Magilatin generating functions and constituents

(general form, with cubic data)

Notation:

L, S: magilatin, semimagic squares (all positive values).

ml: magilatin, except in g.f.'s.

l, s: normalized squares (symmetry types).

R: reduced squares (least element is 0).

r: reduced normalized squares (reduced symmetry types).

n: semimagic r.

gf: generating function in some form.

gfsum: generating function as a sum of simple terms.

c: Cubic (fixed strict upper bound; weak upper bound for reduced).

a: Affine (fixed magic sum).

We start by recomputing rs from the semimagic count. From the Latte results we get the closed Ehrhart g.f. of each flat, which depends on whether we're doing cubic or affine. We also need

p = period (or truncated period in affine),

d = degree/dimension,

RtoLfactor = the rational function that multiplies Rgf to Lgf and rgf to lgf.

This is for cubic: set up main constants.

```
> d:=5; p:=60;
  RtoLfactor:=x^2/(1-x)^2;
```

$$d := 5$$

$$p := 60$$

$$RtoLfactor := \frac{x^2}{(1-x)^2}$$

This is also for cubic: set up simplex data.

```
> simplexname[1]:="OABC": ehrgf[1]:= 1/((1-x)^3*(1-x^2)) : dimen[1]:=3:
  simplexname[2]:="OEE2": ehrgf[2]:= 1/((1-x)*(1-x^2)*(1-x^3)) :
  dimen[2]:=2:
  simplexname[3]:="OAE2": ehrgf[3]:= 1/((1-x)*(1-x^2)^2) : dimen[3]:=2:
  simplexname[4]:="ADE2": ehrgf[4]:= 1/((1-x^2)^3) : dimen[4]:=2:
  simplexname[5]:="DE1E2": ehrgf[5]:= 1/((1-x^2)^2*(1-x^3)) : dimen[5]:=2:
  simplexname[6]:="OCE": ehrgf[6]:= 1/((1-x)^2*(1-x^3)) : dimen[6]:=2:
  simplexname[7]:="BDE1": ehrgf[7]:= 1/((1-x)*(1-x^2)*(1-x^3)) :
  dimen[7]:=2:
  simplexname[8]:="ABD": ehrgf[8]:= 1/((1-x)*(1-x^2)^2) : dimen[8]:=2:
  simplexname[9]:="FG1": ehrgf[9]:= 1/((1-x^3)*(1-x^5)) : dimen[9]:=1:
  simplexname[10]:="EF": ehrgf[10]:= 1/((1-x^3)^2) : dimen[10]:=1:
  simplexname[11]:="OG": ehrgf[11]:= 1/((1-x)*(1-x^4)) : dimen[11]:=1:
```

```
simplexname[12]:="FG": ehrgf[12]:= 1/((1-x^3)*(1-x^4)) : dimen[12]:=1:
simplexname[13]:="AF": ehrgf[13]:= 1/((1-x^2)*(1-x^3)) : dimen[13]:=1:
simplexname[14]:="DG": ehrgf[14]:= 1/((1-x^2)*(1-x^4)) : dimen[14]:=1:
simplexname[15]:="DG2": ehrgf[15]:= 1/((1-x^2)*(1-x^5)) : dimen[15]:=1:
simplexname[16]:="DE": ehrgf[16]:= 1/((1-x^2)*(1-x^3)) : dimen[16]:=1:
simplexname[17]:="H": ehrgf[17] := 1/(1-x^5) : dimen[17]:=0:
for n from 1 to 17 do print(simplexname[n], dimen[n], ehrgf[n]); od;
```

$$\text{"OABC"}, 3, \frac{1}{(1-x)^3 (1-x^2)}$$

$$\text{"OEE2"}, 2, \frac{1}{(1-x)(1-x^2)(1-x^3)}$$

$$\text{"OAE2"}, 2, \frac{1}{(1-x)(1-x^2)^2}$$

$$\text{"ADE2"}, 2, \frac{1}{(1-x^2)^3}$$

$$\text{"DE1E2"}, 2, \frac{1}{(1-x^2)^2 (1-x^3)}$$

$$\text{"OCE"}, 2, \frac{1}{(1-x)^2 (1-x^3)}$$

$$\text{"BDE1"}, 2, \frac{1}{(1-x)(1-x^2)(1-x^3)}$$

$$\text{"ABD"}, 2, \frac{1}{(1-x)(1-x^2)^2}$$

$$\text{"FG1"}, 1, \frac{1}{(1-x^3)(1-x^5)}$$

$$\text{"EF"}, 1, \frac{1}{(1-x^3)^2}$$

$$\text{"OG"}, 1, \frac{1}{(1-x)(1-x^4)}$$

$$\text{"FG"}, 1, \frac{1}{(1-x^3)(1-x^4)}$$

$$\text{"AF"}, 1, \frac{1}{(1-x^2)(1-x^3)}$$

$$\text{"DG"}, 1, \frac{1}{(1-x^2)(1-x^4)}$$

$$\text{"DG2"}, 1, \frac{1}{(1-x^2)(1-x^5)}$$

$$"DE", 1, \frac{1}{(1-x^2)(1-x^3)}$$

$$"H", 0, \frac{1}{1-x^5}$$

The closed E.g.f. is converted to the open E.g.f.

```
> for n from 1 to 17 do
    mu[n]:=(-1)^(dimen[1]-dimen[n]):
  od:
  mu[14]:=2*mu[14]:
  for n from 1 to 17 do
    openehrgf[n]:=simplify(-(-1)^dimen[n]*subs(x=1/x,ehrgf[n])):
  od:
  for n from 1 to 17 do
    rsgfterm[n]:=openehrgf[n]:
  od:
  rsgfsum:=sum(mu[nn]*rsgfterm[nn],nn=1..17);
  rsgf:=rsgfsum:
  sgfsum:=RtoLfactor*rsgf;
  sgf:=sgfsum:
```

$$rsgfsum := \frac{x^5}{(x-1)^3(x^2-1)} + \frac{2x^6}{(x-1)(x^2-1)(x^3-1)} + \frac{2x^5}{(x-1)(x^2-1)^2} + \frac{x^6}{(x^2-1)^3} + \frac{x^7}{(x^2-1)^2(x^3-1)} + \frac{x^5}{(x-1)^2(x^3-1)} + \frac{x^8}{(x^3-1)(x^5-1)} + \frac{x^6}{(x^3-1)^2} + \frac{x^5}{(x-1)(x^4-1)} + \frac{x^7}{(x^3-1)(x^4-1)} + \frac{2x^5}{(x^2-1)(x^3-1)} + \frac{2x^6}{(x^2-1)(x^4-1)} + \frac{x^7}{(x^2-1)(x^5-1)} + \frac{x^5}{x^5-1}$$

$$sgfsum := \frac{1}{(1-x)^2}\left(x^2\left(\frac{x^5}{(x-1)^3(x^2-1)} + \frac{2x^6}{(x-1)(x^2-1)(x^3-1)} + \frac{2x^5}{(x-1)(x^2-1)^2} + \frac{x^6}{(x^2-1)^3} + \frac{x^7}{(x^2-1)^2(x^3-1)} + \frac{x^5}{(x-1)^2(x^3-1)} + \frac{x^8}{(x^3-1)(x^5-1)} + \frac{x^6}{(x^3-1)^2} + \frac{x^5}{(x-1)(x^4-1)} + \frac{x^7}{(x^3-1)(x^4-1)} + \frac{2x^5}{(x^2-1)(x^3-1)} + \frac{2x^6}{(x^2-1)(x^4-1)} + \frac{x^7}{(x^2-1)(x^5-1)} + \frac{x^5}{x^5-1}\right)\right)$$

