

Semimagic generating functions and constituents

(general form, with affine data)

Notation:

S: semimagic squares (all positive values).
s: normalized squares (symmetry types).
R: reduced squares (least element is 0).
r: reduced normalized squares (reduced symmetry types).
gf: generating function in some form.
gfsum: generating function as a sum of simple terms.
c: Cubic (fixed strict upper bound; weak upper bound for reduced).
a: Affine (fixed magic sum).
p = period,
pno7 = truncated period (in affine),
p7 = period of the H term in the g.f.'s, with the denominator factor (1-x^d7).
d = degree (and dimension, in the general and no-H cases),
d7 = degree in the H term.
RtoSfactor = the rational function that multiplies Rgf to Sgf and rgf to sgf.

This is for **affine**: set up main constants.

```
> d:=4; d7:=1;
  p:=840; pno7:=120; p7:=21;
  RtoSfactor:=x^3/(1-x^3);
               d:= 4
               d7:= 1
               p:= 840
               pno7:= 120
               p7:= 21
               RtoSfactor:=  $\frac{x^3}{1 - x^3}$ 
```

From the Latte results we get the closed Ehrhart g.f. of each flat, which depends on whether we're doing cubic or affine.

This is for **affine**: set up simplex data.

```
> simplexname[1]:="OABC": ehrgf[1]:= 1/((1-x)*(1-x^2)^3) : dimen
  [1]:=3:
  simplexname[2]:="OEE2": ehrgf[2]:= 1/((1-x)*(1-x^4)^2) : dimen
  [2]:=2:
  simplexname[3]:="OAE2": ehrgf[3]:= 1/((1-x)*(1-x^2)*(1-x^4)) :
  dimen[3]:=2:
  simplexname[4]:="ADE2": ehrgf[4]:= 1/((1-x^2)*(1-x^3)*(1-x^4)) :
  dimen[4]:=2:
  simplexname[5]:="DE1E2": ehrgf[5]:= 1/((1-x^3)*(1-x^4)^2) :
```

```

dimen[5]:=2:
simplexname[6]:="OCE": ehrgf[6]:= 1/((1-x)*(1-x^2)*(1-x^4)) :
dimen[6]:=2:
simplexname[7]:="BDE1": ehrgf[7]:= 1/((1-x^2)*(1-x^3)*(1-x^4)) :
: dimen[7]:=2:
simplexname[8]:="ABD": ehrgf[8]:= 1/((1-x^2)^2*(1-x^3)) : dimen
[8]:=2:
simplexname[9]:="FG1": ehrgf[9]:= 1/((1-x^5)*(1-x^8)) : dimen
[9]:=1:
simplexname[10]:="EF": ehrgf[10]:= 1/((1-x^4)*(1-x^5)) : dimen
[10]:=1:
simplexname[11]:="OG": ehrgf[11]:= 1/((1-x)*(1-x^6)) : dimen
[11]:=1:
simplexname[12]:="FG": ehrgf[12]:= 1/((1-x^5)*(1-x^6)) : dimen
[12]:=1:
simplexname[13]:="AF": ehrgf[13]:= 1/((1-x^2)*(1-x^5)) : dimen
[13]:=1:
simplexname[14]:="DG": ehrgf[14]:= 1/((1-x^3)*(1-x^6)) : dimen
[14]:=1:
simplexname[15]:="DG2": ehrgf[15]:= 1/((1-x^3)*(1-x^8)) : dimen
[15]:=1:
simplexname[16]:="DE": ehrgf[16]:= 1/((1-x^3)*(1-x^4)) : dimen
[16]:=1:
simplexname[17]:="H": ehrgf[17] := 1/(1-x^7) : dimen[17]:=0:
for n from 1 to 17 do print(simplexname[n], dimen[n], ehrgf[n])
; od;

```

$$\text{"OABC", } 3, \frac{1}{(1-x)(1-x^2)^3}$$

$$\text{"OEE2", } 2, \frac{1}{(1-x)(1-x^4)^2}$$

$$\text{"OAE2", } 2, \frac{1}{(1-x)(1-x^2)(1-x^4)}$$

$$\text{"ADE2", } 2, \frac{1}{(1-x^2)(1-x^3)(1-x^4)}$$

$$\text{"DE1E2", } 2, \frac{1}{(1-x^3)(1-x^4)^2}$$

$$\text{"OCE", } 2, \frac{1}{(1-x)(1-x^2)(1-x^4)}$$

$$\text{"BDE1", } 2, \frac{1}{(1-x^2)(1-x^3)(1-x^4)}$$

$$\text{"ABD", } 2, \frac{1}{(1-x^2)^2(1-x^3)}$$

$$\begin{aligned}
& \text{"FG1", } 1, \frac{1}{(1-x^5)(1-x^8)} \\
& \text{"EF", } 1, \frac{1}{(1-x^4)(1-x^5)} \\
& \text{"OG", } 1, \frac{1}{(1-x)(1-x^6)} \\
& \text{"FG", } 1, \frac{1}{(1-x^5)(1-x^6)} \\
& \text{"AF", } 1, \frac{1}{(1-x^2)(1-x^5)} \\
& \text{"DG", } 1, \frac{1}{(1-x^3)(1-x^6)} \\
& \text{"DG2", } 1, \frac{1}{(1-x^3)(1-x^8)} \\
& \text{"DE", } 1, \frac{1}{(1-x^3)(1-x^4)} \\
& \text{"H", } 0, \frac{1}{1-x^7}
\end{aligned}$$

The closed E.g.f. is converted to the open E.g.f.

```

> for n from 1 to 17 do
    mu[n]:=(-1)^(dimen[1]-dimen[n]):
  od:
mu[14]:=2*mu[14]:
for n from 1 to 17 do
  openehrgf[n]:=simplify(-(-1)^dimen[n]*subs(x=1/x,ehrgf[n])):
  od:

```

Set up basic g.f.'s. for reduced symmetry types.

```

> for n from 1 to 17 do
    rgfterm[n]:=openehrgf[n]:
  od:
rgfsum:=sum(mu[nn]*rgfterm[nn],nn=1..17);
rgf:=simplify(rgfsum):
rno7gfsum:=sum(mu[nn]*rgfterm[nn],nn=1..16);
rno7gf:=simplify(rno7gfsum):
r7gf:=rgfterm[17];

```

$$\begin{aligned}
rgfsum := & \frac{x^7}{(x-1)(x^2-1)^3} + \frac{x^9}{(x-1)(x^4-1)^2} + \frac{2x^7}{(x-1)(x^2-1)(x^4-1)} \\
& + \frac{2x^9}{(x^2-1)(x^3-1)(x^4-1)} + \frac{x^{11}}{(x^3-1)(x^4-1)^2} + \frac{x^7}{(x^2-1)^2(x^3-1)}
\end{aligned}$$

$$\begin{aligned}
& + \frac{x^{13}}{(x^5 - 1)(x^8 - 1)} + \frac{x^9}{(x^4 - 1)(x^5 - 1)} + \frac{x^7}{(x - 1)(x^6 - 1)} \\
& + \frac{x^{11}}{(x^5 - 1)(x^6 - 1)} + \frac{x^7}{(x^2 - 1)(x^5 - 1)} + \frac{2x^9}{(x^3 - 1)(x^6 - 1)} \\
& + \frac{x^{11}}{(x^3 - 1)(x^8 - 1)} + \frac{x^7}{(x^3 - 1)(x^4 - 1)} + \frac{x^7}{x^7 - 1} \\
rno7gfsum := & \frac{x^7}{(x - 1)(x^2 - 1)^3} + \frac{x^9}{(x - 1)(x^4 - 1)^2} + \frac{2x^7}{(x - 1)(x^2 - 1)(x^4 - 1)} \\
& + \frac{2x^9}{(x^2 - 1)(x^3 - 1)(x^4 - 1)} + \frac{x^{11}}{(x^3 - 1)(x^4 - 1)^2} + \frac{x^7}{(x^2 - 1)^2(x^3 - 1)} \\
& + \frac{x^{13}}{(x^5 - 1)(x^8 - 1)} + \frac{x^9}{(x^4 - 1)(x^5 - 1)} + \frac{x^7}{(x - 1)(x^6 - 1)} \\
& + \frac{x^{11}}{(x^5 - 1)(x^6 - 1)} + \frac{x^7}{(x^2 - 1)(x^5 - 1)} + \frac{2x^9}{(x^3 - 1)(x^6 - 1)} \\
& + \frac{x^{11}}{(x^3 - 1)(x^8 - 1)} + \frac{x^7}{(x^3 - 1)(x^4 - 1)} \\
r7gf := & -\frac{x^7}{x^7 - 1}
\end{aligned}$$

To compute Rgf, we need rgf:

$$\begin{aligned}
> \text{Rgfsun:=72*rgfsum;} \\
\text{Rgf:=simplify(Rgfsun);} \\
\text{Rno7gfsun:=72*rno7gfsun;} \\
\text{Rno7gf:=simplify(Rno7gfsun);} \\
\text{R7gf:=72*r7gf;} \\
Rgfsun := & \frac{72x^7}{(x - 1)(x^2 - 1)^3} + \frac{72x^9}{(x - 1)(x^4 - 1)^2} + \frac{144x^7}{(x - 1)(x^2 - 1)(x^4 - 1)} \\
& + \frac{144x^9}{(x^2 - 1)(x^3 - 1)(x^4 - 1)} + \frac{72x^{11}}{(x^3 - 1)(x^4 - 1)^2} + \frac{72x^7}{(x^2 - 1)^2(x^3 - 1)} \\
& + \frac{72x^{13}}{(x^5 - 1)(x^8 - 1)} + \frac{72x^9}{(x^4 - 1)(x^5 - 1)} + \frac{72x^7}{(x - 1)(x^6 - 1)} \\
& + \frac{72x^{11}}{(x^5 - 1)(x^6 - 1)} + \frac{72x^7}{(x^2 - 1)(x^5 - 1)} + \frac{144x^9}{(x^3 - 1)(x^6 - 1)} \\
& + \frac{72x^{11}}{(x^3 - 1)(x^8 - 1)} + \frac{72x^7}{(x^3 - 1)(x^4 - 1)} + \frac{72x^7}{x^7 - 1}
\end{aligned}$$

$$\begin{aligned}
Rno7gfs := & \frac{72x^7}{(x-1)(x^2-1)^3} + \frac{72x^9}{(x-1)(x^4-1)^2} + \frac{144x^7}{(x-1)(x^2-1)(x^4-1)} \\
& + \frac{144x^9}{(x^2-1)(x^3-1)(x^4-1)} + \frac{72x^{11}}{(x^3-1)(x^4-1)^2} + \frac{72x^7}{(x^2-1)^2(x^3-1)} \\
& + \frac{72x^{13}}{(x^5-1)(x^8-1)} + \frac{72x^9}{(x^4-1)(x^5-1)} + \frac{72x^7}{(x-1)(x^6-1)} \\
& + \frac{72x^{11}}{(x^5-1)(x^6-1)} + \frac{72x^7}{(x^2-1)(x^5-1)} + \frac{144x^9}{(x^3-1)(x^6-1)} \\
& + \frac{72x^{11}}{(x^3-1)(x^8-1)} + \frac{72x^7}{(x^3-1)(x^4-1)} \\
R7gf := & -\frac{72x^7}{x^7-1}
\end{aligned}$$

Hence Sgf, the g.f. of the number of semimagic squares, equals

```

> Sgfsum:=RtoSfactor*Rgfs;
Sgf:=simplify(Sgfsum);
Sno7gfs := RtoSfactor*Rno7gfs;
Sno7gf:=simplify(Sno7gfs);
S7gf:=72*s7gf;

Sgfsum := 
$$\begin{aligned}
& \frac{1}{1-x^3} \left( x^3 \left( \frac{72x^7}{(x-1)(x^2-1)^3} + \frac{72x^9}{(x-1)(x^4-1)^2} \right. \right. \\
& + \frac{144x^7}{(x-1)(x^2-1)(x^4-1)} + \frac{144x^9}{(x^2-1)(x^3-1)(x^4-1)} \\
& + \frac{72x^{11}}{(x^3-1)(x^4-1)^2} + \frac{72x^7}{(x^2-1)^2(x^3-1)} + \frac{72x^{13}}{(x^5-1)(x^8-1)} \\
& + \frac{72x^9}{(x^4-1)(x^5-1)} + \frac{72x^7}{(x-1)(x^6-1)} + \frac{72x^{11}}{(x^5-1)(x^6-1)} \\
& + \frac{72x^7}{(x^2-1)(x^5-1)} + \frac{144x^9}{(x^3-1)(x^6-1)} + \frac{72x^{11}}{(x^3-1)(x^8-1)} \\
& \left. \left. + \frac{72x^7}{(x^3-1)(x^4-1)} + \frac{72x^7}{x^7-1} \right) \right)
\end{aligned}$$


```

$$\begin{aligned}
Sno7gfs := & \frac{1}{1-x^3} \left(x^3 \left(\frac{72x^7}{(x-1)(x^2-1)^3} + \frac{72x^9}{(x-1)(x^4-1)^2} \right. \right. \\
& + \frac{144x^7}{(x-1)(x^2-1)(x^4-1)} + \frac{144x^9}{(x^2-1)(x^3-1)(x^4-1)}
\end{aligned}$$

$$\begin{aligned}
& + \frac{72x^{11}}{(x^3-1)(x^4-1)^2} + \frac{72x^7}{(x^2-1)^2(x^3-1)} + \frac{72x^{13}}{(x^5-1)(x^8-1)} \\
& + \frac{72x^9}{(x^4-1)(x^5-1)} + \frac{72x^7}{(x-1)(x^6-1)} + \frac{72x^{11}}{(x^5-1)(x^6-1)} \\
& + \frac{72x^7}{(x^2-1)(x^5-1)} + \frac{144x^9}{(x^3-1)(x^6-1)} + \frac{72x^{11}}{(x^3-1)(x^8-1)} \\
& + \frac{72x^7}{(x^3-1)(x^4-1)} \Big) \Big) \\
S7gf := & \frac{1}{(1-x^{21})^2} (72x^{10}(x^{20}+x^{19}+x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+x^{11} \\
& + x^{10}+x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1)(x^{12}-x^{11}+x^9-x^8+x^6 \\
& -x^4+x^3-x+1))
\end{aligned}$$

