac{1}{8640}\,(t-8)\,(36\,t^4 - 837\,t^3 + 6544\,t^2 - 19516\,t + 19552)$$

$$sconstituent_9 := \frac{949}{48}\,t - \frac{11933}{1440}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5 - \frac{4983}{320}$$

$$\frac{1}{8640}\,(t-9)\,(36\,t^4 - 801\,t^3 + 6031\,t^2 - 17319\,t + 14949)$$

$$sconstituent_{10} := \frac{182}{9}\,t - \frac{5989}{720}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5 - \frac{443}{27}$$

$$\frac{182}{9}\,t - \frac{5989}{720}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5 - \frac{443}{27}$$

$$sconstituent_{11} := \frac{949}{48} t - \frac{130253}{8640} - \frac{11933}{1440} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5$$

$$\frac{949}{48} t - \frac{130253}{8640} - \frac{11933}{1440} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5$$

$$sconstituent_{12} := -18 + \frac{61}{3} t - \frac{5989}{720} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5$$

$$-18 + \frac{61}{3} t - \frac{5989}{720} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5$$

$$sconstituent_{13} := \frac{2831}{144} t - \frac{11933}{1440} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5 - \frac{120781}{8640}$$

$$\frac{2831}{144} t - \frac{11933}{1440} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5 - \frac{120781}{8640}$$

$$sconstituent_{14} := \frac{61}{3} t - \frac{5989}{720} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5 - \frac{2444}{135}$$

$$\frac{1}{8640} (t - 8) (36 t^4 - 837 t^3 + 6544 t^2 - 19516 t + 19552)$$

$$sconstituent_{15} := \frac{949}{48} t - \frac{11933}{1440} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5 - \frac{1035}{64}$$

$$\frac{949}{48} t - \frac{11933}{1440} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5 - \frac{1035}{64}$$

$$sconstituent_{16} := \frac{182}{9} t - \frac{5989}{720} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5 - \frac{2107}{135}$$

$$\frac{182}{9} t - \frac{5989}{720} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5 - \frac{2107}{135}$$

$$sconstituent_{17} := \frac{949}{48} t - \frac{11933}{1440} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5 - \frac{25705}{1728}$$

$$\frac{1}{8640} (t - 5) (36 t^4 - 945 t^3 + 8515 t^2 - 29023 t + 25705)$$

$$sconstituent_{18} := \frac{61}{3} t - \frac{5989}{720} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5 - \frac{92}{5}$$

$$\frac{61}{3} t - \frac{5989}{720} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5 - \frac{92}{5}$$

$$sconstituent_{19} := \frac{2831}{144} t - \frac{11933}{1440} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5 - \frac{129421}{8640}$$

$$\frac{2831}{144} t - \frac{11933}{1440} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5 - \frac{129421}{8640}$$

$$sconstituent_{20} := \frac{61}{3}\,t - \frac{5989}{720}\,t^2 - \frac{478}{27} + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5$$

$$\frac{1}{8640}\,(t-2)\,(36\,t^4 - 1053\,t^3 + 11134\,t^2 - 49600\,t + 76480)$$

$$sconstituent_{21} := \frac{949}{48}\,t - \frac{4599}{320} - \frac{11933}{1440}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5$$

$$\frac{949}{48}\,t - \frac{4599}{320} - \frac{11933}{1440}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5$$

$$sconstituent_{22} := \frac{182}{9}\,t - \frac{5989}{720}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5 - \frac{443}{27}$$

$$\frac{182}{9}\,t - \frac{5989}{720}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5 - \frac{443}{27}$$

$$sconstituent_{23} := -\frac{140621}{8640} + \frac{949}{48}\,t - \frac{11933}{1440}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5$$

$$-\frac{140621}{8640} + \frac{949}{48}\,t - \frac{11933}{1440}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5$$

$$sconstituent_{24} := \frac{61}{3}\,t - \frac{5989}{720}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5 - \frac{92}{5}$$

$$\frac{61}{3}\,t - \frac{5989}{720}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5 - \frac{92}{5}$$

$$sconstituent_{25} := \frac{2831}{144}\,t - \frac{23465}{1728} - \frac{11933}{1440}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5$$

$$\frac{2831}{144}\,t - \frac{23465}{1728} - \frac{11933}{1440}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5$$

$$sconstituent_{26} := \frac{61}{3}\,t - \frac{5989}{720}\,t^2 - \frac{2282}{135} + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5$$

$$\frac{61}{3}\,t - \frac{5989}{720}\,t^2 - \frac{2282}{135} + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5$$

$$sconstituent_{27} := \frac{949}{48}\,t - \frac{11933}{1440}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5 - \frac{1035}{64}$$