The additional faces and intersection polytopes involved in the magilatin computation. These depend on whether we're cubic or affine.

```
> mlsimplexname[1]:="OAB": mlehrgf[1]:= 1 / ((1-x)^2*(1-x^2)) :
  mldimen[1]:=2  :
  mlsimplexname[2]:="OE": mlehrgf[2]:= 1 / ((1-x)*(1-x^3)) : mldimen[2]:=1
  :
  mlsimplexname[3]:="OAC": mlehrgf[3]:= 1 / ((1-x)^2*(1-x^2)) :
  mldimen[3]:=2  :
  mlsimplexname[4]:="AD": mlehrgf[4]:= 1/(1-x^2)^2 : mldimen[4]:=1 :
  mlsimplexname[5]:="DE1": mlehrgf[5]:= 1 / ((1-x^2)*(1-x^3)) :
```

```
mldimen[5]:=1  :
mlsimplexname[6]:="OBC": mlehrgf[6]:= 1/(1-x)^3 : mldimen[6]:=2 :
mlsimplexname[7]:="OE2": mlehrgf[7]:= 1 / ((1-x)*(1-x^2)) :
mldimen[7]:=1  :
mlsimplexname[8]:="BD": mlehrgf[8]:= 1 / ((1-x)*(1-x^2)) : mldimen[8]:=1
:
mlsimplexname[9]:="DE2": mlehrgf[9]:= 1/(1-x^2)^2 : mldimen[9]:=1 :
mlsimplexname[10]:="F": mlehrgf[10]:= 1/(1-x^3) : mldimen[10]:=0 :
mlsimplexname[11]:="OB": mlehrgf[11]:= 1/(1-x)^2 : mldimen[11]:=1 :
for n from 1 to 11 do print(mlsimplexname[n], mldimen[n], mlehrgf[n]);
od;
```

$$\text{"OAB"}, 2, \frac{1}{(1-x)^2 (1-x^2)}$$

$$\text{"OE"}, 1, \frac{1}{(1-x)(1-x^3)}$$

$$\text{"OAC"}, 2, \frac{1}{(1-x)^2 (1-x^2)}$$

$$\text{"AD"}, 1, \frac{1}{(1-x^2)^2}$$

$$\text{"DE1"}, 1, \frac{1}{(1-x^2)(1-x^3)}$$

$$\text{"OBC"}, 2, \frac{1}{(1-x)^3}$$

$$\text{"OE2"}, 1, \frac{1}{(1-x)(1-x^2)}$$

$$\text{"BD"}, 1, \frac{1}{(1-x)(1-x^2)}$$

$$\text{"DE2"}, 1, \frac{1}{(1-x^2)^2}$$

$$\text{"F"}, 0, \frac{1}{1-x^3}$$

$$\text{"OB"}, 1, \frac{1}{(1-x)^2}$$

Now a general computation. First, open Ehrhart g.f.'s.

```
> for n from 1 to 11 do
    openmlehrgf[n]:=simplify(-(-1)^mldimen[n]*subs(x=1/x,mlehrgf[n]));
  od:
```

(-1)^3 n_OAB(1/x) equals mlehrgf[1]+mlehrgf[2], and hence n_OAB(x) is,   by another method that gives a nicer appearance, summing mu(,)E^o(x):

```
> mlnnew[1] := openmlehrgf[1]-openmlehrgf[2];
```

$$mlnnew_1 := -\frac{x^4}{(x-1)^2(x^2-1)} - \frac{x^4}{(x-1)(x^3-1)}$$

(-1)^3 n_OAC(1/x) equals mlehrgf[3]+mlehrgf[4]+mlehrgf[5]. Hence n_OAC(x) equals

```
> mlnnew[2] := openmlehrgf[3]-openmlehrgf[4]-openmlehrgf[5];
```

$$mlnnew_2 := -\frac{x^4}{(x-1)^2(x^2-1)} - \frac{x^4}{(x^2-1)^2} - \frac{x^5}{(x^2-1)(x^3-1)}$$

(-1)^3 n_OBC(1/x) equals mlehrgf[6]+mlehrgf[7]+mlehrgf[8]+mlehrgf[9]+mlehrgf[10]. So n_OBC(x) equals

```
> mlnnew[3] :=
  openmlehrgf[6]-openmlehrgf[7]-openmlehrgf[8]-openmlehrgf[9]+openmlehrgf[
  10];
```

$$mlnnew_3 := -\frac{x^3}{(x-1)^3} - \frac{2x^3}{(x-1)(x^2-1)} - \frac{x^4}{(x^2-1)^2} - \frac{x^3}{x^3-1}$$

Finally, OB gives mlehrgf[11], so that n_OB(x) is

```
> mlnnew[4] := openmlehrgf[11];
```

$$mlnnew_4 := \frac{x^2}{(x-1)^2}$$

To compute r, we need rs=n from semimagic, which equals rgf:

```
> Rgfsum:=72*rsgf+36*(mlnnew[1]+mlnnew[2]+mlnnew[3])+12*mlnnew[4];
  Rgf:=simplify(Rgfsum);
```

$$Rgfsum := \frac{72x^5}{(x-1)^3(x^2-1)} + \frac{144x^6}{(x-1)(x^2-1)(x^3-1)} + \frac{144x^5}{(x-1)(x^2-1)^2} + \frac{72x^6}{(x^2-1)^3} + \frac{72x^7}{(x^2-1)^2(x^3-1)} + \frac{72x^5}{(x-1)^2(x^3-1)} + \frac{72x^8}{(x^3-1)(x^5-1)} + \frac{72x^6}{(x^3-1)^2} + \frac{72x^5}{(x-1)(x^4-1)} + \frac{72x^7}{(x^3-1)(x^4-1)} + \frac{108x^5}{(x^2-1)(x^3-1)} + \frac{144x^6}{(x^2-1)(x^4-1)} + \frac{72x^7}{(x^2-1)(x^5-1)} + \frac{72x^5}{x^5-1} - \frac{72x^4}{(x-1)^2(x^2-1)} - \frac{36x^4}{(x-1)(x^3-1)} - \frac{72x^4}{(x^2-1)^2} - \frac{36x^3}{(x-1)^3} - \frac{72x^3}{(x-1)(x^2-1)} - \frac{36x^3}{x^3-1} + \frac{12x^2}{(x-1)^2}$$

$$Rgf := \frac{1}{(x^4-1)(x^5-1)(x^3-1)^2(x+1)^2}(12x^2(79x^{15}+190x^{14}+260x^{13}+250x^{12}+211x^{11}+179x^{10} + 181x^9+198x^8+210x^7+181x^6+125x^5+61x^4+22x^3+8x^2+4x+1))$$

Hence L, the g.f. of the number of magilatin squares, equals

```
> Lgfsum:=RtoLfactor*Rgfsum;
  Lgf:=simplify(Lgfsum);
```

$$Lgfsum := \frac{1}{(1-x)^2}\left(x^2\left(\frac{72\,x^5}{(x-1)^3\,(x^2-1)} + \frac{144\,x^6}{(x-1)\,(x^2-1)\,(x^3-1)} + \frac{144\,x^5}{(x-1)\,(x^2-1)^2} + \frac{72\,x^6}{(x^2-1)^3} + \frac{72\,x^7}{(x^2-1)^2\,(x^3-1)} + \frac{72\,x^5}{(x-1)^2\,(x^3-1)} + \frac{72\,x^8}{(x^3-1)\,(x^5-1)} + \frac{72\,x^6}{(x^3-1)^2} + \frac{72\,x^5}{(x-1)\,(x^4-1)} + \frac{72\,x^7}{(x^3-1)\,(x^4-1)} + \frac{108\,x^5}{(x^2-1)\,(x^3-1)} + \frac{144\,x^6}{(x^2-1)\,(x^4-1)} + \frac{72\,x^7}{(x^2-1)\,(x^5-1)} + \frac{72\,x^5}{x^5-1} - \frac{72\,x^4}{(x-1)^2\,(x^2-1)} - \frac{36\,x^4}{(x-1)\,(x^3-1)} - \frac{72\,x^4}{(x^2-1)^2} - \frac{36\,x^3}{(x-1)^3} - \frac{72\,x^3}{(x-1)\,(x^2-1)} - \frac{36\,x^3}{x^3-1} + \frac{12\,x^2}{(x-1)^2}\right)\right)$$

$$Lgf := \frac{1}{(x^4-1)\,(x^5-1)\,(x^3-1)^2\,(x^2-1)^2}(12\,x^4\,(79\,x^{15} + 190\,x^{14} + 260\,x^{13} + 250\,x^{12} + 211\,x^{11} + 179\,x^{10} + 181\,x^9 + 198\,x^8 + 210\,x^7 + 181\,x^6 + 125\,x^5 + 61\,x^4 + 22\,x^3 + 8\,x^2 + 4\,x + 1))$$