The g.f. of the total number of symmetry types, l_ml ("sgf"):

```

> sgfsum:=RtoSfactor*rgfsum;
  sgf:=simplify(sgfsum);
  sno7gfsum:=RtoSfactor*rno7gfsum;
  sno7gf:=simplify(sno7gfsum);
  s7gf:=RtoSfactor*r7gf;

sgfsum:= 
$$\begin{aligned}
& \frac{1}{1-x^3} \left( x^3 \left( \frac{x^7}{(x-1)(x^2-1)^3} + \frac{x^9}{(x-1)(x^4-1)^2} \right. \right. \\
& + \frac{2x^7}{(x-1)(x^2-1)(x^4-1)} + \frac{2x^9}{(x^2-1)(x^3-1)(x^4-1)} \\
& + \frac{x^{11}}{(x^3-1)(x^4-1)^2} + \frac{x^7}{(x^2-1)^2(x^3-1)} + \frac{x^{13}}{(x^5-1)(x^8-1)} \\
& + \frac{x^9}{(x^4-1)(x^5-1)} + \frac{x^7}{(x-1)(x^6-1)} + \frac{x^{11}}{(x^5-1)(x^6-1)} \\
& + \frac{x^7}{(x^2-1)(x^5-1)} + \frac{2x^9}{(x^3-1)(x^6-1)} + \frac{x^{11}}{(x^3-1)(x^8-1)} \\
& \left. \left. + \frac{x^7}{(x^3-1)(x^4-1)} + \frac{x^7}{x^7-1} \right) \right)
\end{aligned}$$


```

$$\begin{aligned}
sno7gfsum:= & \frac{1}{1-x^3} \left(x^3 \left(\frac{x^7}{(x-1)(x^2-1)^3} + \frac{x^9}{(x-1)(x^4-1)^2} \right. \right. \\
& + \frac{2x^7}{(x-1)(x^2-1)(x^4-1)} + \frac{2x^9}{(x^2-1)(x^3-1)(x^4-1)}
\end{aligned}$$

$$\begin{aligned}
& + \frac{x^{11}}{(x^3 - 1)(x^4 - 1)^2} + \frac{x^7}{(x^2 - 1)^2(x^3 - 1)} + \frac{x^{13}}{(x^5 - 1)(x^8 - 1)} \\
& + \frac{x^9}{(x^4 - 1)(x^5 - 1)} + \frac{x^7}{(x - 1)(x^6 - 1)} + \frac{x^{11}}{(x^5 - 1)(x^6 - 1)} \\
& + \frac{x^7}{(x^2 - 1)(x^5 - 1)} + \frac{2x^9}{(x^3 - 1)(x^6 - 1)} + \frac{x^{11}}{(x^3 - 1)(x^8 - 1)} \\
& + \frac{x^7}{(x^3 - 1)(x^4 - 1)} \Big) \Big) \\
s7gf & := -\frac{x^{10}}{(1 - x^3)(x^7 - 1)}
\end{aligned}$$

Generate the series expansions of the g.f.'s.

Expressing the rational function with standard denominator gives an orders-of-magnitude speedup in the series expansion.

enddegree: The number of terms of the sequences to show.

```
> enddegree:=20;
      enddegree:= 20
```

Standard denominator $(1-x^p)^{d+1}$.

```
> pdenom:=(1-x^p):
  standenom:=pdenom^(d+1);
  pno7denom:=(1-x^pno7):
  stanno7denom:=pno7denom^(d+1);
  p7denom:=(1-x^p7):
  stan7denom:=p7denom^(d7+1);
  standenom:= (1 - x840)5
  stanno7denom:= (1 - x120)5
  stan7denom:= (1 - x21)2
```

G.f. as rational function with standard denominator.

```
> Sgfstandnum:=simplify(numer(Sgf)*simplify(standenom/denom(Sgf))
  ):
  Sgf:=Sgfstandnum/standenom;
> Sno7gfstandnum:=simplify(numer(Sno7gf)*simplify
  (stanno7denom/denom(Sno7gf))):
  Sno7gf:=Sno7gfstandnum/stanno7denom;
Sno7gf:=  $\frac{1}{(1 - x^{120})^5} (72 x^{10} (17 x^{18} + 22 x^{17} + 37 x^{16} + 38 x^{15} + 50 x^{14} + 43 x^{13} + 45 x^{12} + 38 x^{11} + 30 x^{10} + 21 x^9 + 12 x^8 + 6 x^7 - x^5 - x^4 + x^2 + x + 1) (1 - x$ 
```

$$\begin{aligned}
& + x^4 + x^8 - x^6 - x^{11} + x^{12} + x^{20} - x^{21} - x^{26} + x^{24} + x^{28} - x^{31} + x^{32} - x^{41} + x^{40} \\
& + x^{112} - x^{91} + x^{84} + x^{72} + x^{64} - x^{66} + x^{108} + x^{48} - x^{71} - x^{106} + x^{92} + x^{60} - x^{111} \\
& + x^{104} - x^{101} + x^{100} + x^{88} - x^{86} - x^{81} + x^{80} + x^{68} - x^{61} + x^{52} - x^{51} - x^{46} + x^{44}) \\
& (1 + x^2 + x^4 + x^8 + x^6 + x^{14} + x^{10} + x^{12} + x^{20} + x^{16} + x^{18} + x^{26} + x^{22} + x^{24} + x^{28} \\
& + x^{32} + x^{30} + x^{34} + x^{36} + x^{40} + x^{38} + x^{112} + x^{98} + x^{84} + x^{70} + x^{56} + x^{42} + x^{72} + x^{64} \\
& + x^{90} + x^{96} + x^{66} + x^{54} + x^{50} + x^{102} + x^{108} + x^{48} + x^{106} + x^{114} + x^{92} + x^{60} + x^{118} \\
& + x^{116} + x^{110} + x^{104} + x^{100} + x^{94} + x^{88} + x^{86} + x^{82} + x^{80} + x^{78} + x^{76} + x^{74} + x^{68} \\
& + x^{62} + x^{58} + x^{52} + x^{46} + x^{44}) (1 + x^3 + x^6 + x^9 + x^{15} + x^{12} + x^{18} + x^{21} + x^{24} \\
& + x^{27} + x^{33} + x^{30} + x^{36} + x^{39} + x^{105} + x^{84} + x^{42} + x^{63} + x^{72} + x^{90} + x^{57} + x^{96} + x^{66} \\
& + x^{54} + x^{99} + x^{102} + x^{108} + x^{48} + x^{114} + x^{60} + x^{117} + x^{111} + x^{93} + x^{87} + x^{81} + x^{78} \\
& + x^{75} + x^{69} + x^{51} + x^{45})^2 (1 - x^2 + x^6 - x^{10} + x^{12} - x^{26} + x^{24} + x^{30} - x^{34} + x^{36} \\
& - x^{98} + x^{84} + x^{72} + x^{96} + x^{54} - x^{50} + x^{102} + x^{108} + x^{48} - x^{106} + x^{60} - x^{82} + x^{78} \\
& - x^{74} - x^{58}))
\end{aligned}$$

```

> S7gfstandnum:=simplify(numer(S7gf)*simplify(stan7denom/denom
(S7gf)));
S7gf:=S7gfstandnum/stan7denom;
S7gfstandnum:= 72 x^10 (x^20 + x^19 + x^18 + x^17 + x^16 + x^15 + x^14 + x^13 + x^12 + x^11
+ x^10 + x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1) (x^12 - x^11 + x^9 - x^8 + x^6
- x^4 + x^3 - x + 1)

```

G.f. as rational function with standard denominator.

```

> Rgfstandnum:=simplify(numer(Rgf)*standenom/denom(Rgf));
Rgf:=Rgfstandnum/standenom;

```

G.f. as rational function with standard denominator.

```

> sgfstandnum:=simplify(numer(sgf)*simplify(standenom/denom(sgf))
):
sgf:=sgfstandnum/standenom;

```

```

> sno7gfstandnum:=simplify(numer(sno7gf)*simplify
(stanno7denom/denom(sno7gf))):
sno7gf:=sno7gfstandnum/stanno7denom;

```

```

sno7gf:= 
$$\frac{1}{(1-x^{120})^5} (x^{10} (17 x^{18} + 22 x^{17} + 37 x^{16} + 38 x^{15} + 50 x^{14} + 43 x^{13} \\
+ 45 x^{12} + 38 x^{11} + 30 x^{10} + 21 x^9 + 12 x^8 + 6 x^7 - x^5 - x^4 + x^2 + x + 1) (1 - x \\
+ x^4 + x^8 - x^6 - x^{11} + x^{12} + x^{20} - x^{21} - x^{26} + x^{24} + x^{28} - x^{31} + x^{32} - x^{41} + x^{40})$$


```

$$\begin{aligned}
& + x^{112} - x^{91} + x^{84} + x^{72} + x^{64} - x^{66} + x^{108} + x^{48} - x^{71} - x^{106} + x^{92} + x^{60} - x^{111} \\
& + x^{104} - x^{101} + x^{100} + x^{88} - x^{86} - x^{81} + x^{80} + x^{68} - x^{61} + x^{52} - x^{51} - x^{46} + x^{44}) \\
& (1 + x^2 + x^4 + x^8 + x^6 + x^{14} + x^{10} + x^{12} + x^{20} + x^{16} + x^{18} + x^{26} + x^{22} + x^{24} + x^{28} \\
& + x^{32} + x^{30} + x^{34} + x^{36} + x^{40} + x^{38} + x^{112} + x^{98} + x^{84} + x^{70} + x^{56} + x^{42} + x^{72} + x^{64} \\
& + x^{90} + x^{96} + x^{66} + x^{54} + x^{50} + x^{102} + x^{108} + x^{48} + x^{106} + x^{114} + x^{92} + x^{60} + x^{118} \\
& + x^{116} + x^{110} + x^{104} + x^{100} + x^{94} + x^{88} + x^{86} + x^{82} + x^{80} + x^{78} + x^{76} + x^{74} + x^{68} \\
& + x^{62} + x^{58} + x^{52} + x^{46} + x^{44}) (1 + x^3 + x^6 + x^9 + x^{15} + x^{12} + x^{18} + x^{21} + x^{24} \\
& + x^{27} + x^{33} + x^{30} + x^{36} + x^{39} + x^{105} + x^{84} + x^{42} + x^{63} + x^{72} + x^{90} + x^{57} + x^{96} + x^{66} \\
& + x^{54} + x^{99} + x^{102} + x^{108} + x^{48} + x^{114} + x^{60} + x^{117} + x^{111} + x^{93} + x^{87} + x^{81} + x^{78} \\
& + x^{75} + x^{69} + x^{51} + x^{45})^2 (1 - x^2 + x^6 - x^{10} + x^{12} - x^{26} + x^{24} + x^{30} - x^{34} + x^{36} \\
& - x^{98} + x^{84} + x^{72} + x^{96} + x^{54} - x^{50} + x^{102} + x^{108} + x^{48} - x^{106} + x^{60} - x^{82} + x^{78} \\
& - x^{74} - x^{58})))
\end{aligned}$$

```

> s7gfstandnum:=simplify(numer(s7gf)*simplify(stan7denom/denom
(s7gf))):  

s7gf:=s7gfstandnum/stan7denom;  

s7gf:= 
$$\frac{1}{(1-x^{21})^2} (x^{10}(x^{20}+x^{19}+x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+x^{11}$$
  


$$+x^{10}+x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1)(x^{12}-x^{11}+x^9-x^8+x^6$$
  


$$-x^4+x^3-x+1))$$

```

G.f. as rational function with standard denominator.

```

> rgfstandnum:=simplify(numer(rgf)*standenom/denom(rgf)):  

rgf:=rgfstandnum/standenom;

```

Expand the series to find the first few values of the number of squares.

```

> Sseries:=series(Sgf,x=0,enddegree+1);  

Sseries:= 
$$72x^{15} + 144x^{16} + 288x^{17} + 576x^{18} + 864x^{19} + 1440x^{20} + O(x^{21})$$

```

Expand the series to find the first few values of the number of reduced squares.

```

> Rseries:=series(Rgf,x=0,enddegree+1);  

Rseries:= 
$$72x^{12} + 144x^{13} + 288x^{14} + 504x^{15} + 720x^{16} + 1152x^{17} + 1512x^{18}$$
  


$$+ 2160x^{19} + 2448x^{20} + O(x^{21})$$

```

Expand the series to find the first few values of the number of symmetry types.

```

> sseries:=series(sgf,x=0,enddegree+1);  

sseries:= 
$$x^{15} + 2x^{16} + 4x^{17} + 8x^{18} + 12x^{19} + 20x^{20} + O(x^{21})$$

```

Expand the series to find the first few values of the number of reduced symmetry types.

```

> rseries:=series(rgf,x=0,enddegree+1);
rseries:= $x^{12} + 2x^{13} + 4x^{14} + 7x^{15} + 10x^{16} + 16x^{17} + 21x^{18} + 30x^{19} + 34x^{20}$ 
+ O( $x^{21}$ )

```

Find the constituents

First, the true 0th constituent.

```

> Szeroth:=expand(
  sum( coeff(Sgfstandnum,x,p*j)*binomial(d+t/p-j,d) ,j=0..d+1) );
print(subs(t=0,Szeroth)):
Szeroth:= - $\frac{13896}{35}t + \frac{243}{4}t^2 - \frac{9}{2}t^3 + \frac{1}{8}t^4 + 1296$ 
1296

```

Second, the truncated constituents, with no H term (denominator power 7).