$$\frac{949}{48}\,t - \frac{11933}{1440}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5 - \frac{1035}{64}$$

$$sconstituent_{28} := \frac{182}{9}\,t - \frac{2269}{135} - \frac{5989}{720}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5$$

$$\frac{1}{8640}\,(t-4)\,(36\,t^4 - 981\,t^3 + 9316\,t^2 - 34604\,t + 36304)$$

$$sconstituent_{29} := \frac{949}{48}\, t - \frac{131981}{8640} - \frac{11933}{1440}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5$$

$$\frac{949}{48}\, t - \frac{131981}{8640} - \frac{11933}{1440}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5$$

$$sconstituent_{30} := -18 + \frac{61}{3}\, t - \frac{5989}{720}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5$$

$$-18 + \frac{61}{3}\, t - \frac{5989}{720}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5$$

$$sconstituent_{31} := \frac{2831}{144}\, t - \frac{11933}{1440}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5 - \frac{119053}{8640}$$

$$\frac{2831}{144}\, t - \frac{11933}{1440}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5 - \frac{119053}{8640}$$

$$sconstituent_{32} := \frac{61}{3}\, t - \frac{5989}{720}\, t^2 - \frac{478}{27} + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5$$

$$\frac{1}{8640}\, (t-2)\,(36\, t^4 - 1053\, t^3 + 11134\, t^2 - 49600\, t + 76480)$$

$$sconstituent_{33} := \frac{949}{48}\, t - \frac{11933}{1440}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5 - \frac{4983}{320}$$

$$\frac{1}{8640}\, (t-9)\,(36\, t^4 - 801\, t^3 + 6031\, t^2 - 17319\, t + 14949)$$

$$sconstituent_{34} := \frac{182}{9}\, t - \frac{2269}{135} - \frac{5989}{720}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5$$

$$\frac{1}{8640}\, (t-4)\,(36\, t^4 - 981\, t^3 + 9316\, t^2 - 34604\, t + 36304)$$

$$sconstituent_{35} := \frac{949}{48}\, t - \frac{27433}{1728} - \frac{11933}{1440}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5$$

$$\frac{949}{48}\, t - \frac{27433}{1728} - \frac{11933}{1440}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5$$

$$sconstituent_{36} := \frac{61}{3}\, t - \frac{5989}{720}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5 - \frac{86}{5}$$

$$\frac{1}{8640}\, (t-6)\,(36\, t^4 - 909\, t^3 + 7786\, t^2 - 25152\, t + 24768)$$

$$sconstituent_{37} := \frac{2831}{144}\, t - \frac{23465}{1728} - \frac{11933}{1440}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5$$

$$\frac{2831}{144}\, t - \frac{23465}{1728} - \frac{11933}{1440}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5$$

$$sconstituent_{38} := \frac{61}{3}\, t - \frac{5989}{720}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5 - \frac{2444}{135}$$

$$\frac{1}{8640}\, (t-8)\, (36\, t^4 - 837\, t^3 + 6544\, t^2 - 19516\, t + 19552)$$

$$sconstituent_{39} := \frac{949}{48}\, t - \frac{11933}{1440}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5 - \frac{5303}{320}$$

$$\frac{1}{8640}\, (t-3)\, (36\, t^4 - 1017\, t^3 + 10189\, t^2 - 41031\, t + 47727)$$

$$sconstituent_{40} := \frac{182}{9}\, t - \frac{5989}{720}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5 - \frac{443}{27}$$

$$\frac{182}{9}\, t - \frac{5989}{720}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5 - \frac{443}{27}$$

$$sconstituent_{41} := \frac{949}{48}\, t - \frac{11933}{1440}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5 - \frac{121613}{8640}$$

$$\frac{949}{48}\, t - \frac{11933}{1440}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5 - \frac{121613}{8640}$$

$$sconstituent_{42} := -18 + \frac{61}{3}\, t - \frac{5989}{720}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5$$

$$-18 + \frac{61}{3}\, t - \frac{5989}{720}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5$$

$$sconstituent_{43} := \frac{2831}{144}\, t - \frac{11933}{1440}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5 - \frac{129421}{8640}$$

$$\frac{2831}{144}\, t - \frac{11933}{1440}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5 - \frac{129421}{8640}$$

$$sconstituent_{44} := \frac{61}{3}\, t - \frac{5989}{720}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5 - \frac{2444}{135}$$

$$\frac{1}{8640}\, (t-8)\, (36\, t^4 - 837\, t^3 + 6544\, t^2 - 19516\, t + 19552)$$

$$sconstituent_{45} := \frac{949}{48}\, t - \frac{11933}{1440}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5 - \frac{971}{64}$$