Now compute the number of reduced symmetry types:

```
> rgfsum:=rsgf+mlnnew[1]+mlnnew[2]+mlnnew[3]+mlnnew[4];
  rgf:=simplify(rgfsum);
```

$$rgfsum := \frac{x^5}{(x-1)^3\,(x^2-1)} + \frac{2\,x^6}{(x-1)\,(x^2-1)\,(x^3-1)} + \frac{2\,x^5}{(x-1)\,(x^2-1)^2} + \frac{x^6}{(x^2-1)^3} + \frac{x^7}{(x^2-1)^2\,(x^3-1)} + \frac{x^5}{(x-1)^2\,(x^3-1)} + \frac{x^8}{(x^3-1)\,(x^5-1)} + \frac{x^6}{(x^3-1)^2} + \frac{x^5}{(x-1)\,(x^4-1)} + \frac{x^7}{(x^3-1)\,(x^4-1)} + \frac{x^5}{(x^2-1)\,(x^3-1)} + \frac{2\,x^6}{(x^2-1)\,(x^4-1)} + \frac{x^7}{(x^2-1)\,(x^5-1)} + \frac{x^5}{x^5-1} - \frac{2\,x^4}{(x-1)^2\,(x^2-1)} - \frac{x^4}{(x-1)\,(x^3-1)} - \frac{2\,x^4}{(x^2-1)^2} - \frac{x^3}{(x-1)^3} - \frac{2\,x^3}{(x-1)\,(x^2-1)} - \frac{x^3}{x^3-1} + \frac{x^2}{(x-1)^2}$$

$$rgf := \frac{1}{(x^4-1)\,(x^5-1)\,(x^3-1)^2\,(x+1)^2}(x^2\,(9\,x^{15} + 20\,x^{14} + 23\,x^{13} + 16\,x^{12} + 10\,x^{11} + 13\,x^{10} + 27\,x^9 + 43\,x^8 + 54\,x^7 + 52\,x^6 + 41\,x^5 + 25\,x^4 + 14\,x^3 + 8\,x^2 + 4\,x + 1))$$

The g.f. of the total number of symmetry types, l_ml ("lgf"):

```
> lgfsum:=RtoLfactor*rgfsum;
  lgf:=simplify(lgfsum);
```

$$lgfsum := \frac{1}{(1-x)^2}\left(x^2\left(\frac{x^5}{(x-1)^3\,(x^2-1)} + \frac{2\,x^6}{(x-1)\,(x^2-1)\,(x^3-1)} + \frac{2\,x^5}{(x-1)\,(x^2-1)^2} + \frac{x^6}{(x^2-1)^3} + \frac{x^7}{(x^2-1)^2\,(x^3-1)} + \frac{x^5}{(x-1)^2\,(x^3-1)} + \frac{x^8}{(x^3-1)\,(x^5-1)} + \frac{x^6}{(x^3-1)^2} + \frac{x^5}{(x-1)\,(x^4-1)} + \frac{x^7}{(x^3-1)\,(x^4-1)}\right.\right.$$

$$+\frac{x^5}{(x^2-1)(x^3-1)}+\frac{2x^6}{(x^2-1)(x^4-1)}+\frac{x^7}{(x^2-1)(x^5-1)}+\frac{x^5}{x^5-1}-\frac{2x^4}{(x-1)^2(x^2-1)}-\frac{x^4}{(x-1)(x^3-1)}$$

$$-\frac{2x^4}{(x^2-1)^2}-\frac{x^3}{(x-1)^3}-\frac{2x^3}{(x-1)(x^2-1)}-\frac{x^3}{x^3-1}+\frac{x^2}{(x-1)^2}\Bigg)\Bigg)$$

$$lgf := \frac{1}{(x^4-1)(x^5-1)(x^3-1)^2(x^2-1)^2}(x^4(9x^{15}+20x^{14}+23x^{13}+16x^{12}+10x^{11}+13x^{10}+27x^9$$

$$+43x^8+54x^7+52x^6+41x^5+25x^4+14x^3+8x^2+4x+1))$$

Generate the series expansions of the g.f.'s.

Expressing the rational function with standard denominator gives an orders-of-magnitude speedup in the series expansion.

enddegree: The number of terms of the L and l sequences to print out.

```
> enddegree:=100;
```

$$enddegree := 100$$

Standard denominator (1-x^p)^{d+1}.

```
> pdenom:=(1-x^p):
  standenom:=pdenom^(d+1);
```

$$standenom := (1-x^{60})^6$$

G.f. as rational function with standard denominator.

```
> Lgfstandnum:=simplify(numer(Lgf)*simplify(standenom/denom(Lgf)));
  Lgf:=Lgfstandnum/standenom;
```

$$Lgfstandnum := 12x^4(79x^{15}+190x^{14}+260x^{13}+250x^{12}+211x^{11}+179x^{10}+181x^9+198x^8$$

$$+210x^7+181x^6+125x^5+61x^4+22x^3+8x^2+4x+1)(x^{57}+x^{54}+x^{51}+x^{48}+x^{45}+x^{42}$$

$$+x^{39}+x^{36}+x^{33}+x^{30}+x^{27}+x^{24}+x^{21}+x^{18}+x^{15}+x^{12}+x^9+x^6+x^3+1)^2(1+x+x^2+x^3$$

$$+x^5+x^4+x^6+x^7+x^8+x^{10}+x^{15}+x^{12}+x^9+x^{13}+x^{11}+x^{19}+x^{14}+x^{16}+x^{17}+x^{22}+x^{20}$$

$$+x^{18}+x^{21}+x^{56}+x^{48}+x^{52}+x^{44}+x^{40}+x^{36}+x^{32}+x^{28}+x^{24}+x^{46}+x^{41}+x^{51}+x^{26}+x^{31}$$

$$+x^{59}+x^{57}+x^{58}+x^{55}+x^{54}+x^{53}+x^{50}+x^{49}+x^{47}+x^{45}+x^{43}+x^{42}+x^{39}+x^{38}+x^{37}+x^{35}$$

$$+x^{34}+x^{33}+x^{30}+x^{29}+x^{27}+x^{25}+x^{23})(1+x^2+x^4+x^6+x^8+x^{10}+x^{12}+x^{14}+x^{16}+x^{22}$$

$$+x^{20}+x^{18}+x^{56}+x^{48}+x^{52}+x^{44}+x^{40}+x^{36}+x^{32}+x^{28}+x^{24}+x^{46}+x^{26}+x^{58}+x^{54}+x^{50}$$

$$+x^{42}+x^{38}+x^{34}+x^{30})^2(x^{52}-x^{51}+x^{48}-x^{46}+x^{44}-x^{41}+x^{40}+x^{32}-x^{31}+x^{28}-x^{26}+x^{24}$$

$$-x^{21}+x^{20}+x^{12}-x^{11}+x^8-x^6+x^4-x+1)$$

$$Lgf := \frac{1}{(1-x^{60})^6}(12x^4(79x^{15}+190x^{14}+260x^{13}+250x^{12}+211x^{11}+179x^{10}+181x^9+198x^8$$

$$+210x^7+181x^6+125x^5+61x^4+22x^3+8x^2+4x+1)(x^{57}+x^{54}+x^{51}+x^{48}+x^{45}+x^{42}$$

$$+x^{39}+x^{36}+x^{33}+x^{30}+x^{27}+x^{24}+x^{21}+x^{18}+x^{15}+x^{12}+x^9+x^6+x^3+1)^2(1+x+x^2+x^3$$

$$+x^5+x^4+x^6+x^7+x^8+x^{10}+x^{15}+x^{12}+x^9+x^{13}+x^{11}+x^{19}+x^{14}+x^{16}+x^{17}+x^{22}+x^{20}$$

$$+x^{18}+x^{21}+x^{56}+x^{48}+x^{52}+x^{44}+x^{40}+x^{36}+x^{32}+x^{28}+x^{24}+x^{46}+x^{41}+x^{51}+x^{26}+x^{31}$$

$$+ x^{59} + x^{57} + x^{58} + x^{55} + x^{54} + x^{53} + x^{50} + x^{49} + x^{47} + x^{45} + x^{43} + x^{42} + x^{39} + x^{38} + x^{37} + x^{35}$$
$$+ x^{34} + x^{33} + x^{30} + x^{29} + x^{27} + x^{25} + x^{23}) (1 + x^{2} + x^{4} + x^{6} + x^{8} + x^{10} + x^{12} + x^{14} + x^{16} + x^{22}$$
$$+ x^{20} + x^{18} + x^{56} + x^{48} + x^{52} + x^{44} + x^{40} + x^{36} + x^{32} + x^{28} + x^{24} + x^{46} + x^{26} + x^{58} + x^{54} + x^{50}$$
$$+ x^{42} + x^{38} + x^{34} + x^{30})\hat{}2 (x^{52} - x^{51} + x^{48} - x^{46} + x^{44} - x^{41} + x^{40} + x^{32} - x^{31} + x^{28} - x^{26} + x^{24}$$
$$- x^{21} + x^{20} + x^{12} - x^{11} + x^{8} - x^{6} + x^{4} - x + 1))$$