Calculate the zeroth constituent of the truncated semimagic counting function.
Find its constant term.

```

> Sno7zeroth:=expand(
  sum(coeff(Sno7gfstandnum,x,pno7*j)*binomial(d+t/pno7-j,d),j=0..
d+1) );
print(subs(t=0,Sno7zeroth)):
Sno7zeroth:= - $\frac{1968}{5}t + \frac{243}{4}t^2 - \frac{9}{2}t^3 + \frac{1}{8}t^4 + 1224$ 
1224

```

Extract the constituents of the truncated total semimagic counting function.

```

> Sno7constituent[0]:=Sno7zeroth:
for r from 1 to pno7 do
  Sno7constituent[r]:=expand(sum( coeff(Sno7gfstandnum,x,pno7*
j+r)*binomial(d+(t-r)/pno7-j,d), j=0..d));
# print(r):
# print( Sno7constituent[r] ):
  print( factor(Sno7constituent[r]) ):
od;

```

$$Sno7constituent_1 := -\frac{1158}{5}t + \frac{109}{2}t^2 - \frac{9}{2}t^3 + \frac{1}{8}t^4 + \frac{7259}{40}$$

$$\frac{1}{40}(t-1)(5t^3 - 175t^2 + 2005t - 7259)$$

$$Sno7constituent_2 := -\frac{1383}{5}t + \frac{227}{4}t^2 - \frac{9}{2}t^3 + \frac{1}{8}t^4 + \frac{1801}{5}$$

$$\frac{1}{40}(t-2)(5t^3 - 170t^2 + 1930t - 7204)$$

$$Sno7constituent_3 := -\frac{1653}{5} t + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{23067}{40}$$

$$\frac{1}{40} (t-3) (5 t^3 - 165 t^2 + 1845 t - 7689)$$

$$Sno7constituent_4 := -\frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2452}{5}$$

$$\frac{1}{40} (t-4) (5 t^3 - 160 t^2 + 1630 t - 4904)$$

$$Sno7constituent_5 := -\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2239}{8}$$

$$\frac{1}{40} (t-5) (5 t^3 - 155 t^2 + 1405 t - 2239)$$

$$Sno7constituent_6 := -\frac{1923}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{4653}{5}$$

$$\frac{1}{40} (t-6) (5 t^3 - 150 t^2 + 1530 t - 6204)$$

$$Sno7constituent_7 := -\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{5243}{40}$$

$$\frac{1}{40} (t-7) (5 t^3 - 145 t^2 + 1165 t - 749)$$

$$Sno7constituent_8 := -\frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2224}{5}$$

$$\frac{1}{40} (t-8) (5 t^3 - 140 t^2 + 1150 t - 2224)$$

$$Sno7constituent_9 := -\frac{1698}{5} t + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{31131}{40}$$

$$\frac{1}{40} (t-9) (5 t^3 - 135 t^2 + 1125 t - 3459)$$

$$Sno7constituent_{10} := 413 - \frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$413 - \frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{11} := -\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{539}{40}$$

$$\frac{1}{40} (t-11) (5 t^3 - 125 t^2 + 805 t - 49)$$

$$Sno7constituent_{12} := -\frac{1968}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{5796}{5} \frac{1}{40} (t-12) (5 t^3 - 120 t^2 + 990 t - 3864)$$

$$Sno7constituent_{13} := -\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{7547}{40} - \frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{7547}{40}$$

$$Sno7constituent_{14} := -\frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{1477}{5} \frac{1}{40} (t-14) (5 t^3 - 110 t^2 + 730 t - 844)$$

$$Sno7constituent_{15} := -\frac{1653}{5} t + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{5823}{8} - \frac{1653}{5} t + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{5823}{8}$$

$$Sno7constituent_{16} := -\frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2488}{5} - \frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2488}{5}$$

$$Sno7constituent_{17} := -\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{8603}{40} - \frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{8603}{40}$$

$$Sno7constituent_{18} := -\frac{1923}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{4689}{5} - \frac{1923}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{4689}{5}$$

$$Sno7constituent_{19} := -\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2651}{40} - \frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2651}{40}$$

$$Sno7constituent_{20} := 524 - \frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 \\ 524 - \frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{21} := -\frac{1698}{5} t + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{31419}{40}$$

$$-\frac{1698}{5} t + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{31419}{40}$$

$$Sno7constituent_{22} := -\frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{1741}{5}$$

$$-\frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{1741}{5}$$

$$Sno7constituent_{23} := -\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{827}{40}$$

$$-\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{827}{40}$$

$$Sno7constituent_{24} := -\frac{1968}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{5832}{5}$$

$$-\frac{1968}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{5832}{5}$$

$$Sno7constituent_{25} := -\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2143}{8}$$

$$-\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2143}{8}$$

$$Sno7constituent_{26} := -\frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{1513}{5}$$

$$-\frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{1513}{5}$$

$$Sno7constituent_{27} := -\frac{1653}{5} t + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{26523}{40}$$

$$-\frac{1653}{5} t + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{26523}{40}$$

$$Sno7constituent_{28} := -\frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2164}{5}$$

$$-\frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2164}{5}$$

$$Sno7constituent_{29} := -\frac{1158}{5} t + \frac{109}{2} t^2 + \frac{8891}{40} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1158}{5} t + \frac{109}{2} t^2 + \frac{8891}{40} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{30} := 1017 - \frac{1923}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$1017 - \frac{1923}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{31} := -\frac{1113}{5} t + \frac{109}{2} t^2 + \frac{2939}{40} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1113}{5} t + \frac{109}{2} t^2 + \frac{2939}{40} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{32} := -\frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2656}{5}$$

$$-\frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2656}{5}$$

$$Sno7constituent_{33} := -\frac{1698}{5} t + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{28827}{40}$$

$$-\frac{1698}{5} t + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{28827}{40}$$

$$Sno7constituent_{34} := -\frac{1383}{5} t + \frac{1777}{5} + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1383}{5} t + \frac{1777}{5} + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{35} := -\frac{1113}{5} t + \frac{799}{8} + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1113}{5} t + \frac{799}{8} + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{36} := -\frac{1968}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{5508}{5}$$

$$-\frac{1968}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{5508}{5}$$

$$Sno7constituent_{37} := -\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{11003}{40}$$

$$-\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{11003}{40}$$

$$Sno7constituent_{38} := -\frac{1383}{5} t + \frac{1189}{5} + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1383}{5} t + \frac{1189}{5} + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{39} := -\frac{1653}{5} t + \frac{117}{2} t^2 + \frac{26811}{40} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1653}{5} t + \frac{117}{2} t^2 + \frac{26811}{40} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{40} := 584 - \frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$584 - \frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{41} := -\frac{1158}{5} t + \frac{6299}{40} + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1158}{5} t + \frac{6299}{40} + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{42} := -\frac{1923}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{5121}{5}$$

$$-\frac{1923}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{5121}{5}$$

$$Sno7constituent_{43} := -\frac{1113}{5} t + \frac{109}{2} t^2 + \frac{347}{40} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1113}{5} t + \frac{109}{2} t^2 + \frac{347}{40} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{44} := -\frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2332}{5}$$

$$-\frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2332}{5}$$

$$Sno7constituent_{45} := -\frac{1698}{5} t + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{6975}{8}$$

$$-\frac{1698}{5} t + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{6975}{8}$$

$$Sno7constituent_{46} := -\frac{1383}{5} t + \frac{1453}{5} + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1383}{5} t + \frac{1453}{5} + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{47} := -\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{4283}{40}$$

$$-\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{4283}{40}$$

$$Sno7constituent_{48} := -\frac{1968}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{5544}{5}$$

$$-\frac{1968}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{5544}{5}$$

$$Sno7constituent_{49} := -\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{8411}{40}$$

$$-\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{8411}{40}$$

$$Sno7constituent_{50} := 389 - \frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$389 - \frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{51} := -\frac{1653}{5} t + \frac{24219}{40} + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1653}{5} t + \frac{24219}{40} + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{52} := -\frac{1428}{5} t + \frac{227}{4} t^2 + \frac{2596}{5} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1428}{5} t + \frac{227}{4} t^2 + \frac{2596}{5} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{53} := -\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{6587}{40}$$

$$-\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{6587}{40}$$

$$Sno7constituent_{54} := -\frac{1923}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{4797}{5}$$

$$-\frac{1923}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{4797}{5}$$

$$Sno7constituent_{55} := -\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{1279}{8}$$

$$-\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{1279}{8}$$

$$Sno7constituent_{56} := -\frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{2368}{5} + \frac{1}{8} t^4$$

$$-\frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{2368}{5} + \frac{1}{8} t^4$$

$$Sno7constituent_{57} := -\frac{1698}{5} t + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{32283}{40}$$

$$-\frac{1698}{5} t + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{32283}{40}$$

$$Sno7constituent_{58} := -\frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{1489}{5}$$

$$-\frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{1489}{5}$$

$$Sno7constituent_{59} := -\frac{1113}{5} t + \frac{1691}{40} + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1113}{5} t + \frac{1691}{40} + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{60} := 1188 - \frac{1968}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$1188 - \frac{1968}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{61} := -\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{8699}{40} + \frac{1}{8} t^4$$

$$-\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{8699}{40} + \frac{1}{8} t^4$$

$$Sno7constituent_{62} := -\frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{1621}{5}$$

$$-\frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{1621}{5}$$

$$Sno7constituent_{63} := -\frac{1653}{5} t + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{24507}{40}$$

$$-\frac{1653}{5} t + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{24507}{40}$$

$$Sno7constituent_{64} := -\frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2632}{5}$$

$$-\frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2632}{5}$$

$$Sno7constituent_{65} := -\frac{1158}{5} t + \frac{109}{2} t^2 + \frac{1951}{8} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1158}{5} t + \frac{109}{2} t^2 + \frac{1951}{8} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{66} := -\frac{1923}{5} t + \frac{243}{4} t^2 + \frac{4833}{5} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1923}{5} t + \frac{243}{4} t^2 + \frac{4833}{5} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{67} := -\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{3803}{40}$$

$$-\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{3803}{40}$$

$$Sno7constituent_{68} := -\frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2044}{5}$$

$$-\frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2044}{5}$$

$$Sno7constituent_{69} := -\frac{1698}{5} t + \frac{117}{2} t^2 + \frac{32571}{40} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1698}{5} t + \frac{117}{2} t^2 + \frac{32571}{40} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{70} := 377 - \frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$377 - \frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{71} := -\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{1979}{40}$$

$$-\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{1979}{40}$$

$$Sno7constituent_{72} := -\frac{1968}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{5976}{5}$$

$$-\frac{1968}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{5976}{5}$$

$$Sno7constituent_{73} := -\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{6107}{40}$$

$$-\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{6107}{40}$$

$$Sno7constituent_{74} := -\frac{1383}{5} t + \frac{227}{4} t^2 + \frac{1657}{5} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1383}{5} t + \frac{227}{4} t^2 + \frac{1657}{5} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{75} := -\frac{1653}{5} t + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{5535}{8} + \frac{1}{8} t^4$$

$$-\frac{1653}{5} t + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{5535}{8} + \frac{1}{8} t^4$$

$$Sno7constituent_{76} := -\frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2308}{5}$$

$$-\frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2308}{5}$$

$$Sno7constituent_{77} := -\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{10043}{40}$$

$$-\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{10043}{40}$$

$$Sno7constituent_{78} := -\frac{1923}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{4509}{5}$$

$$-\frac{1923}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{4509}{5}$$

$$Sno7constituent_{79} := -\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{4091}{40}$$

$$-\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{4091}{40}$$

$$Sno7constituent_{80} := 560 - \frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$560 - \frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{81} := -\frac{1698}{5} t + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{29979}{40}$$

$$-\frac{1698}{5} t + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{29979}{40}$$

$$Sno7constituent_{82} := -\frac{1383}{5} t + \frac{227}{4} t^2 + \frac{1921}{5} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1383}{5} t + \frac{227}{4} t^2 + \frac{1921}{5} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{83} := -\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 - \frac{613}{40}$$

$$-\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 - \frac{613}{40}$$

$$Sno7constituent_{84} := -\frac{1968}{5} t + \frac{5652}{5} + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1968}{5} t + \frac{5652}{5} + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{85} := -\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2431}{8}$$

$$-\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2431}{8}$$

$$Sno7constituent_{86} := -\frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{1333}{5}$$

$$-\frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{1333}{5}$$

$$Sno7constituent_{87} := -\frac{1653}{5} t + \frac{117}{2} t^2 + \frac{27963}{40} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1653}{5} t + \frac{117}{2} t^2 + \frac{27963}{40} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{88} := -\frac{1428}{5} t + \frac{227}{4} t^2 + \frac{2344}{5} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1428}{5} t + \frac{227}{4} t^2 + \frac{2344}{5} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{89} := -\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{7451}{40}$$

$$-\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{7451}{40}$$

$$Sno7constituent_{90} := 1053 - \frac{1923}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$1053 - \frac{1923}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{91} := -\frac{1113}{5} t + \frac{109}{2} t^2 + \frac{1499}{40} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1113}{5} t + \frac{109}{2} t^2 + \frac{1499}{40} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{92} := -\frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2476}{5}$$

$$-\frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2476}{5}$$

$$Sno7constituent_{93} := -\frac{1698}{5} t + \frac{117}{2} t^2 + \frac{30267}{40} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1698}{5} t + \frac{117}{2} t^2 + \frac{30267}{40} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{94} := -\frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1597}{5} + \frac{1}{8} t^4$$

$$-\frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1597}{5} + \frac{1}{8} t^4$$

$$Sno7constituent_{95} := -\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{1087}{8}$$

$$-\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{1087}{8}$$

$$Sno7constituent_{96} := -\frac{1968}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{5688}{5}$$

$$-\frac{1968}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{5688}{5}$$

$$Sno7constituent_{97} := -\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{9563}{40}$$

$$-\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{9563}{40}$$

$$Sno7constituent_{98} := -\frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{1369}{5}$$

$$-\frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{1369}{5}$$

$$Sno7constituent_{99} := -\frac{1653}{5} t + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{25371}{40}$$

$$-\frac{1653}{5} t + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{25371}{40}$$

$$Sno7constituent_{100} := 548 - \frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$548 - \frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{101} := -\frac{1158}{5} t + \frac{7739}{40} + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1158}{5} t + \frac{7739}{40} + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{102} := -\frac{1923}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{4941}{5}$$

$$-\frac{1923}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{4941}{5}$$

$$Sno7constituent_{103} := -\frac{1113}{5} t + \frac{109}{2} t^2 + \frac{1787}{40} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1113}{5} t + \frac{109}{2} t^2 + \frac{1787}{40} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{104} := -\frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2512}{5}$$

$$-\frac{1428}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2512}{5}$$

$$Sno7constituent_{105} := -\frac{1698}{5} t + \frac{6687}{8} + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1698}{5} t + \frac{6687}{8} + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{106} := -\frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{1633}{5}$$

$$-\frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{1633}{5}$$

$$Sno7constituent_{107} := -\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2843}{40}$$

$$-\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{2843}{40}$$

$$Sno7constituent_{108} := -\frac{1968}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{5364}{5} + \frac{1}{8} t^4$$

$$-\frac{1968}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{5364}{5} + \frac{1}{8} t^4$$

$$Sno7constituent_{109} := -\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{9851}{40}$$

$$-\frac{1158}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{9851}{40}$$

$$Sno7constituent_{110} := 353 - \frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$353 - \frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{111} := -\frac{1653}{5} t + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{25659}{40}$$

$$-\frac{1653}{5} t + \frac{117}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{25659}{40}$$

$$Sno7constituent_{112} := -\frac{1428}{5} t + \frac{227}{4} t^2 + \frac{2776}{5} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1428}{5} t + \frac{227}{4} t^2 + \frac{2776}{5} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{113} := -\frac{1158}{5} t + \frac{109}{2} t^2 + \frac{5147}{40} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1158}{5} t + \frac{109}{2} t^2 + \frac{5147}{40} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{114} := -\frac{1923}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{4977}{5}$$

$$-\frac{1923}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{4977}{5}$$

$$Sno7constituent_{115} := -\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{991}{8}$$

$$-\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{991}{8}$$

$$Sno7constituent_{116} := -\frac{1428}{5} t + \frac{227}{4} t^2 + \frac{2188}{5} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1428}{5} t + \frac{227}{4} t^2 + \frac{2188}{5} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{117} := -\frac{1698}{5} t + \frac{117}{2} t^2 + \frac{33723}{40} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$-\frac{1698}{5} t + \frac{117}{2} t^2 + \frac{33723}{40} - \frac{9}{2} t^3 + \frac{1}{8} t^4$$

$$Sno7constituent_{118} := -\frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{1309}{5}$$

$$-\frac{1383}{5} t + \frac{227}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{1309}{5}$$

$$Sno7constituent_{119} := -\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{3131}{40}$$

$$-\frac{1113}{5} t + \frac{109}{2} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + \frac{3131}{40}$$

$$\begin{aligned} Sno7constituent_{120} := & -\frac{1968}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + 1224 \\ & -\frac{1968}{5} t + \frac{243}{4} t^2 - \frac{9}{2} t^3 + \frac{1}{8} t^4 + 1224 \end{aligned}$$