$$\frac{949}{48}\, t - \frac{11933}{1440}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5 - \frac{971}{64}$$

$$sconstituent_{46} := \frac{182}{9}\, t - \frac{5989}{720}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5 - \frac{2107}{135}$$

$$\frac{182}{9}\, t - \frac{5989}{720}\, t^2 + \frac{331}{216}\, t^3 - \frac{25}{192}\, t^4 + \frac{1}{240}\, t^5 - \frac{2107}{135}$$

$$sconstituent_{47} := \frac{949}{48} t - \frac{27433}{1728} - \frac{11933}{1440} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5$$

$$\frac{949}{48} t - \frac{27433}{1728} - \frac{11933}{1440} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5$$

$$sconstituent_{48} := \frac{61}{3} t - \frac{5989}{720} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5 - \frac{92}{5}$$

$$\frac{61}{3} t - \frac{5989}{720} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5 - \frac{92}{5}$$

$$sconstituent_{49} := \frac{2831}{144} t - \frac{11933}{1440} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5 - \frac{120781}{8640}$$

$$\frac{2831}{144} t - \frac{11933}{1440} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5 - \frac{120781}{8640}$$

$$sconstituent_{50} := \frac{61}{3} t - \frac{5989}{720} t^2 - \frac{478}{27} + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5$$

$$\frac{1}{8640} (t - 2) (36 t^4 - 1053 t^3 + 11134 t^2 - 49600 t + 76480)$$

$$sconstituent_{51} := \frac{949}{48} t - \frac{11933}{1440} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5 - \frac{4919}{320}$$

$$\frac{949}{48} t - \frac{11933}{1440} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5 - \frac{4919}{320}$$

$$sconstituent_{52} := \frac{182}{9} t - \frac{5989}{720} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5 - \frac{443}{27}$$

$$\frac{182}{9} t - \frac{5989}{720} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5 - \frac{443}{27}$$

$$sconstituent_{53} := \frac{949}{48} t - \frac{131981}{8640} - \frac{11933}{1440} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5$$

$$\frac{949}{48} t - \frac{131981}{8640} - \frac{11933}{1440} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5$$

$$sconstituent_{54} := \frac{61}{3} t - \frac{5989}{720} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5 - \frac{92}{5}$$

$$\frac{61}{3} t - \frac{5989}{720} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5 - \frac{92}{5}$$

$$sconstituent_{55} := \frac{2831}{144} t - \frac{11933}{1440} t^2 + \frac{331}{216} t^3 - \frac{25}{192} t^4 + \frac{1}{240} t^5 - \frac{25193}{1728}$$

$$\frac{1}{8640} (t - 7) (36 t^4 - 873 t^3 + 7129 t^2 - 21695 t + 17995)$$

$$sconstituent_{56} := \frac{61}{3}\,t - \frac{5989}{720}\,t^2 - \frac{2282}{135} + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5$$

$$\frac{61}{3}\,t - \frac{5989}{720}\,t^2 - \frac{2282}{135} + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5$$

$$sconstituent_{57} := \frac{949}{48}\,t - \frac{11933}{1440}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5 - \frac{971}{64}$$

$$\frac{949}{48}\,t - \frac{11933}{1440}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5 - \frac{971}{64}$$

$$sconstituent_{58} := \frac{182}{9}\,t - \frac{2269}{135} - \frac{5989}{720}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5$$

$$\frac{1}{8640}\,(t-4)\,(36\,t^4 - 981\,t^3 + 9316\,t^2 - 34604\,t + 36304)$$

$$sconstituent_{59} := -\frac{140621}{8640} + \frac{949}{48}\,t - \frac{11933}{1440}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5$$

$$-\frac{140621}{8640} + \frac{949}{48}\,t - \frac{11933}{1440}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5$$

$$sconstituent_{60} := -18 + \frac{61}{3}\,t - \frac{5989}{720}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5$$

$$-18 + \frac{61}{3}\,t - \frac{5989}{720}\,t^2 + \frac{331}{216}\,t^3 - \frac{25}{192}\,t^4 + \frac{1}{240}\,t^5$$

Extract the coefficients of the constituents.

```
> for r from 1 to p do
    for coeffdeg from 0 to d do
     sc[coeffdeg,r]:=coeff(sconstituent[r],t,coeffdeg):
     #print( r, sc[coeffdeg,r] ):
    od:
  od:
```