G.f. as rational function with standard denominator.

```
> Rgfstandnum:=simplify(numer(Rgf)*standenom/denom(Rgf));
  Rgf:=Rgfstandnum/standenom;
```

$$Rgfstandnum := 12 (79 x^{15} + 190 x^{14} + 260 x^{13} + 250 x^{12} + 211 x^{11} + 179 x^{10} + 181 x^{9} + 198 x^{8} + 210 x^{7}$$
$$+ 181 x^{6} + 125 x^{5} + 61 x^{4} + 22 x^{3} + 8 x^{2} + 4 x + 1) x^{2} (x^{57} + x^{54} + x^{51} + x^{48} + x^{45} + x^{42} + x^{39}$$
$$+ x^{36} + x^{33} + x^{30} + x^{27} + x^{24} + x^{21} + x^{18} + x^{15} + x^{12} + x^{9} + x^{6} + x^{3} + 1)\hat{}2 (-1 + x - x^{2} + x^{3} + x^{5}$$
$$- x^{4} - x^{6} + x^{7} - x^{8} - x^{10} + x^{15} - x^{12} + x^{9} + x^{13} + x^{11} + x^{19} - x^{14} - x^{16} + x^{17} - x^{22} - x^{20} - x^{18} + x^{21}$$
$$- x^{56} - x^{48} - x^{52} - x^{44} - x^{40} - x^{36} - x^{32} - x^{28} - x^{24} - x^{46} + x^{41} + x^{51} - x^{26} + x^{31} + x^{59} + x^{57} - x^{58}$$
$$+ x^{55} - x^{54} + x^{53} - x^{50} + x^{49} + x^{47} + x^{45} + x^{43} - x^{42} + x^{39} - x^{38} + x^{37} + x^{35} - x^{34} + x^{33} - x^{30} + x^{29}$$
$$+ x^{27} + x^{25} + x^{23}) (-1 + x^{60}) (1 + x^{2} + x^{4} + x^{6} + x^{8} + x^{10} + x^{12} + x^{14} + x^{16} + x^{22} + x^{20} + x^{18} + x^{56}$$
$$+ x^{48} + x^{52} + x^{44} + x^{40} + x^{36} + x^{32} + x^{28} + x^{24} + x^{46} + x^{26} + x^{58} + x^{54} + x^{50} + x^{42} + x^{38} + x^{34}$$
$$+ x^{30}) (x^{52} - x^{51} + x^{48} - x^{46} + x^{44} - x^{41} + x^{40} + x^{32} - x^{31} + x^{28} - x^{26} + x^{24} - x^{21} + x^{20} + x^{12} - x^{11}$$
$$+ x^{8} - x^{6} + x^{4} - x + 1)$$

$$Rgf := \frac{1}{(1 - x^{60})^{6}} (12 (79 x^{15} + 190 x^{14} + 260 x^{13} + 250 x^{12} + 211 x^{11} + 179 x^{10} + 181 x^{9} + 198 x^{8} + 210 x^{7}$$
$$+ 181 x^{6} + 125 x^{5} + 61 x^{4} + 22 x^{3} + 8 x^{2} + 4 x + 1) x^{2} (x^{57} + x^{54} + x^{51} + x^{48} + x^{45} + x^{42} + x^{39}$$
$$+ x^{36} + x^{33} + x^{30} + x^{27} + x^{24} + x^{21} + x^{18} + x^{15} + x^{12} + x^{9} + x^{6} + x^{3} + 1)\hat{}2 (-1 + x - x^{2} + x^{3} + x^{5}$$
$$- x^{4} - x^{6} + x^{7} - x^{8} - x^{10} + x^{15} - x^{12} + x^{9} + x^{13} + x^{11} + x^{19} - x^{14} - x^{16} + x^{17} - x^{22} - x^{20} - x^{18} + x^{21}$$
$$- x^{56} - x^{48} - x^{52} - x^{44} - x^{40} - x^{36} - x^{32} - x^{28} - x^{24} - x^{46} + x^{41} + x^{51} - x^{26} + x^{31} + x^{59} + x^{57} - x^{58}$$
$$+ x^{55} - x^{54} + x^{53} - x^{50} + x^{49} + x^{47} + x^{45} + x^{43} - x^{42} + x^{39} - x^{38} + x^{37} + x^{35} - x^{34} + x^{33} - x^{30} + x^{29}$$
$$+ x^{27} + x^{25} + x^{23}) (-1 + x^{60}) (1 + x^{2} + x^{4} + x^{6} + x^{8} + x^{10} + x^{12} + x^{14} + x^{16} + x^{22} + x^{20} + x^{18} + x^{56}$$
$$+ x^{48} + x^{52} + x^{44} + x^{40} + x^{36} + x^{32} + x^{28} + x^{24} + x^{46} + x^{26} + x^{58} + x^{54} + x^{50} + x^{42} + x^{38} + x^{34}$$
$$+ x^{30}) (x^{52} - x^{51} + x^{48} - x^{46} + x^{44} - x^{41} + x^{40} + x^{32} - x^{31} + x^{28} - x^{26} + x^{24} - x^{21} + x^{20} + x^{12} - x^{11}$$
$$+ x^{8} - x^{6} + x^{4} - x + 1))$$

G.f. as rational function with standard denominator.

```
> lgfstandnum:=simplify(numer(lgf)*simplify(standenom/denom(lgf)));
  lgf:=lgfstandnum/standenom;
```

$$lgfstandnum := x^{4} (9 x^{15} + 20 x^{14} + 23 x^{13} + 16 x^{12} + 10 x^{11} + 13 x^{10} + 27 x^{9} + 43 x^{8} + 54 x^{7} + 52 x^{6}$$
$$+ 41 x^{5} + 25 x^{4} + 14 x^{3} + 8 x^{2} + 4 x + 1) (x^{57} + x^{54} + x^{51} + x^{48} + x^{45} + x^{42} + x^{39} + x^{36} + x^{33} + x^{30}$$
$$+ x^{27} + x^{24} + x^{21} + x^{18} + x^{15} + x^{12} + x^{9} + x^{6} + x^{3} + 1)\hat{}2 (1 + x + x^{2} + x^{3} + x^{5} + x^{4} + x^{6} + x^{7} + x^{8}$$
$$+ x^{10} + x^{15} + x^{12} + x^{9} + x^{13} + x^{11} + x^{19} + x^{14} + x^{16} + x^{17} + x^{22} + x^{20} + x^{18} + x^{21} + x^{56} + x^{48}$$
$$+ x^{52} + x^{44} + x^{40} + x^{36} + x^{32} + x^{28} + x^{24} + x^{46} + x^{41} + x^{51} + x^{26} + x^{31} + x^{59} + x^{57} + x^{58} + x^{55}$$
$$+ x^{54} + x^{53} + x^{50} + x^{49} + x^{47} + x^{45} + x^{43} + x^{42} + x^{39} + x^{38} + x^{37} + x^{35} + x^{34} + x^{33} + x^{30} + x^{29}$$
$$+ x^{27} + x^{25} + x^{23}) (1 + x^{2} + x^{4} + x^{6} + x^{8} + x^{10} + x^{12} + x^{14} + x^{16} + x^{22} + x^{20} + x^{18} + x^{56} + x^{48}$$
$$+ x^{52} + x^{44} + x^{40} + x^{36} + x^{32} + x^{28} + x^{24} + x^{46} + x^{26} + x^{58} + x^{54} + x^{50} + x^{42} + x^{38} + x^{34} + x^{30})\hat{}2$$
$$(x^{52} - x^{51} + x^{48} - x^{46} + x^{44} - x^{41} + x^{40} + x^{32} - x^{31} + x^{28} - x^{26} + x^{24} - x^{21} + x^{20} + x^{12} - x^{11} + x^{8} - x^{6}$$