Extract the coefficients of the constituents.%?

```
> for r from 1 to pno7 do
    for coeffdeg from 0 to d do
        Sno7c[coeffdeg,r]:=coeff(Sno7constituent[r],t,coeffdeg):
        #print( r, Sno7c[coeffdeg,r] ):
    od:
    od:
    ?
```

Print and analyze the constituent coefficients for periods. First the higher coefficients, which ought to be constant. Print the first coefficient, then any that don't repeat the preceding value.

```
> for coeffdeg from 3 to d do
    print("degree", coeffdeg, "coeff", Sno7c[coeffdeg,1]):
    print(1,Sno7c[coeffdeg,1]);
    for r from 2 to pno7 do
        stepdifference:=Sno7c[coeffdeg,r]-Sno7c[coeffdeg,r-1]:
        if( stepdifference<>0 ) then
            print(r,Sno7c[coeffdeg,r],stepdifference):
        fi:
    od:
    print("Compared all coefficients of degree", coeffdeg);
od:
```

$$\begin{aligned} & "degree", 3, "coeff", -\frac{9}{2} \\ & 1, -\frac{9}{2} \end{aligned}$$

"Compared all coefficients of degree", 3

$$"degree", 4, "coeff", \frac{1}{8}$$

$$1, \frac{1}{8}$$

"Compared all coefficients of degree", 4

Next, the constant terms, whose period is expected to be pno7. Print all constant terms up to the presumed period "stepsize". Print the difference (at step "stepsize") if they are not repeating.

Note that the even terms repeat at step 30 (a period of 15, half the expected period).

```
> stepsize:=30;
for r from 1 to stepsize do
    print(r, Sno7c[0,r]);
```

```

od:
for r from stepsize+1 to pno7 do
  stepdifference:=Sno7c[0,r]-Sno7c[0,r-stepsize]:
  if( stepdifference<>0 ) then print(r,Sno7c[0,r],
stepdifference): fi:
  print(r,Sno7c[0,r],stepdifference);
od:
print("Constant terms completed.");
stepsize:= 30

```

$$1, \frac{7259}{40}$$

$$2, \frac{1801}{5}$$

$$3, \frac{23067}{40}$$

$$4, \frac{2452}{5}$$

$$5, \frac{2239}{8}$$

$$6, \frac{4653}{5}$$

$$7, \frac{5243}{40}$$

$$8, \frac{2224}{5}$$

$$9, \frac{31131}{40}$$

$$10, 413$$

$$11, \frac{539}{40}$$

$$12, \frac{5796}{5}$$

$$13, \frac{7547}{40}$$

$$14, \frac{1477}{5}$$

$$15, \frac{5823}{8}$$

$$16, \frac{2488}{5}$$

$$17, \frac{8603}{40}$$

$$18, \frac{4689}{5}$$

$$19, \frac{2651}{40}$$

$$20, 524$$

$$21, \frac{31419}{40}$$

$$22, \frac{1741}{5}$$

$$23, \frac{827}{40}$$

$$24, \frac{5832}{5}$$

$$25, \frac{2143}{8}$$

$$26, \frac{1513}{5}$$

$$27, \frac{26523}{40}$$

$$28, \frac{2164}{5}$$

$$29, \frac{8891}{40}$$

$$30, 1017$$

$$31, \frac{2939}{40}, -108$$

$$31, \frac{2939}{40}, -108$$

$$32, \frac{2656}{5}, 171$$

$$32, \frac{2656}{5}, 171$$

$$33, \frac{28827}{40}, 144$$

$$33, \frac{28827}{40}, 144$$

$$34, \frac{1777}{5}, -135$$

$$34, \frac{1777}{5}, -135$$

$$35, \frac{799}{8}, -180$$

$$35, \frac{799}{8}, -180$$

$$36, \frac{5508}{5}, 171$$

$$36, \frac{5508}{5}, 171$$

$$37, \frac{11003}{40}, 144$$

$$37, \frac{11003}{40}, 144$$

$$38, \frac{1189}{5}, -207$$

$$38, \frac{1189}{5}, -207$$

$$39, \frac{26811}{40}, -108$$

$$39, \frac{26811}{40}, -108$$

$$40, 584, 171$$

$$40, 584, 171$$

$$41, \frac{6299}{40}, 144$$

$$41, \frac{6299}{40}, 144$$

$$42, \frac{5121}{5}, -135$$

$$42, \frac{5121}{5}, -135$$

$$43, \frac{347}{40}, -180$$

$$43, \frac{347}{40}, -180$$

$$44, \frac{2332}{5}, 171$$

$$44, \frac{2332}{5}, 171$$

$$45, \frac{6975}{8}, 144$$

$$45, \frac{6975}{8}, 144$$

$$46, \frac{1453}{5}, -207$$

$$46, \frac{1453}{5}, -207$$

$$47, \frac{4283}{40}, -108$$

$$47, \frac{4283}{40}, -108$$

$$48, \frac{5544}{5}, 171$$

$$48, \frac{5544}{5}, 171$$

$$49, \frac{8411}{40}, 144$$

$$49, \frac{8411}{40}, 144$$

$$50, 389, -135$$

$$50, 389, -135$$

$$51, \frac{24219}{40}, -180$$

$$51, \frac{24219}{40}, -180$$

$$52, \frac{2596}{5}, 171$$

$$52, \frac{2596}{5}, 171$$

$$53, \frac{6587}{40}, 144$$

$$53, \frac{6587}{40}, 144$$

$$54, \frac{4797}{5}, -207$$

$$54, \frac{4797}{5}, -207$$

$$55, \frac{1279}{8}, -108$$

$$55, \frac{1279}{8}, -108$$

$$56, \frac{2368}{5}, 171$$

$$56, \frac{2368}{5}, 171$$

$$57, \frac{32283}{40}, 144$$

$$57, \frac{32283}{40}, 144$$

$$58, \frac{1489}{5}, -135$$

$$58, \frac{1489}{5}, -135$$

$$59, \frac{1691}{40}, -180$$

$$59, \frac{1691}{40}, -180$$

$$60, 1188, 171$$

$$60, 1188, 171$$

$$61, \frac{8699}{40}, 144$$

$$61, \frac{8699}{40}, 144$$

$$62, \frac{1621}{5}, -207$$

$$62, \frac{1621}{5}, -207$$

$$63, \frac{24507}{40}, -108$$

$$63, \frac{24507}{40}, -108$$

$$64, \frac{2632}{5}, 171$$

$$64, \frac{2632}{5}, 171$$

$$65, \frac{1951}{8}, 144$$

$$65, \frac{1951}{8}, 144$$

$$66, \frac{4833}{5}, -135$$

$$66, \frac{4833}{5}, -135$$

$$67, \frac{3803}{40}, -180$$

$$67, \frac{3803}{40}, -180$$

$$68, \frac{2044}{5}, 171$$

$$68, \frac{2044}{5}, 171$$

$$69, \frac{32571}{40}, 144$$

$$69, \frac{32571}{40}, 144$$

$$70, 377, -207$$

$$70, 377, -207$$

$$71, \frac{1979}{40}, -108$$

$$71, \frac{1979}{40}, -108$$

$$72, \frac{5976}{5}, 171$$

$$72, \frac{5976}{5}, 171$$

$$73, \frac{6107}{40}, 144$$

$$73, \frac{6107}{40}, 144$$

$$74, \frac{1657}{5}, -135$$

$$74, \frac{1657}{5}, -135$$

$$75, \frac{5535}{8}, -180$$

$$75, \frac{5535}{8}, -180$$

$$76, \frac{2308}{5}, 171$$

$$76, \frac{2308}{5}, 171$$

$$77, \frac{10043}{40}, 144$$

$$77, \frac{10043}{40}, 144$$

$$78, \frac{4509}{5}, -207$$

$$78, \frac{4509}{5}, -207$$

$$79, \frac{4091}{40}, -108$$

$$79, \frac{4091}{40}, -108$$

$$80, 560, 171$$

$$80, \frac{560}{40}, 171$$

$$81, \frac{29979}{40}, 144$$

$$81, \frac{29979}{40}, 144$$

$$82, \frac{1921}{5}, -135$$

$$82, \frac{1921}{5}, -135$$

$$83, -\frac{613}{40}, -180$$

$$83, -\frac{613}{40}, -180$$

$$84, \frac{5652}{5}, 171$$

$$84, \frac{5652}{5}, 171$$

$$85, \frac{2431}{8}, 144$$

$$85, \frac{2431}{8}, 144$$

$$86, \frac{1333}{5}, -207$$

$$86, \frac{1333}{5}, -207$$

$$87, \frac{27963}{40}, -108$$

$$87, \frac{27963}{40}, -108$$

$$88, \frac{2344}{5}, 171$$

$$88, \frac{2344}{5}, 171$$

$$89, \frac{7451}{40}, 144$$

$$89, \frac{7451}{40}, 144$$

- 90, 1053, -135
90, 1053, -135
91, $\frac{1499}{40}$, -180
91, $\frac{1499}{40}$, -180
92, $\frac{2476}{5}$, 171
92, $\frac{2476}{5}$, 171
93, $\frac{30267}{40}$, 144
93, $\frac{30267}{40}$, 144
94, $\frac{1597}{5}$, -207
94, $\frac{1597}{5}$, -207
95, $\frac{1087}{8}$, -108
95, $\frac{1087}{8}$, -108
96, $\frac{5688}{5}$, 171
96, $\frac{5688}{5}$, 171
97, $\frac{9563}{40}$, 144
97, $\frac{9563}{40}$, 144
98, $\frac{1369}{5}$, -135
98, $\frac{1369}{5}$, -135
99, $\frac{25371}{40}$, -180

$$99, \frac{25371}{40}, -180$$

$$100, 548, 171$$

$$100, 548, 171$$

$$101, \frac{7739}{40}, 144$$

$$101, \frac{7739}{40}, 144$$

$$102, \frac{4941}{5}, -207$$

$$102, \frac{4941}{5}, -207$$

$$103, \frac{1787}{40}, -108$$

$$103, \frac{1787}{40}, -108$$

$$104, \frac{2512}{5}, 171$$

$$104, \frac{2512}{5}, 171$$

$$105, \frac{6687}{8}, 144$$

$$105, \frac{6687}{8}, 144$$

$$106, \frac{1633}{5}, -135$$

$$106, \frac{1633}{5}, -135$$

$$107, \frac{2843}{40}, -180$$

$$107, \frac{2843}{40}, -180$$

$$108, \frac{5364}{5}, 171$$

$$108, \frac{5364}{5}, 171$$

$$109, \frac{9851}{40}, 144$$

$$109, \frac{9851}{40}, 144$$

$$110, 353, -207$$

$$110, 353, -207$$

$$111, \frac{25659}{40}, -108$$

$$111, \frac{25659}{40}, -108$$

$$112, \frac{2776}{5}, 171$$

$$112, \frac{2776}{5}, 171$$

$$113, \frac{5147}{40}, 144$$

$$113, \frac{5147}{40}, 144$$

$$114, \frac{4977}{5}, -135$$

$$114, \frac{4977}{5}, -135$$

$$115, \frac{991}{8}, -180$$

$$115, \frac{991}{8}, -180$$

$$116, \frac{2188}{5}, 171$$

$$116, \frac{2188}{5}, 171$$

$$117, \frac{33723}{40}, 144$$

$$117, \frac{33723}{40}, 144$$

$$118, \frac{1309}{5}, -207$$

$$118, \frac{1309}{5}, -207$$

$$119, \frac{3131}{40}, -108$$

$$119, \frac{3131}{40}, -108$$

$$120, 1224, 171$$

$$120, 1224, 171$$

"Constant terms completed."