Print and analyze the constituent coefficients for periods. First the higher coefficients, which are constant. Print the first coefficient, then any that don't repeat the preceding value (there are none).

```
> for coeffdeg from 3 to d do
    print("degree", coeffdeg, "coeff", sc[coeffdeg,1]):
    for r from 2 to p do
      stepdifference:=sc[coeffdeg,r]-sc[coeffdeg,r-1]:
      if( stepdifference<>0 ) then
      print(r,sc[coeffdeg,r],stepdifference):
      fi:
    od:
  od:
```

$$\text{"degree", 3, "coeff", } \frac{331}{216}$$

$$\text{"degree", 4, "coeff", } -\frac{25}{192}$$

$$\text{"degree", 5, "coeff", } \frac{1}{240}$$

Next, the constant terms, whose period is expected to be 60. Print all constant terms up to the presumed period "stepsize". Print the difference (at step "stepsize") if they are not repeating.

```
> stepsize:=30;
  for r from 1 to stepsize do
    print(r, sc[0,r]);
  od:
  for r from stepsize+1 to p do
    stepdifference:=sc[0,r]-sc[0,r-stepsize]:
    if( stepdifference<>0 ) then print(r,sc[0,r],stepdifference):
  fi:
  od:
  print("Constant terms completed.");
```

$$stepsize := 30$$

$$1, \ -\frac{110413}{8640}$$

$$2, \ -\frac{478}{27}$$

$$3, \ -\frac{5303}{320}$$

$$4, \ -\frac{2269}{135}$$

$$5, \ -\frac{25705}{1728}$$

$$6, \ -\frac{86}{5}$$

$$7, \ -\frac{25193}{1728}$$

$$8, \ -\frac{2444}{135}$$

$$9, \ -\frac{4983}{320}$$

$$10, \ -\frac{443}{27}$$

$$11, \ -\frac{130253}{8640}$$

27

12, -18

13, $-\dfrac{120781}{8640}$

14, $-\dfrac{2444}{135}$

15, $-\dfrac{1035}{64}$

16, $-\dfrac{2107}{135}$

17, $-\dfrac{25705}{1728}$

18, $-\dfrac{92}{5}$

19, $-\dfrac{129421}{8640}$

20, $-\dfrac{478}{27}$

21, $-\dfrac{4599}{320}$

22, $-\dfrac{443}{27}$

23, $-\dfrac{140621}{8640}$

24, $-\dfrac{92}{5}$

25, $-\dfrac{23465}{1728}$

26, $-\dfrac{2282}{135}$

27, $-\dfrac{1035}{64}$

28, $-\dfrac{2269}{135}$

29, $-\dfrac{131981}{8640}$

30, -18

$$31, -\frac{119053}{8640}, -1$$

$$33, -\frac{4983}{320}, 1$$

$$35, -\frac{27433}{1728}, -1$$

$$37, -\frac{23465}{1728}, 1$$

$$39, -\frac{5303}{320}, -1$$

$$41, -\frac{121613}{8640}, 1$$

$$43, -\frac{129421}{8640}, -1$$

$$45, -\frac{971}{64}, 1$$

$$47, -\frac{27433}{1728}, -1$$

$$49, -\frac{120781}{8640}, 1$$

$$51, -\frac{4919}{320}, -1$$

$$53, -\frac{131981}{8640}, 1$$

$$55, -\frac{25193}{1728}, -1$$

$$57, -\frac{971}{64}, 1$$

$$59, -\frac{140621}{8640}, -1$$

"Constant terms completed."

Now, the linear terms. First print all linear coefficients up to the presumed period "stepsize". Then analyze for period and print the difference (at step "stepsize") if they are not repeating.

```
> stepsize:=6;
  for r from 1 to stepsize do
    print(r, sc[1,r]);
  od:
```

```
for r from stepsize+1 to p do
  stepdifference:=sc[1,r]-sc[1,r-stepsize]:
  if( stepdifference<>0 ) then print(r,sc[1,r],stepdifference):
fi:
od:
print("Linear coefficients completed.");
```

$$stepsize := 6$$

$$1, \frac{2831}{144}$$

$$2, \frac{61}{3}$$

$$3, \frac{949}{48}$$

$$4, \frac{182}{9}$$

$$5, \frac{949}{48}$$

$$6, \frac{61}{3}$$

"Linear coefficients completed."

The quadratic terms. First print all quadratic coefficients up to the presumed period "stepsize". Then analyze for period and print the difference (at step "stepsize") if they are not repeating.

```
> stepsize:=2;
  for r from 1 to stepsize do
    print(r, sc[2,r]);
  od:
  for r from stepsize+1 to p do
    stepdifference:=sc[2,r]-sc[2,r-stepsize]:
    if( stepdifference<>0 ) then print(r,sc[2,r],stepdifference):
  fi:
  od:
  print("Quadratic coefficients completed.");
```

$$stepsize := 2$$

$$1, -\frac{11933}{1440}$$

$$2, -\frac{5989}{720}$$

"Quadratic coefficients completed."