$$+ x^4 - x + 1)$$

$$lgf := \frac{1}{(1 - x^{60})^6} (x^4 (9 x^{15} + 20 x^{14} + 23 x^{13} + 16 x^{12} + 10 x^{11} + 13 x^{10} + 27 x^9 + 43 x^8 + 54 x^7 + 52 x^6 + 41 x^5 + 25 x^4 + 14 x^3 + 8 x^2 + 4 x + 1) (x^{57} + x^{54} + x^{51} + x^{48} + x^{45} + x^{42} + x^{39} + x^{36} + x^{33} + x^{30} + x^{27} + x^{24} + x^{21} + x^{18} + x^{15} + x^{12} + x^9 + x^6 + x^3 + 1)\verb|^|2 (1 + x + x^2 + x^3 + x^5 + x^4 + x^6 + x^7 + x^8 + x^{10} + x^{15} + x^{12} + x^9 + x^{13} + x^{11} + x^{19} + x^{14} + x^{16} + x^{17} + x^{22} + x^{20} + x^{18} + x^{21} + x^{56} + x^{48} + x^{52} + x^{44} + x^{40} + x^{36} + x^{32} + x^{28} + x^{24} + x^{46} + x^{41} + x^{51} + x^{26} + x^{31} + x^{59} + x^{57} + x^{58} + x^{55} + x^{54} + x^{53} + x^{50} + x^{49} + x^{47} + x^{45} + x^{43} + x^{42} + x^{39} + x^{38} + x^{37} + x^{35} + x^{34} + x^{33} + x^{30} + x^{29} + x^{27} + x^{25} + x^{23}) (1 + x^2 + x^4 + x^6 + x^8 + x^{10} + x^{12} + x^{14} + x^{16} + x^{22} + x^{20} + x^{18} + x^{56} + x^{48} + x^{52} + x^{44} + x^{40} + x^{36} + x^{32} + x^{28} + x^{24} + x^{46} + x^{26} + x^{58} + x^{54} + x^{50} + x^{42} + x^{38} + x^{34} + x^{30})\verb|^|2 (x^{52} - x^{51} + x^{48} - x^{46} + x^{44} - x^{41} + x^{40} + x^{32} - x^{31} + x^{28} - x^{26} + x^{24} - x^{21} + x^{20} + x^{12} - x^{11} + x^8 - x^6 + x^4 - x + 1))$$

G.f. as rational function with standard denominator.

```
> rgfstandnum:=simplify(numer(rgf)*standenom/denom(rgf));
  rgf:=rgfstandnum/standenom;
```

$$rgfstandnum := (9 x^{15} + 20 x^{14} + 23 x^{13} + 16 x^{12} + 10 x^{11} + 13 x^{10} + 27 x^9 + 43 x^8 + 54 x^7 + 52 x^6 + 41 x^5 + 25 x^4 + 14 x^3 + 8 x^2 + 4 x + 1) x^2 (x^{57} + x^{54} + x^{51} + x^{48} + x^{45} + x^{42} + x^{39} + x^{36} + x^{33} + x^{30} + x^{27} + x^{24} + x^{21} + x^{18} + x^{15} + x^{12} + x^9 + x^6 + x^3 + 1)\verb|^|2 (-1 + x - x^2 + x^3 + x^5 - x^4 - x^6 + x^7 - x^8 - x^{10} + x^{15} - x^{12} + x^9 + x^{13} + x^{11} + x^{19} - x^{14} - x^{16} + x^{17} - x^{22} - x^{20} - x^{18} + x^{21} - x^{56} - x^{48} - x^{52} - x^{44} - x^{40} - x^{36} - x^{32} - x^{28} - x^{24} - x^{46} + x^{41} + x^{51} - x^{26} + x^{31} + x^{59} + x^{57} - x^{58} + x^{55} - x^{54} + x^{53} - x^{50} + x^{49} + x^{47} + x^{45} + x^{43} - x^{42} + x^{39} - x^{38} + x^{37} + x^{35} - x^{34} + x^{33} - x^{30} + x^{29} + x^{27} + x^{25} + x^{23}) (-1 + x^{60}) (1 + x^2 + x^4 + x^6 + x^8 + x^{10} + x^{12} + x^{14} + x^{16} + x^{22} + x^{20} + x^{18} + x^{56} + x^{48} + x^{52} + x^{44} + x^{40} + x^{36} + x^{32} + x^{28} + x^{24} + x^{46} + x^{26} + x^{58} + x^{54} + x^{50} + x^{42} + x^{38} + x^{34} + x^{30}) (x^{52} - x^{51} + x^{48} - x^{46} + x^{44} - x^{41} + x^{40} + x^{32} - x^{31} + x^{28} - x^{26} + x^{24} - x^{21} + x^{20} + x^{12} - x^{11} + x^8 - x^6 + x^4 - x + 1)$$

$$rgf := \frac{1}{(1 - x^{60})^6} ((9 x^{15} + 20 x^{14} + 23 x^{13} + 16 x^{12} + 10 x^{11} + 13 x^{10} + 27 x^9 + 43 x^8 + 54 x^7 + 52 x^6 + 41 x^5 + 25 x^4 + 14 x^3 + 8 x^2 + 4 x + 1) x^2 (x^{57} + x^{54} + x^{51} + x^{48} + x^{45} + x^{42} + x^{39} + x^{36} + x^{33} + x^{30} + x^{27} + x^{24} + x^{21} + x^{18} + x^{15} + x^{12} + x^9 + x^6 + x^3 + 1)\verb|^|2 (-1 + x - x^2 + x^3 + x^5 - x^4 - x^6 + x^7 - x^8 - x^{10} + x^{15} - x^{12} + x^9 + x^{13} + x^{11} + x^{19} - x^{14} - x^{16} + x^{17} - x^{22} - x^{20} - x^{18} + x^{21} - x^{56} - x^{48} - x^{52} - x^{44} - x^{40} - x^{36} - x^{32} - x^{28} - x^{24} - x^{46} + x^{41} + x^{51} - x^{26} + x^{31} + x^{59} + x^{57} - x^{58} + x^{55} - x^{54} + x^{53} - x^{50} + x^{49} + x^{47} + x^{45} + x^{43} - x^{42} + x^{39} - x^{38} + x^{37} + x^{35} - x^{34} + x^{33} - x^{30} + x^{29} + x^{27} + x^{25} + x^{23}) (-1 + x^{60}) (1 + x^2 + x^4 + x^6 + x^8 + x^{10} + x^{12} + x^{14} + x^{16} + x^{22} + x^{20} + x^{18} + x^{56} + x^{48} + x^{52} + x^{44} + x^{40} + x^{36} + x^{32} + x^{28} + x^{24} + x^{46} + x^{26} + x^{58} + x^{54} + x^{50} + x^{42} + x^{38} + x^{34} + x^{30}) (x^{52} - x^{51} + x^{48} - x^{46} + x^{44} - x^{41} + x^{40} + x^{32} - x^{31} + x^{28} - x^{26} + x^{24} - x^{21} + x^{20} + x^{12} - x^{11} + x^8 - x^6 + x^4 - x + 1))$$

Expand the series to find the first few values of the number of squares.

```
> Lseries:=series(Lgf,x=0,enddegree+1);
```

$$Lseries := 12 x^4 + 48 x^5 + 120 x^6 + 384 x^7 + 1068 x^8 + 2472 x^9 + 4896 x^{10} + 9072 x^{11} + 15516 x^{12} + 25608 x^{13} + 40296 x^{14} + 61608 x^{15} + 91068 x^{16} + 131640 x^{17} + 185136 x^{18} + 255960 x^{19}$$