Now, the linear terms. First print all linear coefficients up to the presumed period "stepsize" .. Then analyze for period and print the difference (at step "stepsize") if they are not repeating.

```
> stepsize:=6;
for r from 1 to stepsize do
  print(r, Sno7c[1,r]);
od;
for r from stepsize+1 to pno7 do
  stepdifference:=Sno7c[1,r]-Sno7c[1,r-stepsize];
  if( stepdifference<>0 ) then print(r,Sno7c[1,r],
stepdifference); fi;
od;
print("Sno7linear coefficients completed.");
stepsize:= 6
```

$$1, -\frac{1158}{5}$$

$$2, -\frac{1383}{5}$$

$$3, -\frac{1653}{5}$$

$$4, -\frac{1428}{5}$$

$$5, -\frac{1158}{5}$$

$$6, -\frac{1923}{5}$$

$$7, -\frac{1113}{5}, 9$$

$$8, -\frac{1428}{5}, -9$$

$$9, -\frac{1698}{5}, -9$$

$$10, -\frac{1383}{5}, 9$$

$$11, -\frac{1113}{5}, 9$$

$$12, -\frac{1968}{5}, -9$$

$$13, -\frac{1158}{5}, -9$$

$$14, -\frac{1383}{5}, 9$$

$$15, -\frac{1653}{5}, 9$$

$$16, -\frac{1428}{5}, -9$$

$$17, -\frac{1158}{5}, -9$$

$$18, -\frac{1923}{5}, 9$$

$$19, -\frac{1113}{5}, 9$$

$$20, -\frac{1428}{5}, -9$$

$$21, -\frac{1698}{5}, -9$$

$$22, -\frac{1383}{5}, 9$$

$$23, -\frac{1113}{5}, 9$$

$$24, -\frac{1968}{5}, -9$$

$$25, -\frac{1158}{5}, -9$$

$$26, -\frac{1383}{5}, 9$$

$$27, -\frac{1653}{5}, 9$$

$$28, -\frac{1428}{5}, -9$$

$$29, -\frac{1158}{5}, -9$$

$$30, -\frac{1923}{5}, 9$$

$$31, -\frac{1113}{5}, 9$$

$$32, -\frac{1428}{5}, -9$$

$$33, -\frac{1698}{5}, -9$$

$$34, -\frac{1383}{5}, 9$$

$$35, -\frac{1113}{5}, 9$$

$$36, -\frac{1968}{5}, -9$$

$$37, -\frac{1158}{5}, -9$$

$$38, -\frac{1383}{5}, 9$$

$$39, -\frac{1653}{5}, 9$$

$$40, -\frac{1428}{5}, -9$$

$$41, -\frac{1158}{5}, -9$$

$$42, -\frac{1923}{5}, 9$$

$$43, -\frac{1113}{5}, 9$$

$$44, -\frac{1428}{5}, -9$$

$$45, -\frac{1698}{5}, -9$$

$$46, -\frac{1383}{5}, 9$$

$$47, -\frac{1113}{5}, 9$$

$$48, -\frac{1968}{5}, -9$$

$$49, -\frac{1158}{5}, -9$$

$$50, -\frac{1383}{5}, 9$$

$$51, -\frac{1653}{5}, 9$$

$$52, -\frac{1428}{5}, -9$$

$$53, -\frac{1158}{5}, -9$$

$$54, -\frac{1923}{5}, 9$$

$$55, -\frac{1113}{5}, 9$$

$$56, -\frac{1428}{5}, -9$$

$$57, -\frac{1698}{5}, -9$$

$$58, -\frac{1383}{5}, 9$$

$$59, -\frac{1113}{5}, 9$$

$$60, -\frac{1968}{5}, -9$$

$$61, -\frac{1158}{5}, -9$$

$$62, -\frac{1383}{5}, 9$$

$$63, -\frac{1653}{5}, 9$$

$$64, -\frac{1428}{5}, -9$$

$$65, -\frac{1158}{5}, -9$$

$$66, -\frac{1923}{5}, 9$$

$$67, -\frac{1113}{5}, 9$$

$$68, -\frac{1428}{5}, -9$$

$$69, -\frac{1698}{5}, -9$$

$$70, -\frac{1383}{5}, 9$$

$$71, -\frac{1113}{5}, 9$$

$$72, -\frac{1968}{5}, -9$$

$$73, -\frac{1158}{5}, -9$$

$$74, -\frac{1383}{5}, 9$$

$$75, -\frac{1653}{5}, 9$$

$$76, -\frac{1428}{5}, -9$$

$$77, -\frac{1158}{5}, -9$$

$$78, -\frac{1923}{5}, 9$$

$$79, -\frac{1113}{5}, 9$$

$$80, -\frac{1428}{5}, -9$$

$$81, -\frac{1698}{5}, -9$$

$$82, -\frac{1383}{5}, 9$$

$$83, -\frac{1113}{5}, 9$$

$$84, -\frac{1968}{5}, -9$$

$$85, -\frac{1158}{5}, -9$$

$$86, -\frac{1383}{5}, 9$$

$$87, -\frac{1653}{5}, 9$$

$$88, -\frac{1428}{5}, -9$$

$$89, -\frac{1158}{5}, -9$$

$$90, -\frac{1923}{5}, 9$$

$$91, -\frac{1113}{5}, 9$$

$$92, -\frac{1428}{5}, -9$$

$$93, -\frac{1698}{5}, -9$$

$$94, -\frac{1383}{5}, 9$$

$$95, -\frac{1113}{5}, 9$$

$$96, -\frac{1968}{5}, -9$$

$$97, -\frac{1158}{5}, -9$$

$$98, -\frac{1383}{5}, 9$$

$$99, -\frac{1653}{5}, 9$$

$$100, -\frac{1428}{5}, -9$$

$$101, -\frac{1158}{5}, -9$$

$$102, -\frac{1923}{5}, 9$$

$$103, -\frac{1113}{5}, 9$$

$$104, -\frac{1428}{5}, -9$$

$$105, -\frac{1698}{5}, -9$$

$$106, -\frac{1383}{5}, 9$$

$$107, -\frac{1113}{5}, 9$$

$$108, -\frac{1968}{5}, -9$$

$$109, -\frac{1158}{5}, -9$$

$$110, -\frac{1383}{5}, 9$$

$$111, -\frac{1653}{5}, 9$$

$$112, -\frac{1428}{5}, -9$$

$$113, -\frac{1158}{5}, -9$$

$$114, -\frac{1923}{5}, 9$$

$$115, -\frac{1113}{5}, 9$$

$$116, -\frac{1428}{5}, -9$$

$$117, -\frac{1698}{5}, -9$$

$$118, -\frac{1383}{5}, 9$$

$$119, -\frac{1113}{5}, 9$$

$$120, -\frac{1968}{5}, -9$$

"Sno7inear coefficients completed."

The quadratic terms. First print all quadratic coefficients up to the presumed period "stepsize".... Then analyze for period and print the difference (at step "stepsize") if they are not repeating.

```
> stepsize:=2;
  for r from 1 to stepsize do
    print(r, Sno7c[2,r]);
  od:
  for r from stepsize+1 to pno7 do
    stepdifference:=Sno7c[2,r]-Sno7c[2,r-stepsize]:
    if( stepdifference<>0 ) then print(r,Sno7c[2,r],
    stepdifference): fi:
  od:
print("Quadratic coefficients completed.");
stepsize:= 2
```

$$1, \frac{109}{2}$$

$$2, \frac{227}{4}$$

$$3, \frac{117}{2}, 4$$

$$5, \frac{109}{2}, -4$$

$$6, \frac{243}{4}, 4$$

$$8, \frac{227}{4}, -4$$

$$9, \frac{117}{2}, 4$$

$$11, \frac{109}{2}, -4$$

$$12, \frac{243}{4}, 4$$

$$14, \frac{227}{4}, -4$$

$$15, \frac{117}{2}, 4$$

$$17, \frac{109}{2}, -4$$

$$18, \frac{243}{4}, 4$$

$$20, \frac{227}{4}, -4$$

$$21, \frac{117}{2}, 4$$

$$23, \frac{109}{2}, -4$$

$$24, \frac{243}{4}, 4$$

$$26, \frac{227}{4}, -4$$

$$27, \frac{117}{2}, 4$$

$$29, \frac{109}{2}, -4$$

$$30, \frac{243}{4}, 4$$

$$32, \frac{227}{4}, -4$$

$$33, \frac{117}{2}, 4$$

$$35, \frac{109}{2}, -4$$

$$36, \frac{243}{4}, 4$$

$$38, \frac{227}{4}, -4$$

$$39, \frac{117}{2}, 4$$

$$41, \frac{109}{2}, -4$$

$$42, \frac{243}{4}, 4$$

$$44, \frac{227}{4}, -4$$

$$45, \frac{117}{2}, 4$$

$$47, \frac{109}{2}, -4$$

$$48, \frac{243}{4}, 4$$

$$50, \frac{227}{4}, -4$$

$$51, \frac{117}{2}, 4$$

$$53, \frac{109}{2}, -4$$

$$54, \frac{243}{4}, 4$$

$$56, \frac{227}{4}, -4$$

$$57, \frac{117}{2}, 4$$

$$59, \frac{109}{2}, -4$$

$$60, \frac{243}{4}, 4$$

$$62, \frac{227}{4}, -4$$

$$63, \frac{117}{2}, 4$$

$$65, \frac{109}{2}, -4$$

$$66, \frac{243}{4}, 4$$

$$68, \frac{227}{4}, -4$$

$$69, \frac{117}{2}, 4$$

$$71, \frac{109}{2}, -4$$

$$72, \frac{243}{4}, 4$$

$$74, \frac{227}{4}, -4$$

$$75, \frac{117}{2}, 4$$

$$77, \frac{109}{2}, -4$$

$$78, \frac{243}{4}, 4$$

$$80, \frac{227}{4}, -4$$

$$81, \frac{117}{2}, 4$$

$$83, \frac{109}{2}, -4$$

$$84, \frac{243}{4}, 4$$

$$86, \frac{227}{4}, -4$$

$$87, \frac{117}{2}, 4$$

$$89, \frac{109}{2}, -4$$

$$90, \frac{243}{4}, 4$$

$$92, \frac{227}{4}, -4$$

$$93, \frac{117}{2}, 4$$

$$95, \frac{109}{2}, -4$$

$$96, \frac{243}{4}, 4$$

$$98, \frac{227}{4}, -4$$

$$99, \frac{117}{2}, 4$$

$$101, \frac{109}{2}, -4$$

$$102, \frac{243}{4}, 4$$

$$104, \frac{227}{4}, -4$$

$$105, \frac{117}{2}, 4$$

$$107, \frac{109}{2}, -4$$

$$108, \frac{243}{4}, 4$$

$$110, \frac{227}{4}, -4$$

$$111, \frac{117}{2}, 4$$

$$113, \frac{109}{2}, -4$$

$$114, \frac{243}{4}, 4$$

$$116, \frac{227}{4}, -4$$

$$117, \frac{117}{2}, 4$$

$$119, \frac{109}{2}, -4$$

$$120, \frac{243}{4}, 4$$

"Quadratic coefficients completed."

Calculate the zeroth constituent of the **truncated semimagic symmetry-type counting function**. Find its constant term.

```
> sno7zeroth:=expand(
  sum(coeff(sno7gfstandnum,x,pno7*j)*binomial(d+t/pno7-j,d),j=0..
d+1) );
print(subs(t=0,sno7zeroth));
sno7zeroth:=-\frac{82}{15} t+\frac{27}{32} t^2-\frac{1}{16} t^3+\frac{1}{576} t^4+17
```

17

Extract the constituents of the **truncated semimagic symmetry-type counting function**.

```
> sno7constituent[0]:=sno7zeroth;
for r from 1 to pno7 do
  sno7constituent[r]:=expand(sum( coeff(sno7gfstandnum,x,pno7*
j+r)*binomial(d+(t-r)/pno7-j,d), j=0..d));
# print(r);
# print( sno7constituent[r] );
print( factor(sno7constituent[r]) );
od;
sno7constituent1:= -\frac{193}{60} t+\frac{109}{144} t^2-\frac{1}{16} t^3+\frac{1}{576} t^4+\frac{7259}{2880}
```

$$\frac{1}{2880} (t-1) (5 t^3 - 175 t^2 + 2005 t - 7259)$$

```
sno7constituent2:= -\frac{461}{120} t+\frac{227}{288} t^2-\frac{1}{16} t^3+\frac{1}{576} t^4+\frac{1801}{360}
```

$$\frac{1}{2880} (t-2) (5 t^3 - 170 t^2 + 1930 t - 7204)$$

```
sno7constituent3:= -\frac{551}{120} t+\frac{13}{16} t^2-\frac{1}{16} t^3+\frac{1}{576} t^4+\frac{2563}{320}
```

$$\frac{1}{2880} (t-3) (5 t^3 - 165 t^2 + 1845 t - 7689)$$

```
sno7constituent4:= -\frac{119}{30} t+\frac{227}{288} t^2-\frac{1}{16} t^3+\frac{1}{576} t^4+\frac{613}{90}
```

$$\frac{1}{2880} (t-4) (5 t^3 - 160 t^2 + 1630 t - 4904)$$

```
sno7constituent5:= -\frac{193}{60} t+\frac{109}{144} t^2-\frac{1}{16} t^3+\frac{1}{576} t^4+\frac{2239}{576}
```

$$\frac{1}{2880} (t-5) (5 t^3 - 155 t^2 + 1405 t - 2239)$$

$$sno7constituent_6 := -\frac{641}{120} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{517}{40}$$

$$\frac{1}{2880} (t-6) (5 t^3 - 150 t^2 + 1530 t - 6204)$$

$$sno7constituent_7 := -\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{5243}{2880}$$

$$\frac{1}{2880} (t-7) (5 t^3 - 145 t^2 + 1165 t - 749)$$

$$sno7constituent_8 := -\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{278}{45}$$

$$\frac{1}{2880} (t-8) (5 t^3 - 140 t^2 + 1150 t - 2224)$$

$$sno7constituent_9 := -\frac{283}{60} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{3459}{320}$$

$$\frac{1}{2880} (t-9) (5 t^3 - 135 t^2 + 1125 t - 3459)$$

$$sno7constituent_{10} := -\frac{461}{120} t + \frac{227}{288} t^2 + \frac{413}{72} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{461}{120} t + \frac{227}{288} t^2 + \frac{413}{72} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{11} := -\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{539}{2880}$$

$$\frac{1}{2880} (t-11) (5 t^3 - 125 t^2 + 805 t - 49)$$

$$sno7constituent_{12} := -\frac{82}{15} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{161}{10}$$

$$\frac{1}{2880} (t-12) (5 t^3 - 120 t^2 + 990 t - 3864)$$

$$sno7constituent_{13} := -\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{7547}{2880}$$

$$-\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{7547}{2880}$$

$$sno7constituent_{14} := -\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1477}{360}$$

$$\frac{1}{2880} (t-14) (5 t^3 - 110 t^2 + 730 t - 844)$$

$$sno7constituent_{15} := -\frac{551}{120} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{647}{64}$$

$$-\frac{551}{120} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{647}{64}$$

$$sno7constituent_{16} := -\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{311}{45}$$

$$-\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{311}{45}$$

$$sno7constituent_{17} := -\frac{193}{60} t + \frac{109}{144} t^2 + \frac{8603}{2880} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{193}{60} t + \frac{109}{144} t^2 + \frac{8603}{2880} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{18} := -\frac{641}{120} t + \frac{27}{32} t^2 + \frac{521}{40} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{641}{120} t + \frac{27}{32} t^2 + \frac{521}{40} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{19} := -\frac{371}{120} t + \frac{109}{144} t^2 + \frac{2651}{2880} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{371}{120} t + \frac{109}{144} t^2 + \frac{2651}{2880} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{20} := -\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{131}{18}$$

$$-\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{131}{18}$$

$$sno7constituent_{21} := -\frac{283}{60} t + \frac{3491}{320} + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{283}{60} t + \frac{3491}{320} + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{22} := -\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1741}{360} + \frac{1}{576} t^4$$

$$-\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1741}{360} + \frac{1}{576} t^4$$