$+ 346860\, x^{20} + 463248\, x^{21} + 608088\, x^{22} + 789240\, x^{23} + 1010316\, x^{24} + 1280544\, x^{25} + 1604832\, x^{26} + 1994064\, x^{27} + 2454012\, x^{28} + 2998656\, x^{29} + 3633912\, x^{30} + 4376064\, x^{31} + 5232972\, x^{32} + 6223080\, x^{33} + 7354896\, x^{34} + 8650896\, x^{35} + 10120236\, x^{36} + 11787768\, x^{37} + 13665096\, x^{38} + 15780024\, x^{39} + 18144876\, x^{40} + 20792280\, x^{41} + 23734848\, x^{42} + 27008664\, x^{43} + 30629580\, x^{44} + 34636560\, x^{45} + 39046104\, x^{46} + 43903080\, x^{47} + 49224924\, x^{48} + 55060176\, x^{49} + 61429440\, x^{50} + 68385072\, x^{51} + 75948540\, x^{52} + 84179040\, x^{53} + 93098760\, x^{54} + 102771072\, x^{55} + 113222268\, x^{56} + 124519608\, x^{57} + 136690320\, x^{58} + 149809728\, x^{59} + 163905852\, x^{60} + 179058552\, x^{61} + 195300024\, x^{62} + 212715096\, x^{63} + 231337260\, x^{64} + 251259768\, x^{65} + 272516832\, x^{66} + 295206888\, x^{67} + 319369404\, x^{68} + 345108000\, x^{69} + 372462936\, x^{70} + 401547624\, x^{71} + 432403260\, x^{72} + 465148944\, x^{73} + 499831344\, x^{74} + 536575248\, x^{75} + 575428620\, x^{76} + 616526544\, x^{77} + 659917992\, x^{78} + 705744528\, x^{79} + 754061100\, x^{80} + 805015320\, x^{81} + 858663072\, x^{82} + 915163776\, x^{83} + 974574684\, x^{84} + 1037061624\, x^{85} + 1102688184\, x^{86} + 1171626888\, x^{87} + 1243942908\, x^{88} + 1319821224\, x^{89} + 1399327872\, x^{90} + 1482655176\, x^{91} + 1569876156\, x^{92} + 1661190336\, x^{93} + 1756672104\, x^{94} + 1856534376\, x^{95} + 1960852908\, x^{96} + 2069848032\, x^{97} + 2183602992\, x^{98} + 2302346112\, x^{99} + 2426162076\, x^{100} + O(x^{101})$

Expand the series to find the first few values of the number of reduced squares.

```
> Rseries:=series(Rgf,x=0,enddegree+1);
```

$Rseries := 12\, x^{2} + 24\, x^{3} + 36\, x^{4} + 192\, x^{5} + 420\, x^{6} + 720\, x^{7} + 1020\, x^{8} + 1752\, x^{9} + 2268\, x^{10} + 3648\, x^{11} + 4596\, x^{12} + 6624\, x^{13} + 8148\, x^{14} + 11112\, x^{15} + 12924\, x^{16} + 17328\, x^{17} + 20076\, x^{18} + 25488\, x^{19} + 28452\, x^{20} + 36312\, x^{21} + 39924\, x^{22} + 49152\, x^{23} + 54060\, x^{24} + 64944\, x^{25} + 70716\, x^{26} + 84696\, x^{27} + 90612\, x^{28} + 106896\, x^{29} + 114756\, x^{30} + 133200\, x^{31} + 141708\, x^{32} + 164184\, x^{33} + 173340\, x^{34} + 198192\, x^{35} + 209796\, x^{36} + 237600\, x^{37} + 249924\, x^{38} + 282552\, x^{39} + 295164\, x^{40} + 331248\, x^{41} + 347100\, x^{42} + 386064\, x^{43} + 402564\, x^{44} + 447432\, x^{45} + 464868\, x^{46} + 513408\, x^{47} + 534012\, x^{48} + 586368\, x^{49} + 607836\, x^{50} + 667032\, x^{51} + 689220\, x^{52} + 752592\, x^{53} + 778884\, x^{54} + 846144\, x^{55} + 873372\, x^{56} + 948696\, x^{57} + 976716\, x^{58} + 1056576\, x^{59} + 1088772\, x^{60} + 1173600\, x^{61} + 1207092\, x^{62} + 1300344\, x^{63} + 1334556\, x^{64} + 1432992\, x^{65} + 1472460\, x^{66} + 1576080\, x^{67} + 1616340\, x^{68} + 1729752\, x^{69} + 1770948\, x^{70} + 1890048\, x^{71} + 1936716\, x^{72} + 2061504\, x^{73} + 2109468\, x^{74} + 2244552\, x^{75} + 2293524\, x^{76} + 2435088\, x^{77} + 2490036\, x^{78} + 2637648\, x^{79} + 2693532\, x^{80} + 2852952\, x^{81} + 2910204\, x^{82} + 3076032\, x^{83} + 3139620\, x^{84} + 3312144\, x^{85} + 3377316\, x^{86} + 3562296\, x^{87} + 3628332\, x^{88} + 3820656\, x^{89} + 3893676\, x^{90} + 4093200\, x^{91} + 4167588\, x^{92} + 4380504\, x^{93} + 4456260\, x^{94} + 4676592\, x^{95} + 4759836\, x^{96} + 4988160\, x^{97} + 5072844\, x^{98} + 5315352\, x^{99} + 5401044\, x^{100} + O(x^{101})$

Expand the series to find the first few values of the number of symmetry types.

```
> lseries:=series(lgf,x=0,enddegree+1);
```

$lseries := x^{4} + 4\, x^{5} + 10\, x^{6} + 24\, x^{7} + 53\, x^{8} + 106\, x^{9} + 191\, x^{10} + 328\, x^{11} + 528\, x^{12} + 822\, x^{13} + 1230\, x^{14} + 1794\, x^{15} + 2542\, x^{16} + 3534\, x^{17} + 4802\, x^{18} + 6428\, x^{19} + 8460\, x^{20} + 10996\, x^{21} + 14087\, x^{22} + 17870\, x^{23} + 22405\, x^{24} + 27850\, x^{25} + 34286\, x^{26} + 41896\, x^{27} + 50773\, x^{28} + 61148\, x^{29} + 73116\, x^{30} + 86942\, x^{31} + 102751\, x^{32} + 120840\, x^{33} + 141343\, x^{34} + 164618\, x^{35} + 190808\, x^{36} + 220306\, x^{37} + 253292\, x^{38} + 290202\, x^{39} + 331226\, x^{40} + 376872\, x^{41} + 427334\, x^{42} + 483170\, x^{43} + 544622\, x^{44} + 612290\, x^{45} + 686425\, x^{46} + 767714\, x^{47} + 856421\, x^{48} + 953286\, x^{49} + 1058620\, x^{50} + 1173218\, x^{51} + 1297403\, x^{52} + 1432070\, x^{53} + 1577552\, x^{54} + 1734804\, x^{55} + 1904219\, x^{56} + 2086808\, x^{57}$

$$+ 2282977 x^{58} + 2493854 x^{59} + 2719856 x^{60} + 2962176 x^{61} + 3221292 x^{62} + 3498468 x^{63} + 3794200 x^{64} + 4109874 x^{65} + 4445996 x^{66} + 4804026 x^{67} + 5184546 x^{68} + 5589090 x^{69} + 6018251 x^{70} + 6473704 x^{71} + 6956055 x^{72} + 7467060 x^{73} + 8007404 x^{74} + 8578924 x^{75} + 9182323 x^{76} + 9819586 x^{77} + 10491430 x^{78} + 11199932 x^{79} + 11945895 x^{80} + 12731482 x^{81} + 13557509 x^{82} + 14426308 x^{83} + 15338714 x^{84} + 16297150 x^{85} + 17302542 x^{86} + 18357408 x^{87} + 19462696 x^{88} + 20621102 x^{89} + 21833586 x^{90} + 23102948 x^{91} + 24430248 x^{92} + 25818388 x^{93} + 27268447 x^{94} + 28783518 x^{95} + 30364699 x^{96} + 32015188 x^{97} + 33736190 x^{98} + 35531016 x^{99} + 37400891 x^{100} + O(x^{101})$$

Expand the series to find the first few values of the number of reduced symmetry types.

```
> rseries:=series(rgf,x=0,enddegree+1);
```

$$rseries := x^2 + 2 x^3 + 3 x^4 + 8 x^5 + 15 x^6 + 24 x^7 + 32 x^8 + 52 x^9 + 63 x^{10} + 94 x^{11} + 114 x^{12} + 156 x^{13} + 184 x^{14} + 244 x^{15} + 276 x^{16} + 358 x^{17} + 406 x^{18} + 504 x^{19} + 555 x^{20} + 692 x^{21} + 752 x^{22} + 910 x^{23} + 991 x^{24} + 1174 x^{25} + 1267 x^{26} + 1498 x^{27} + 1593 x^{28} + 1858 x^{29} + 1983 x^{30} + 2280 x^{31} + 2414 x^{32} + 2772 x^{33} + 2915 x^{34} + 3308 x^{35} + 3488 x^{36} + 3924 x^{37} + 4114 x^{38} + 4622 x^{39} + 4816 x^{40} + 5374 x^{41} + 5616 x^{42} + 6216 x^{43} + 6467 x^{44} + 7154 x^{45} + 7418 x^{46} + 8158 x^{47} + 8469 x^{48} + 9264 x^{49} + 9587 x^{50} + 10482 x^{51} + 10815 x^{52} + 11770 x^{53} + 12163 x^{54} + 13174 x^{55} + 13580 x^{56} + 14708 x^{57} + 15125 x^{58} + 16318 x^{59} + 16796 x^{60} + 18060 x^{61} + 18556 x^{62} + 19942 x^{63} + 20448 x^{64} + 21908 x^{65} + 22490 x^{66} + 24024 x^{67} + 24617 x^{68} + 26292 x^{69} + 26898 x^{70} + 28654 x^{71} + 29339 x^{72} + 31176 x^{73} + 31879 x^{74} + 33864 x^{75} + 34581 x^{76} + 36658 x^{77} + 37461 x^{78} + 39624 x^{79} + 40440 x^{80} + 42772 x^{81} + 43607 x^{82} + 46030 x^{83} + 46956 x^{84} + 49474 x^{85} + 50422 x^{86} + 53118 x^{87} + 54078 x^{88} + 56878 x^{89} + 57938 x^{90} + 60840 x^{91} + 61919 x^{92} + 65012 x^{93} + 66110 x^{94} + 69308 x^{95} + 70513 x^{96} + 73824 x^{97} + 75049 x^{98} + 78562 x^{99} + 79801 x^{100} + O(x^{101})$$

Find the constituents

Calculate the zeroth constituent of the **total magilatin counting function** . Find its constant term.

```
> Lzeroth:=expand(
  sum(coeff(Lgfstandnum,x,p*jj)*binomial(d+t/p-jj,d),jj=0..d+1) );
  print(subs(t=0,Lzeroth)):
```

$$Lzeroth := -948 + 994\, t - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5$$

$$-948$$

Extract the constituents of the total magilatin counting function.

```
> Lconstituent[0]:=Lzeroth:
  for r from 1 to p do
    Lconstituent[r]:=expand(sum(
  coeff(Lgfstandnum,x,p*jj+r)*binomial(d+(t-r)/p-jj,d),   jj=0..d)):
  #  print(r):
  #  print( Lconstituent[r] ):
    print( factor(Lconstituent[r]) ):
  od;
```

$$Lconstituent_1 := \frac{1909}{2} t - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{76933}{120}$$

$$\frac{1}{120} (t - 1) (36 t^4 - 729 t^3 + 7351 t^2 - 37607 t + 76933)$$

$$Lconstituent_2 := 994 t - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{2780}{3}$$

$$\frac{1}{120} (t - 2) (36 t^4 - 693 t^3 + 6694 t^2 - 31840 t + 55600)$$

$$Lconstituent_3 := \frac{1925}{2} t - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{35607}{40}$$

$$\frac{1}{120} (t - 3) (36 t^4 - 657 t^3 + 6109 t^2 - 26631 t + 35607)$$

$$Lconstituent_4 := 986 t - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{13292}{15}$$

$$986 t - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{13292}{15}$$

$$Lconstituent_5 := \frac{1925}{2} t - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{18433}{24}$$

$$\frac{1925}{2} t - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{18433}{24}$$

$$Lconstituent_6 := 994 t - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{4452}{5}$$

$$994 t - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{4452}{5}$$

$$Lconstituent_7 := \frac{1909}{2} t - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{18497}{24}$$

$$\frac{1909}{2} t - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{18497}{24}$$

$$Lconstituent_8 := 994 t - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{14332}{15}$$

$$994 t - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{14332}{15}$$

$$Lconstituent_9 := \frac{1925}{2} t - \frac{32727}{40} - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5$$

$$\frac{1925}{2} t - \frac{32727}{40} - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5$$

$$Lconstituent_{10} := 986 t - \frac{2572}{3} - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5$$

$$986 t - \frac{2572}{3} - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5$$

$$Lconstituent_{11} := \frac{1925}{2}t - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{93893}{120}$$

$$\frac{1925}{2}t - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{93893}{120}$$

$$Lconstituent_{12} := -948 + 994\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$-948 + 994\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$Lconstituent_{13} := \frac{1909}{2}t - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{87301}{120}$$

$$\frac{1909}{2}t - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{87301}{120}$$

$$Lconstituent_{14} := 994\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{14332}{15}$$

$$994\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{14332}{15}$$

$$Lconstituent_{15} := \frac{1925}{2}t - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{6891}{8}$$

$$\frac{1925}{2}t - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{6891}{8}$$

$$Lconstituent_{16} := 986\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{11996}{15}$$

$$986\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{11996}{15}$$

$$Lconstituent_{17} := \frac{1925}{2}t - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{18433}{24}$$

$$\frac{1925}{2}t - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{18433}{24}$$

$$Lconstituent_{18} := -\frac{4884}{5} + 994\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$-\frac{4884}{5} + 994\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$Lconstituent_{19} := \frac{1909}{2}t - \frac{95941}{120} - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$\frac{1909}{2}t - \frac{95941}{120} - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$Lconstituent_{20} := 994\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{2780}{3}$$

$$\frac{1}{120}(t-2)\,(36\,t^4 - 693\,t^3 + 6694\,t^2 - 31840\,t + 55600)$$

$$Lconstituent_{21} := \frac{1925}{2}t - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{29271}{40}$$

$$\frac{1925}{2}t - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{29271}{40}$$

$$Lconstituent_{22} := 986\,t - \frac{2572}{3} - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$986\,t - \frac{2572}{3} - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$Lconstituent_{23} := \frac{1925}{2}t - \frac{104261}{120} - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$\frac{1925}{2}t - \frac{104261}{120} - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$Lconstituent_{24} := -\frac{4884}{5} + 994\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$-\frac{4884}{5} + 994\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$Lconstituent_{25} := \frac{1909}{2}t - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{16769}{24}$$

$$\frac{1909}{2}t - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{16769}{24}$$

$$Lconstituent_{26} := 994\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{13036}{15}$$

$$994\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{13036}{15}$$

$$Lconstituent_{27} := \frac{1925}{2}t - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{6891}{8}$$

$$\frac{1925}{2}t - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{6891}{8}$$

$$Lconstituent_{28} := 986\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{13292}{15}$$

$$986\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{13292}{15}$$

$$Lconstituent_{29} := \frac{1925}{2}t - \frac{95621}{120} - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$\frac{1925}{2}t - \frac{95621}{120} - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$Lconstituent_{30} := -948 + 994\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$-948 + 994\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$Lconstituent_{31} := \frac{1909}{2} t - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{85573}{120}$$

$$\frac{1909}{2} t - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{85573}{120}$$

$$Lconstituent_{32} := 994\, t - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{2780}{3}$$

$$\frac{1}{120} (t - 2) (36\, t^4 - 693\, t^3 + 6694\, t^2 - 31840\, t + 55600)$$

$$Lconstituent_{33} := \frac{1925}{2} t - \frac{32727}{40} - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5$$

$$\frac{1925}{2} t - \frac{32727}{40} - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5$$

$$Lconstituent_{34} := 986\, t - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{13292}{15}$$

$$986\, t - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{13292}{15}$$

$$Lconstituent_{35} := - \frac{20161}{24} + \frac{1925}{2} t - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5$$

$$- \frac{20161}{24} + \frac{1925}{2} t - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5$$

$$Lconstituent_{36} := 994\, t - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{4452}{5}$$