$$sno7constituent_{23} := -\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{827}{2880} + \frac{1}{576} t^4$$

$$-\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{827}{2880} + \frac{1}{576} t^4$$

$$sno7constituent_{24} := -\frac{82}{15} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{81}{5}$$

$$-\frac{82}{15} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{81}{5}$$

$$sno7constituent_{25} := -\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{2143}{576}$$

$$-\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{2143}{576}$$

$$sno7constituent_{26} := -\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1513}{360}$$

$$-\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1513}{360}$$

$$sno7constituent_{27} := -\frac{551}{120} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{2947}{320}$$

$$-\frac{551}{120} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{2947}{320}$$

$$sno7constituent_{28} := -\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{541}{90}$$

$$-\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{541}{90}$$

$$sno7constituent_{29} := -\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{8891}{2880} + \frac{1}{576} t^4$$

$$-\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{8891}{2880} + \frac{1}{576} t^4$$

$$sno7constituent_{30} := -\frac{641}{120} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{113}{8}$$

$$-\frac{641}{120} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{113}{8}$$

$$sno7constituent_{31} := -\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{2939}{2880}$$

$$-\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{2939}{2880}$$

$$sno7constituent_{32} := -\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{332}{45}$$

$$-\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{332}{45}$$

$$sno7constituent_{33} := -\frac{283}{60} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{3203}{320}$$

$$-\frac{283}{60} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{3203}{320}$$

$$sno7constituent_{34} := -\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1777}{360}$$

$$-\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1777}{360}$$

$$sno7constituent_{35} := -\frac{371}{120} t + \frac{109}{144} t^2 + \frac{799}{576} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{371}{120} t + \frac{109}{144} t^2 + \frac{799}{576} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{36} := -\frac{82}{15} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{153}{10}$$

$$-\frac{82}{15} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{153}{10}$$

$$sno7constituent_{37} := -\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{11003}{2880}$$

$$-\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{11003}{2880}$$

$$sno7constituent_{38} := -\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1189}{360}$$

$$-\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1189}{360}$$

$$sno7constituent_{39} := -\frac{551}{120} t + \frac{13}{16} t^2 + \frac{2979}{320} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{551}{120} t + \frac{13}{16} t^2 + \frac{2979}{320} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{40} := -\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{73}{9}$$

$$-\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{73}{9}$$

$$sno7constituent_{41} := -\frac{193}{60} t + \frac{109}{144} t^2 + \frac{6299}{2880} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{193}{60} t + \frac{109}{144} t^2 + \frac{6299}{2880} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{42} := -\frac{641}{120} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{569}{40}$$

$$-\frac{641}{120} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{569}{40}$$

$$sno7constituent_{43} := -\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{347}{2880}$$

$$-\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{347}{2880}$$

$$sno7constituent_{44} := -\frac{119}{30} t + \frac{227}{288} t^2 + \frac{583}{90} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{119}{30} t + \frac{227}{288} t^2 + \frac{583}{90} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{45} := -\frac{283}{60} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{775}{64}$$

$$-\frac{283}{60} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{775}{64}$$

$$sno7constituent_{46} := -\frac{461}{120} t + \frac{227}{288} t^2 + \frac{1453}{360} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{461}{120} t + \frac{227}{288} t^2 + \frac{1453}{360} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{47} := -\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{4283}{2880}$$

$$-\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{4283}{2880}$$

$$sno7constituent_{48} := -\frac{82}{15} t + \frac{77}{5} + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{82}{15} t + \frac{77}{5} + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{49} := -\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{8411}{2880}$$

$$-\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{8411}{2880}$$

$$sno7constituent_{50} := -\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{389}{72}$$

$$-\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{389}{72}$$

$$sno7constituent_{51} := -\frac{551}{120} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{2691}{320}$$

$$-\frac{551}{120} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{2691}{320}$$

$$sno7constituent_{52} := -\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{649}{90}$$

$$-\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{649}{90}$$

$$sno7constituent_{53} := -\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{6587}{2880}$$

$$-\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{6587}{2880}$$

$$sno7constituent_{54} := -\frac{641}{120} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{533}{40}$$

$$-\frac{641}{120} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{533}{40}$$

$$sno7constituent_{55} := -\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1279}{576}$$

$$-\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1279}{576}$$

$$sno7constituent_{56} := -\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{296}{45}$$

$$-\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{296}{45}$$

$$sno7constituent_{57} := -\frac{283}{60} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{3587}{320}$$

$$-\frac{283}{60} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{3587}{320}$$

$$sno7constituent_{58} := -\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1489}{360}$$

$$-\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1489}{360}$$

$$sno7constituent_{59} := -\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1691}{2880}$$

$$-\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1691}{2880}$$

$$sno7constituent_{60} := \frac{33}{2} - \frac{82}{15} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$\frac{33}{2} - \frac{82}{15} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{61} := -\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{8699}{2880}$$

$$-\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{8699}{2880}$$

$$sno7constituent_{62} := -\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1621}{360}$$

$$-\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1621}{360}$$

$$sno7constituent_{63} := -\frac{551}{120} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{2723}{320}$$

$$-\frac{551}{120} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{2723}{320}$$

$$sno7constituent_{64} := -\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{329}{45}$$

$$-\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{329}{45}$$

$$sno7constituent_{65} := -\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1951}{576}$$

$$-\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1951}{576}$$

$$sno7constituent_{66} := -\frac{641}{120} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{537}{40}$$

$$-\frac{641}{120} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{537}{40}$$

$$sno7constituent_{67} := -\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{3803}{2880}$$

$$-\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{3803}{2880}$$

$$sno7constituent_{68} := -\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{511}{90}$$

$$-\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{511}{90}$$

$$sno7constituent_{69} := -\frac{283}{60} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{3619}{320}$$

$$-\frac{283}{60} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{3619}{320}$$

$$sno7constituent_{70} := -\frac{461}{120} t + \frac{227}{288} t^2 + \frac{377}{72} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{461}{120} t + \frac{227}{288} t^2 + \frac{377}{72} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{71} := -\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1979}{2880}$$

$$-\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1979}{2880}$$

$$sno7constituent_{72} := -\frac{82}{15} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{83}{5} + \frac{1}{576} t^4$$

$$-\frac{82}{15} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{83}{5} + \frac{1}{576} t^4$$

$$sno7constituent_{73} := -\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{6107}{2880}$$

$$-\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{6107}{2880}$$

$$sno7constituent_{74} := -\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1657}{360}$$

$$-\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1657}{360}$$

$$sno7constituent_{75} := -\frac{551}{120} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{615}{64}$$

$$-\frac{551}{120} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{615}{64}$$

$$sno7constituent_{76} := -\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{577}{90}$$

$$-\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{577}{90}$$

$$sno7constituent_{77} := -\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{10043}{2880}$$

$$-\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{10043}{2880}$$

$$sno7constituent_{78} := -\frac{641}{120} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{501}{40}$$

$$-\frac{641}{120} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{501}{40}$$

$$sno7constituent_{79} := -\frac{371}{120} t + \frac{109}{144} t^2 + \frac{4091}{2880} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{371}{120} t + \frac{109}{144} t^2 + \frac{4091}{2880} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{80} := -\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{70}{9}$$

$$-\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{70}{9}$$

$$sno7constituent_{81} := -\frac{283}{60} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{3331}{320}$$

$$-\frac{283}{60} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{3331}{320}$$

$$sno7constituent_{82} := -\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1921}{360}$$

$$-\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1921}{360}$$

$$sno7constituent_{83} := -\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 - \frac{613}{2880}$$

$$-\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 - \frac{613}{2880}$$

$$sno7constituent_{84} := -\frac{82}{15} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{157}{10}$$

$$-\frac{82}{15} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{157}{10}$$

$$sno7constituent_{85} := -\frac{193}{60} t + \frac{109}{144} t^2 + \frac{2431}{576} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{193}{60} t + \frac{109}{144} t^2 + \frac{2431}{576} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{86} := -\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1333}{360}$$

$$-\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1333}{360}$$

$$sno7constituent_{87} := -\frac{551}{120} t + \frac{13}{16} t^2 + \frac{3107}{320} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{551}{120} t + \frac{13}{16} t^2 + \frac{3107}{320} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{88} := -\frac{119}{30} t + \frac{227}{288} t^2 + \frac{293}{45} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{119}{30} t + \frac{227}{288} t^2 + \frac{293}{45} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{89} := -\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{7451}{2880}$$

$$-\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{7451}{2880}$$

$$sno7constituent_{90} := -\frac{641}{120} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{117}{8}$$

$$-\frac{641}{120} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{117}{8}$$

$$sno7constituent_{91} := -\frac{371}{120} t + \frac{109}{144} t^2 + \frac{1499}{2880} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{371}{120} t + \frac{109}{144} t^2 + \frac{1499}{2880} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{92} := -\frac{119}{30} t + \frac{227}{288} t^2 + \frac{619}{90} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{119}{30} t + \frac{227}{288} t^2 + \frac{619}{90} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{93} := -\frac{283}{60} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{3363}{320}$$

$$-\frac{283}{60} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{3363}{320}$$

$$sno7constituent_{94} := -\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1597}{360}$$

$$-\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1597}{360}$$

$$sno7constituent_{95} := -\frac{371}{120} t + \frac{1087}{576} + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{371}{120} t + \frac{1087}{576} + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{96} := -\frac{82}{15} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{79}{5}$$

$$-\frac{82}{15} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{79}{5}$$

$$sno7constituent_{97} := -\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{9563}{2880}$$

$$-\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{9563}{2880}$$

$$sno7constituent_{98} := -\frac{461}{120} t + \frac{227}{288} t^2 + \frac{1369}{360} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{461}{120} t + \frac{227}{288} t^2 + \frac{1369}{360} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{99} := -\frac{551}{120} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{2819}{320}$$

$$-\frac{551}{120} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{2819}{320}$$

$$sno7constituent_{100} := -\frac{119}{30} t + \frac{227}{288} t^2 + \frac{137}{18} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{119}{30} t + \frac{227}{288} t^2 + \frac{137}{18} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{101} := -\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{7739}{2880}$$

$$-\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{7739}{2880}$$

$$sno7constituent_{102} := -\frac{641}{120} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{549}{40}$$

$$-\frac{641}{120} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{549}{40}$$

$$sno7constituent_{103} := -\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1787}{2880}$$

$$-\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1787}{2880}$$

$$sno7constituent_{104} := -\frac{119}{30} t + \frac{314}{45} + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{119}{30} t + \frac{314}{45} + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{105} := -\frac{283}{60} t + \frac{13}{16} t^2 + \frac{743}{64} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{283}{60} t + \frac{13}{16} t^2 + \frac{743}{64} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{106} := -\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1633}{360}$$

$$-\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1633}{360}$$

$$sno7constituent_{107} := -\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{2843}{2880}$$

$$-\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{2843}{2880}$$

$$sno7constituent_{108} := -\frac{82}{15} t + \frac{27}{32} t^2 + \frac{149}{10} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{82}{15} t + \frac{27}{32} t^2 + \frac{149}{10} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{109} := -\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{9851}{2880}$$

$$-\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{9851}{2880}$$

$$sno7constituent_{110} := -\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{353}{72} + \frac{1}{576} t^4$$

$$-\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{353}{72} + \frac{1}{576} t^4$$

$$sno7constituent_{111} := -\frac{551}{120} t + \frac{13}{16} t^2 + \frac{2851}{320} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$-\frac{551}{120} t + \frac{13}{16} t^2 + \frac{2851}{320} - \frac{1}{16} t^3 + \frac{1}{576} t^4$$

$$sno7constituent_{112} := -\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{347}{45}$$

$$-\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{347}{45}$$

$$sno7constituent_{113} := -\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{5147}{2880}$$

$$\begin{aligned}
& -\frac{193}{60} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{5147}{2880} \\
sno7constituent_{114} := & -\frac{641}{120} t + \frac{553}{40} + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 \\
& -\frac{641}{120} t + \frac{553}{40} + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 \\
sno7constituent_{115} := & -\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{991}{576} \\
& -\frac{371}{120} t + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{991}{576} \\
sno7constituent_{116} := & -\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{547}{90} \\
& -\frac{119}{30} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{547}{90} \\
sno7constituent_{117} := & -\frac{283}{60} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{3747}{320} \\
& -\frac{283}{60} t + \frac{13}{16} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{3747}{320} \\
sno7constituent_{118} := & -\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1309}{360} \\
& -\frac{461}{120} t + \frac{227}{288} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + \frac{1309}{360} \\
sno7constituent_{119} := & -\frac{371}{120} t + \frac{3131}{2880} + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 \\
& -\frac{371}{120} t + \frac{3131}{2880} + \frac{109}{144} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 \\
sno7constituent_{120} := & -\frac{82}{15} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + 17 \\
& -\frac{82}{15} t + \frac{27}{32} t^2 - \frac{1}{16} t^3 + \frac{1}{576} t^4 + 17
\end{aligned}$$

Extract the coefficients of the constituents.

```

> for r from 1 to pno7 do
  for coeffdeg from 0 to d do
    sno7c[coeffdeg,r]:=coeff(sno7constituent[r],t,coeffdeg):
    #print( r, coeffdeg, sno7c[coeffdeg,r] ):
  od:
od:

```

Print and analyze the constituent coefficients for periods. First the higher

coefficients, which are constant. Print the first coefficient, then any that don't repeat the preceding value (there are none).

```
> for coeffdeg from 3 to d do
    print("degree", coeffdeg, "coeff", sno7c[coeffdeg,1]):
    for r from 2 to pno7 do
        stepdifference:=sno7c[coeffdeg,r]-sno7c[coeffdeg,r-1]:
        if( stepdifference<>0 ) then
            print(r,sno7c[coeffdeg,r],stepdifference):
            fi:
    od:
od:
"degree", 3, "coeff", -  $\frac{1}{16}$ 
"degree", 4, "coeff",  $\frac{1}{576}$ 
```

Next, the constant terms, whose period is expected to be pno7=120. Print all constant terms up to the presumed period "stepsize". Print the difference (at step "stepsize") if they are not repeating.

```
> stepsize:=pno7;
for r from 1 to stepsize do
    print(r, sno7c[0,r]);
od:
for r from stepsize+1 to pno7 do
    stepdifference:=sno7c[0,r]-sno7c[0,r-stepsize]:
    if( stepdifference<>0 ) then print(r,sno7c[0,r],
stepdifference): fi:
od:
print("Constant terms completed.");
stepsize:= 120
1,  $\frac{7259}{2880}$ 
2,  $\frac{1801}{360}$ 
3,  $\frac{2563}{320}$ 
4,  $\frac{613}{90}$ 
5,  $\frac{2239}{576}$ 
6,  $\frac{517}{40}$ 
```

$$7, \frac{5243}{2880}$$

$$8, \frac{278}{45}$$

$$9, \frac{3459}{320}$$

$$10, \frac{413}{72}$$

$$11, \frac{539}{2880}$$

$$12, \frac{161}{10}$$

$$13, \frac{7547}{2880}$$

$$14, \frac{1477}{360}$$

$$15, \frac{647}{64}$$

$$16, \frac{311}{45}$$

$$17, \frac{8603}{2880}$$

$$18, \frac{521}{40}$$

$$19, \frac{2651}{2880}$$

$$20, \frac{131}{18}$$

$$21, \frac{3491}{320}$$

$$22, \frac{1741}{360}$$

$$23, \frac{827}{2880}$$

$$24, \frac{81}{5}$$

$$25, \frac{2143}{576}$$

$$26, \frac{1513}{360}$$

$$27, \frac{2947}{320}$$

$$28, \frac{541}{90}$$

$$29, \frac{8891}{2880}$$

$$30, \frac{113}{8}$$

$$31, \frac{2939}{2880}$$

$$32, \frac{332}{45}$$

$$33, \frac{3203}{320}$$

$$34, \frac{1777}{360}$$

$$35, \frac{799}{576}$$

$$36, \frac{153}{10}$$

$$37, \frac{11003}{2880}$$

$$38, \frac{1189}{360}$$

$$39, \frac{2979}{320}$$

$$40, \frac{73}{9}$$

$$41, \frac{6299}{2880}$$

$$42, \frac{569}{40}$$

$$43, \frac{347}{2880}$$

$$44, \frac{583}{90}$$

$$45, \frac{775}{64}$$

$$46, \frac{1453}{360}$$

$$47, \frac{4283}{2880}$$

$$48, \frac{77}{5}$$

$$49, \frac{8411}{2880}$$

$$50, \frac{389}{72}$$

$$51, \frac{2691}{320}$$

$$52, \frac{649}{90}$$

$$53, \frac{6587}{2880}$$

$$54, \frac{533}{40}$$

$$55, \frac{1279}{576}$$

$$56, \frac{296}{45}$$

$$57, \frac{3587}{320}$$

$$58, \frac{1489}{360}$$

$$59, \frac{1691}{2880}$$

$$60, \frac{33}{2}$$

$$61, \frac{8699}{2880}$$

$$62, \frac{1621}{360}$$

$$63, \frac{2723}{320}$$

$$64, \frac{329}{45}$$

$$65, \frac{1951}{576}$$

$$66, \frac{537}{40}$$

$$67, \frac{3803}{2880}$$

$$68, \frac{511}{90}$$

$$69, \frac{3619}{320}$$

$$70, \frac{377}{72}$$

$$71, \frac{1979}{2880}$$

$$72, \frac{83}{5}$$

$$73, \frac{6107}{2880}$$

$$74, \frac{1657}{360}$$

$$75, \frac{615}{64}$$

$$76, \frac{577}{90}$$

$$77, \frac{10043}{2880}$$

$$78, \frac{501}{40}$$

$$79, \frac{4091}{2880}$$

$$80, \frac{70}{9}$$

$$81, \frac{3331}{320}$$

$$82, \frac{1921}{360}$$

$$83, -\frac{613}{2880}$$

$$84, \frac{157}{10}$$

$$85, \frac{2431}{576}$$

$$86, \frac{1333}{360}$$

$$87, \frac{3107}{320}$$

$$88, \frac{293}{45}$$

$$89, \frac{7451}{2880}$$

$$90, \frac{117}{8}$$

$$91, \frac{1499}{2880}$$

$$92, \frac{619}{90}$$

$$93, \frac{3363}{320}$$

$$94, \frac{1597}{360}$$

$$95, \frac{1087}{576}$$

$$96, \frac{79}{5}$$

$$97, \frac{9563}{2880}$$

$$98, \frac{1369}{360}$$

$$99, \frac{2819}{320}$$

$$100, \frac{137}{18}$$

$$101, \frac{7739}{2880}$$

$$102, \frac{549}{40}$$

$$103, \frac{1787}{2880}$$

$$104, \frac{314}{45}$$

$$105, \frac{743}{64}$$

$$106, \frac{1633}{360}$$

$$107, \frac{2843}{2880}$$

$$108, \frac{149}{10}$$

$$109, \frac{9851}{2880}$$

$$110, \frac{353}{72}$$

$$111, \frac{2851}{320}$$

$$112, \frac{347}{45}$$

$$113, \frac{5147}{2880}$$

$$114, \frac{553}{40}$$

$$115, \frac{991}{576}$$

$$116, \frac{547}{90}$$

$$117, \frac{3747}{320}$$

$$118, \frac{1309}{360}$$

$$119, \frac{3131}{2880}$$

120, 17

"Constant terms completed."

Now, the linear terms. First print all linear coefficients up to the presumed period "stepsize". Then analyze for period and print the difference (at step "stepsize") if they are not repeating.

```
> stepsize:=6;
for r from 1 to stepsize do
  print(r, sno7c[1,r]);
od:
for r from stepsize+1 to pno7 do
  stepdifference:=sno7c[1,r]-sno7c[1,r-stepsize];
  if( stepdifference<>0 ) then print(r,sno7c[1,r],
  stepdifference): fi:
od:
print("Linear coefficients completed.");
stepsize := 6
```

$$1, -\frac{193}{60}$$

$$2, -\frac{461}{120}$$

$$3, -\frac{551}{120}$$

$$4, -\frac{119}{30}$$

$$5, -\frac{193}{60}$$

$$6, -\frac{641}{120}$$

$$7, -\frac{371}{120}, \frac{1}{8}$$

$$8, -\frac{119}{30}, -\frac{1}{8}$$

$$9, -\frac{283}{60}, -\frac{1}{8}$$

$$10, -\frac{461}{120}, \frac{1}{8}$$

$$11, -\frac{371}{120}, \frac{1}{8}$$

$$12, -\frac{82}{15}, -\frac{1}{8}$$

$$13, -\frac{193}{60}, -\frac{1}{8}$$

$$14, -\frac{461}{120}, \frac{1}{8}$$

$$15, -\frac{551}{120}, \frac{1}{8}$$

$$16, -\frac{119}{30}, -\frac{1}{8}$$

$$17, -\frac{193}{60}, -\frac{1}{8}$$

$$18, -\frac{641}{120}, \frac{1}{8}$$

$$19, -\frac{371}{120}, \frac{1}{8}$$

$$20, -\frac{119}{30}, -\frac{1}{8}$$

$$21, -\frac{283}{60}, -\frac{1}{8}$$

$$22, -\frac{461}{120}, \frac{1}{8}$$

$$23, -\frac{371}{120}, \frac{1}{8}$$

$$24, -\frac{82}{15}, -\frac{1}{8}$$

$$25, -\frac{193}{60}, -\frac{1}{8}$$

$$26, -\frac{461}{120}, \frac{1}{8}$$

$$27, -\frac{551}{120}, \frac{1}{8}$$

$$28, -\frac{119}{30}, -\frac{1}{8}$$

$$29, -\frac{193}{60}, -\frac{1}{8}$$

$$30, -\frac{641}{120}, \frac{1}{8}$$

$$31, -\frac{371}{120}, \frac{1}{8}$$

$$32, -\frac{119}{30}, -\frac{1}{8}$$

$$33, -\frac{283}{60}, -\frac{1}{8}$$

$$34, -\frac{461}{120}, \frac{1}{8}$$

$$35, -\frac{371}{120}, \frac{1}{8}$$

$$36, -\frac{82}{15}, -\frac{1}{8}$$

$$37, -\frac{193}{60}, -\frac{1}{8}$$

$$38, -\frac{461}{120}, \frac{1}{8}$$

$$39, -\frac{551}{120}, \frac{1}{8}$$

$$40, -\frac{119}{30}, -\frac{1}{8}$$

$$41, -\frac{193}{60}, -\frac{1}{8}$$

$$42, -\frac{641}{120}, \frac{1}{8}$$

$$43, -\frac{371}{120}, \frac{1}{8}$$

$$44, -\frac{119}{30}, -\frac{1}{8}$$

$$45, -\frac{283}{60}, -\frac{1}{8}$$

$$46, -\frac{461}{120}, \frac{1}{8}$$

$$47, -\frac{371}{120}, \frac{1}{8}$$

$$48, -\frac{82}{15}, -\frac{1}{8}$$

$$49, -\frac{193}{60}, -\frac{1}{8}$$

$$50, -\frac{461}{120}, \frac{1}{8}$$

$$51, -\frac{551}{120}, \frac{1}{8}$$

$$52, -\frac{119}{30}, -\frac{1}{8}$$

$$53, -\frac{193}{60}, -\frac{1}{8}$$

$$54, -\frac{641}{120}, \frac{1}{8}$$

$$55, -\frac{371}{120}, \frac{1}{8}$$

$$56, -\frac{119}{30}, -\frac{1}{8}$$

$$57, -\frac{283}{60}, -\frac{1}{8}$$

$$58, -\frac{461}{120}, \frac{1}{8}$$

$$59, -\frac{371}{120}, \frac{1}{8}$$

$$60, -\frac{82}{15}, -\frac{1}{8}$$

$$61, -\frac{193}{60}, -\frac{1}{8}$$

$$62, -\frac{461}{120}, \frac{1}{8}$$

$$63, -\frac{551}{120}, \frac{1}{8}$$

$$64, -\frac{119}{30}, -\frac{1}{8}$$

$$65, -\frac{193}{60}, -\frac{1}{8}$$

$$66, -\frac{641}{120}, \frac{1}{8}$$

$$67, -\frac{371}{120}, \frac{1}{8}$$

$$68, -\frac{119}{30}, -\frac{1}{8}$$

$$69, -\frac{283}{60}, -\frac{1}{8}$$

$$70, -\frac{461}{120}, \frac{1}{8}$$

$$71, -\frac{371}{120}, \frac{1}{8}$$

$$72, -\frac{82}{15}, -\frac{1}{8}$$

$$73, -\frac{193}{60}, -\frac{1}{8}$$

$$74, -\frac{461}{120}, \frac{1}{8}$$

$$75, -\frac{551}{120}, \frac{1}{8}$$

$$76, -\frac{119}{30}, -\frac{1}{8}$$

$$77, -\frac{193}{60}, -\frac{1}{8}$$

$$78, -\frac{641}{120}, \frac{1}{8}$$

$$79, -\frac{371}{120}, \frac{1}{8}$$

$$80, -\frac{119}{30}, -\frac{1}{8}$$

$$81, -\frac{283}{60}, -\frac{1}{8}$$

$$82, -\frac{461}{120}, \frac{1}{8}$$

$$83, -\frac{371}{120}, \frac{1}{8}$$

$$84, -\frac{82}{15}, -\frac{1}{8}$$

$$85, -\frac{193}{60}, -\frac{1}{8}$$

$$86, -\frac{461}{120}, \frac{1}{8}$$

$$87, -\frac{551}{120}, \frac{1}{8}$$

$$88, -\frac{119}{30}, -\frac{1}{8}$$

$$89, -\frac{193}{60}, -\frac{1}{8}$$

$$90, -\frac{641}{120}, \frac{1}{8}$$

$$91, -\frac{371}{120}, \frac{1}{8}$$

$$92, -\frac{119}{30}, -\frac{1}{8}$$

$$93, -\frac{283}{60}, -\frac{1}{8}$$

$$94, -\frac{461}{120}, \frac{1}{8}$$

$$95, -\frac{371}{120}, \frac{1}{8}$$

$$96, -\frac{82}{15}, -\frac{1}{8}$$

$$97, -\frac{193}{60}, -\frac{1}{8}$$

$$98, -\frac{461}{120}, \frac{1}{8}$$

$$99, -\frac{551}{120}, \frac{1}{8}$$

$$100, -\frac{119}{30}, -\frac{1}{8}$$

$$101, -\frac{193}{60}, -\frac{1}{8}$$

$$102, -\frac{641}{120}, \frac{1}{8}$$

$$103, -\frac{371}{120}, \frac{1}{8}$$

$$104, -\frac{119}{30}, -\frac{1}{8}$$

$$105, -\frac{283}{60}, -\frac{1}{8}$$

$$106, -\frac{461}{120}, \frac{1}{8}$$

$$107, -\frac{371}{120}, \frac{1}{8}$$

$$108, -\frac{82}{15}, -\frac{1}{8}$$

$$109, -\frac{193}{60}, -\frac{1}{8}$$

$$110, -\frac{461}{120}, \frac{1}{8}$$

$$111, -\frac{551}{120}, \frac{1}{8}$$

$$112, -\frac{119}{30}, -\frac{1}{8}$$

$$113, -\frac{193}{60}, -\frac{1}{8}$$

$$114, -\frac{641}{120}, \frac{1}{8}$$

$$115, -\frac{371}{120}, \frac{1}{8}$$

$$116, -\frac{119}{30}, -\frac{1}{8}$$

$$117, -\frac{283}{60}, -\frac{1}{8}$$

$$118, -\frac{461}{120}, \frac{1}{8}$$

$$119, -\frac{371}{120}, \frac{1}{8}$$

$$120, -\frac{82}{15}, -\frac{1}{8}$$

"Linear coefficients completed."

The quadratic terms. First print all quadratic coefficients up to the presumed period "stepsize". Then analyze for period and print the difference (at step "stepsize") if they are not repeating.

```
> stepsize:=2;
for r from 1 to stepsize do
  print(r, sno7c[2,r]);
od;
for r from stepsize+1 to pno7 do
  stepdifference:=sno7c[2,r]-sno7c[2,r-stepsize];
  if( stepdifference<>0 ) then print(r,sno7c[2,r],
stepdifference): fi;
od;
print("Quadratic coefficients completed.");
stepsize:= 2
```

$$1, \frac{109}{144}$$

$$2, \frac{227}{288}$$

$$3, \frac{13}{16}, \frac{1}{18}$$

$$5, \frac{109}{144}, -\frac{1}{18}$$

$$6, \frac{27}{32}, \frac{1}{18}$$

$$8, \frac{227}{288}, -\frac{1}{18}$$

$$9, \frac{13}{16}, \frac{1}{18}$$

$$11, \frac{109}{144}, -\frac{1}{18}$$

$$12, \frac{27}{32}, \frac{1}{18}$$

$$14, \frac{227}{288}, -\frac{1}{18}$$

$$15, \frac{13}{16}, \frac{1}{18}$$

$$17, \frac{109}{144}, -\frac{1}{18}$$

$$18, \frac{27}{32}, \frac{1}{18}$$

$$20, \frac{227}{288}, -\frac{1}{18}$$

$$21, \frac{13}{16}, \frac{1}{18}$$

$$23, \frac{109}{144}, -\frac{1}{18}$$

$$24, \frac{27}{32}, \frac{1}{18}$$

$$26, \frac{227}{288}, -\frac{1}{18}$$

$$27, \frac{13}{16}, \frac{1}{18}$$

$$29, \frac{109}{144}, -\frac{1}{18}$$

$$30, \frac{27}{32}, \frac{1}{18}$$

$$32, \frac{227}{288}, -\frac{1}{18}$$

$$33, \frac{13}{16}, \frac{1}{18}$$

$$35, \frac{109}{144}, -\frac{1}{18}$$

$$36, \frac{27}{32}, \frac{1}{18}$$

$$38, \frac{227}{288}, -\frac{1}{18}$$

$$39, \frac{13}{16}, \frac{1}{18}$$

$$41, \frac{109}{144}, -\frac{1}{18}$$

$$42, \frac{27}{32}, \frac{1}{18}$$

$$44, \frac{227}{288}, -\frac{1}{18}$$

$$45, \frac{13}{16}, \frac{1}{18}$$

$$47, \frac{109}{144}, -\frac{1}{18}$$

$$48, \frac{27}{32}, \frac{1}{18}$$

$$50, \frac{227}{288}, -\frac{1}{18}$$

$$51, \frac{13}{16}, \frac{1}{18}$$

$$53, \frac{109}{144}, -\frac{1}{18}$$

$$54, \frac{27}{32}, \frac{1}{18}$$

$$56, \frac{227}{288}, -\frac{1}{18}$$

$$57, \frac{13}{16}, \frac{1}{18}$$

$$59, \frac{109}{144}, -\frac{1}{18}$$

$$60, \frac{27}{32}, \frac{1}{18}$$

$$62, \frac{227}{288}, -\frac{1}{18}$$

$$63, \frac{13}{16}, \frac{1}{18}$$

$$65, \frac{109}{144}, -\frac{1}{18}$$

$$66, \frac{27}{32}, \frac{1}{18}$$

$$68, \frac{227}{288}, -\frac{1}{18}$$

$$69, \frac{13}{16}, \frac{1}{18}$$

$$71, \frac{109}{144}, -\frac{1}{18}$$

$$72, \frac{27}{32}, \frac{1}{18}$$

$$74, \frac{227}{288}, -\frac{1}{18}$$

$$75, \frac{13}{16}, \frac{1}{18}$$

$$77, \frac{109}{144}, -\frac{1}{18}$$

$$78, \frac{27}{32}, \frac{1}{18}$$

$$80, \frac{227}{288}, -\frac{1}{18}$$

$$81, \frac{13}{16}, \frac{1}{18}$$

$$83, \frac{109}{144}, -\frac{1}{18}$$

$$84, \frac{27}{32}, \frac{1}{18}$$

$$86, \frac{227}{288}, -\frac{1}{18}$$

$$87, \frac{13}{16}, \frac{1}{18}$$

$$89, \frac{109}{144}, -\frac{1}{18}$$

$$90, \frac{27}{32}, \frac{1}{18}$$

$$92, \frac{227}{288}, -\frac{1}{18}$$

$$93, \frac{13}{16}, \frac{1}{18}$$

$$95, \frac{109}{144}, -\frac{1}{18}$$

$$96, \frac{27}{32}, \frac{1}{18}$$

$$98, \frac{227}{288}, -\frac{1}{18}$$

$$99, \frac{13}{16}, \frac{1}{18}$$

$$101, \frac{109}{144}, -\frac{1}{18}$$

$$102, \frac{27}{32}, \frac{1}{18}$$

$$104, \frac{227}{288}, -\frac{1}{18}$$

$$105, \frac{13}{16}, \frac{1}{18}$$

$$107, \frac{109}{144}, -\frac{1}{18}$$

$$108, \frac{27}{32}, \frac{1}{18}$$

$$110, \frac{227}{288}, -\frac{1}{18}$$

$$111, \frac{13}{16}, \frac{1}{18}$$

$$113, \frac{109}{144}, -\frac{1}{18}$$

$$114, \frac{27}{32}, \frac{1}{18}$$

$$116, \frac{227}{288}, -\frac{1}{18}$$

$$117, \frac{13}{16}, \frac{1}{18}$$

$$119, \frac{109}{144}, -\frac{1}{18}$$

$$120, \frac{27}{32}, \frac{1}{18}$$

"Quadratic coefficients completed."

Third, the H constituents (denominator power 7).

```
> S7gfstandnum; p7; d7;
72 x10 (x20 + x19 + x18 + x17 + x16 + x15 + x14 + x13 + x12 + x11 + x10 + x9 + x8 + x7
+ x6 + x5 + x4 + x3 + x2 + x + 1) (x12 - x11 + x9 - x8 + x6 - x4 + x3 - x + 1)
21
1
```

Calculate the zeroth constituent. Find its constant term.

```
> S7zeroth:=expand(
  sum(coeff(S7gfstandnum,x,p7*j)*binomial(d7+t/p7-j,d7),j=0..
d7+1) );
print(subs(t=0,S7zeroth));
S7zeroth := -72 +  $\frac{24}{7} t$ 
-72
```

Extract the constituents of the total semimagic counting function.

```
> S7constituent[0]:=S7zeroth;
for r from 1 to p7 do
  S7constituent[r]:=expand(sum( coeff(S7gfstandnum,x,p7*j+r)*
binomial(d7+(t-r)/p7-j,d7), j=0..d7 )):
# print(r):
# print( S7constituent[r] ):
# print( factor(S7constituent[r]) ):
  print( S7constituent[r]-24/7*(t-r) ):
od;
S7constituent1 :=  $\frac{24}{7} t - \frac{24}{7}$ 
0
S7constituent2 :=  $\frac{24}{7} t - \frac{48}{7}$ 
0
```

$$S7constituent_3 := \frac{24}{7} t - \frac{72}{7}$$

0

$$S7constituent_4 := \frac{24}{7} t - \frac{96}{7}$$

0

$$S7constituent_5 := \frac{24}{7} t - \frac{120}{7}$$

0

$$S7constituent_6 := \frac{24}{7} t - \frac{144}{7}$$

0

$$S7constituent_7 := \frac{24}{7} t - 24$$

0

$$S7constituent_8 := \frac{24}{7} t - \frac{192}{7}$$

0

$$S7constituent_9 := \frac{24}{7} t - \frac{216}{7}$$

0

$$S7constituent_{10} := \frac{264}{7} + \frac{24}{7} t$$

72

$$S7constituent_{11} := \frac{24}{7} t - \frac{264}{7}$$

0

$$S7constituent_{12} := \frac{24}{7} t - \frac{288}{7}$$

0

$$S7constituent_{13} := \frac{192}{7} + \frac{24}{7} t$$

72

$$S7constituent_{14} := \frac{24}{7} t - 48$$

$$\begin{aligned}
& 0 \\
& S7constituent_{15} := \frac{24}{7} t - \frac{360}{7} \\
& 0 \\
& S7constituent_{16} := \frac{120}{7} + \frac{24}{7} t \\
& 72 \\
& S7constituent_{17} := \frac{96}{7} + \frac{24}{7} t \\
& 72 \\
& S7constituent_{18} := \frac{24}{7} t - \frac{432}{7} \\
& 0 \\
& S7constituent_{19} := \frac{48}{7} + \frac{24}{7} t \\
& 72 \\
& S7constituent_{20} := \frac{24}{7} + \frac{24}{7} t \\
& 72 \\
& S7constituent_{21} := -72 + \frac{24}{7} t \\
& 0
\end{aligned}$$

Extract the coefficients of the constituents.

```

> for r from 1 to p7 do
  for coeffdeg from 0 to d7 do
    S7c[coeffdeg,r]:=coeff(S7constituent[r],t,coeffdeg):
    #print( r, S7c[coeffdeg,r] ):
  od:
od:

```

Next, the constant terms, whose period is expected to be $p7=21$. Print all constant terms up to the presumed period "stepsize". Print the difference (at step "stepsize") if they are not repeating.

```

> stepsize:=21;
for r from 1 to stepsize do
  print(r, S7c[0,r]);
od:
for r from stepsize+1 to p7 do
  stepdifference:=S7c[0,r]-S7c[0,r-stepsize]:
  if( stepdifference<>0 ) then print(r,S7c[0,r],stepdifference)
  : fi:

```

```

    #print(r,S7c[0,r],stepdifference);
od:
print("Constant terms completed.");
stepsize:=21

1, -  $\frac{24}{7}$ 
2, -  $\frac{48}{7}$ 
3, -  $\frac{72}{7}$ 
4, -  $\frac{96}{7}$ 
5, -  $\frac{120}{7}$ 
6, -  $\frac{144}{7}$ 
7, -24
8, -  $\frac{192}{7}$ 
9, -  $\frac{216}{7}$ 
10,  $\frac{264}{7}$ 
11, -  $\frac{264}{7}$ 
12, -  $\frac{288}{7}$ 
13,  $\frac{192}{7}$ 
14, -48
15, -  $\frac{360}{7}$ 
16,  $\frac{120}{7}$ 
17,  $\frac{96}{7}$ 

```

$$18, -\frac{432}{7}$$

$$19, \frac{48}{7}$$

$$20, \frac{24}{7}$$

$$21, -72$$

"Constant terms completed."

Calculate the zeroth constituent of the **symmetry-type H term**. Find its constant term.

```
> s7zeroth:=expand(  
    sum(coeff(numer(s7gf),x,p7*j)*binomial(d7+t/p7-j,d7),j=0..d7+1)  
);  
print(subs(t=0,s7zeroth));
```

$$s7zeroth := -1 + \frac{1}{21} t$$
$$-1$$

Extract the constituents of the semimagic symmetry-type counting function.

```
> s7constituent[0]:=s7zeroth:  
for r from 1 to p7 do  
    s7constituent[r]:=expand(sum( coeff(numer(s7gf),x,p7*j+r)*  
binomial(d7+(t-r)/p7-j,d7), j=0..d7)):  
#   print(r):  
#   print( s7constituent[r] ):  
#   print( factor(s7constituent[r]) ):  
    print( s7constituent[r]-1/21*(t-r) ):  
od;
```

$$s7constituent_1 := \frac{1}{21} t - \frac{1}{21}$$
$$0$$

$$s7constituent_2 := \frac{1}{21} t - \frac{2}{21}$$
$$0$$

$$s7constituent_3 := \frac{1}{21} t - \frac{1}{7}$$
$$0$$

$$s7constituent_4 := \frac{1}{21} t - \frac{4}{21}$$
$$0$$

$$s7constituent_5 := \frac{1}{21} t - \frac{5}{21}$$

0

$$s7constituent_6 := \frac{1}{21} t - \frac{2}{7}$$

0

$$s7constituent_7 := \frac{1}{21} t - \frac{1}{3}$$

0

$$s7constituent_8 := \frac{1}{21} t - \frac{8}{21}$$

0

$$s7constituent_9 := \frac{1}{21} t - \frac{3}{7}$$

0

$$s7constituent_{10} := \frac{11}{21} + \frac{1}{21} t$$

1

$$s7constituent_{11} := \frac{1}{21} t - \frac{11}{21}$$

0

$$s7constituent_{12} := \frac{1}{21} t - \frac{4}{7}$$

0

$$s7constituent_{13} := \frac{8}{21} + \frac{1}{21} t$$

1

$$s7constituent_{14} := \frac{1}{21} t - \frac{2}{3}$$

0

$$s7constituent_{15} := \frac{1}{21} t - \frac{5}{7}$$

0

$$s7constituent_{16} := \frac{5}{21} + \frac{1}{21} t$$

1
 $s7constituent_{17} := \frac{4}{21} + \frac{1}{21} t$
 1
 $s7constituent_{18} := \frac{1}{21} t - \frac{6}{7}$
 0
 $s7constituent_{19} := \frac{2}{21} + \frac{1}{21} t$
 1
 $s7constituent_{20} := \frac{1}{21} + \frac{1}{21} t$
 1
 $s7constituent_{21} := -1 + \frac{1}{21} t$
 0

A better form for this is $(t-t')/21+c$ where $t' :=$ least positive residue of $t \bmod p7=21$ (thus, $0' = 21$), and c depends on t' .

```
> for r from 1 to p7 do
    print( r, s7constituent[r]-(t-r)/21 );
od;
```

$1, 0$
 $2, 0$
 $3, 0$
 $4, 0$
 $5, 0$
 $6, 0$
 $7, 0$
 $8, 0$
 $9, 0$
 $10, 1$
 $11, 0$
 $12, 0$
 $13, 1$
 $14, 0$

```

15, 0
16, 1
17, 1
18, 0
19, 1
20, 1
21, 0

```

Extract the coefficients of the constituents.

```

> for r from 1 to p7 do
    for coeffdeg from 0 to d7+1 do
      s7c[coeffdeg,r]:=coeff(s7constituent[r],t,coeffdeg):
      #print( r, coeffdeg, s7c[coeffdeg,r] ):
    od:
  od:

```

Next, the constant terms, whose period is expected to be $p7=21$. Print all constant terms up to the presumed period "stepsize". Print the difference (at step "stepsize") if they are not repeating.

```

> stepsize:=21;
  for r from 1 to stepsize do
    print(r, s7c[0,r]);
  od:
  for r from stepsize+1 to p7 do
    stepdifference:=s7c[0,r]-s7c[0,r-stepsize]:
    if( stepdifference<>0 ) then print(r,s7c[0,r],stepdifference)
    : fi:
  od:
  print("Constant terms completed.");
  stepsize:= 21

```

$$1, -\frac{1}{21}$$

$$2, -\frac{2}{21}$$

$$3, -\frac{1}{7}$$

$$4, -\frac{4}{21}$$

$$5, -\frac{5}{21}$$

$$6, -\frac{2}{7}$$

$$7, -\frac{1}{3}$$

$$8, -\frac{8}{21}$$

$$9, -\frac{3}{7}$$

$$10, \frac{11}{21}$$

$$11, -\frac{11}{21}$$

$$12, -\frac{4}{7}$$

$$13, \frac{8}{21}$$

$$14, -\frac{2}{3}$$

$$15, -\frac{5}{7}$$

$$16, \frac{5}{21}$$

$$17, \frac{4}{21}$$

$$18, -\frac{6}{7}$$

$$19, \frac{2}{21}$$

$$20, \frac{1}{21}$$

$$21, -1$$

"Constant terms completed."