$$994\, t - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{4452}{5}$$

$$Lconstituent_{37} := \frac{1909}{2} t - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{16769}{24}$$

$$\frac{1909}{2} t - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{16769}{24}$$

$$Lconstituent_{38} := 994\, t - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{14332}{15}$$

$$994\, t - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{14332}{15}$$

$$Lconstituent_{39} := \frac{1925}{2} t - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{35607}{40}$$

$$\frac{1}{120} (t - 3) (36\, t^4 - 657\, t^3 + 6109\, t^2 - 26631\, t + 35607)$$

$$Lconstituent_{40} := 986\, t - \frac{2572}{3} - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5$$

$$986\, t - \frac{2572}{3} - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5$$

$$Lconstituent_{41} := \frac{1925}{2}t - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{85253}{120}$$

$$\frac{1925}{2}t - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{85253}{120}$$

$$Lconstituent_{42} := -948 + 994\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$-948 + 994\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$Lconstituent_{43} := \frac{1909}{2}t - \frac{95941}{120} - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$\frac{1909}{2}t - \frac{95941}{120} - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$Lconstituent_{44} := 994\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{14332}{15}$$

$$994\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{14332}{15}$$

$$Lconstituent_{45} := -\frac{6315}{8} + \frac{1925}{2}t - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$-\frac{6315}{8} + \frac{1925}{2}t - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$Lconstituent_{46} := 986\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{11996}{15}$$

$$986\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{11996}{15}$$

$$Lconstituent_{47} := -\frac{20161}{24} + \frac{1925}{2}t - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$-\frac{20161}{24} + \frac{1925}{2}t - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$Lconstituent_{48} := -\frac{4884}{5} + 994\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$-\frac{4884}{5} + 994\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5$$

$$Lconstituent_{49} := \frac{1909}{2}t - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{87301}{120}$$

$$\frac{1909}{2}t - \frac{7493}{20}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{87301}{120}$$

$$Lconstituent_{50} := 994\,t - \frac{3769}{10}t^2 + \frac{202}{3}t^3 - \frac{51}{8}t^4 + \frac{3}{10}t^5 - \frac{2780}{3}$$

$$\frac{1}{120}(t-2)\,(36\,t^4 - 693\,t^3 + 6694\,t^2 - 31840\,t + 55600)$$

$$Lconstituent_{51} := \frac{1925}{2} t - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{32151}{40}$$

$$\frac{1925}{2} t - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{32151}{40}$$

$$Lconstituent_{52} := 986\, t - \frac{2572}{3} - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5$$

$$986\, t - \frac{2572}{3} - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5$$

$$Lconstituent_{53} := \frac{1925}{2} t - \frac{95621}{120} - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5$$

$$\frac{1925}{2} t - \frac{95621}{120} - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5$$

$$Lconstituent_{54} := -\frac{4884}{5} + 994\, t - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5$$

$$-\frac{4884}{5} + 994\, t - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5$$

$$Lconstituent_{55} := \frac{1909}{2} t - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{18497}{24}$$

$$\frac{1909}{2} t - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{18497}{24}$$

$$Lconstituent_{56} := 994\, t - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{13036}{15}$$

$$994\, t - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{13036}{15}$$

$$Lconstituent_{57} := -\frac{6315}{8} + \frac{1925}{2} t - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5$$

$$-\frac{6315}{8} + \frac{1925}{2} t - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5$$

$$Lconstituent_{58} := 986\, t - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{13292}{15}$$

$$986\, t - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5 - \frac{13292}{15}$$

$$Lconstituent_{59} := \frac{1925}{2} t - \frac{104261}{120} - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5$$

$$\frac{1925}{2} t - \frac{104261}{120} - \frac{7493}{20} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5$$

$$Lconstituent_{60} := -948 + 994\, t - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5$$

$$-948 + 994\, t - \frac{3769}{10} t^2 + \frac{202}{3} t^3 - \frac{51}{8} t^4 + \frac{3}{10} t^5$$

Extract the coefficients of the constituents.

```
> for r from 1 to p do
    for coeffdeg from 0 to d do
     Lc[coeffdeg,r]:=coeff(Lconstituent[r],t,coeffdeg):
     #print( r, Lc[coeffdeg,r] ):
    od:
  od:
```

Print and analyze the constituent coefficients for periods. First the higher coefficients, which ought to be constant. Print the first coefficient, then any that don't repeat the preceding value.

```
> for coeffdeg from 3 to d do
     print("degree", coeffdeg, "coeff", Lc[coeffdeg,1]):
     print(1,Lc[coeffdeg,1]);
     for r from 2 to p do
       stepdifference:=Lc[coeffdeg,r]-Lc[coeffdeg,r-1]:
       if( stepdifference<>0 ) then
       print(r,Lc[coeffdeg,r],stepdifference):
       fi:
     od:
     print("Compared all coefficients of degree", coeffdeg);
  od:
```

$$\text{"degree"}, 3, \text{"coeff"}, \frac{202}{3}$$

$$1, \frac{202}{3}$$

"Compared all coefficients of degree", 3

$$\text{"degree"}, 4, \text{"coeff"}, \frac{-51}{8}$$

$$1, \frac{-51}{8}$$

"Compared all coefficients of degree", 4

$$\text{"degree"}, 5, \text{"coeff"}, \frac{3}{10}$$

$$1, \frac{3}{10}$$

"Compared all coefficients of degree", 5

Next, the constant terms, whose period is expected to be p. Print all constant terms up to the presumed period "stepsize". Print the difference (at step "stepsize") if they are not repeating.

Note that the even terms repeat at step 30 (a period of 15, half the expected period).

```
> stepsize:=30;
  for r from 1 to stepsize do
     print(r, Lc[0,r]);
  od:
  for r from stepsize+1 to p do
     stepdifference:=Lc[0,r]-Lc[0,r-stepsize]:
     if( stepdifference<>0 ) then print(r,Lc[0,r],stepdifference): fi:
     #print(r,Lc[0,r],stepdifference);
```

```
od:
print("Constant terms completed.");
```

$$stepsize := 30$$

$$1, \frac{-76933}{120}$$

$$2, \frac{-2780}{3}$$

$$3, \frac{-35607}{40}$$

$$4, \frac{-13292}{15}$$

$$5, \frac{-18433}{24}$$

$$6, \frac{-4452}{5}$$

$$7, \frac{-18497}{24}$$

$$8, \frac{-14332}{15}$$

$$9, \frac{-32727}{40}$$

$$10, \frac{-2572}{3}$$

$$11, \frac{-93893}{120}$$

$$12, -948$$

$$13, \frac{-87301}{120}$$

$$14, \frac{-14332}{15}$$

$$15, \frac{-6891}{8}$$

$$16, \frac{-11996}{15}$$

$$17, \frac{-18433}{24}$$

$$18, \frac{-4884}{5}$$

$$19, \frac{-95941}{120}$$

$$20, \frac{-2780}{3}$$

$$21, \frac{-29271}{40}$$

$$22, \frac{-2572}{3}$$

$$23, \frac{-104261}{120}$$

$$24, \frac{-4884}{5}$$

$$25, \frac{-16769}{24}$$

$$26, \frac{-13036}{15}$$

$$27, \frac{-6891}{8}$$

$$28, \frac{-13292}{15}$$

$$29, \frac{-95621}{120}$$

$$30, -948$$

$$31, \frac{-85573}{120}, -72$$

$$33, \frac{-32727}{40}, 72$$

$$35, \frac{-20161}{24}, -72$$

$$37, \frac{-16769}{24}, 72$$

$$39, \frac{-35607}{40}, -72$$

$$41, \frac{-85253}{120}, 72$$

$$43, \frac{-95941}{120}, -72$$

$$45, \frac{-6315}{8}, 72$$

$$47, \frac{-20161}{24}, -72$$

$$49, \frac{-87301}{120}, 72$$

$$51, \frac{-32151}{40}, -72$$

$$53, \frac{-95621}{120}, 72$$

$$55, \frac{-18497}{24}, -72$$

$$57, \frac{-6315}{8}, 72$$

$$59, \frac{-104261}{120}, -72$$

"Constant terms completed."

Now, the linear terms. First print all linear coefficients up to the presumed period "stepsize".. Then analyze for period and print the difference (at step "stepsize") if they are not repeating.

```
> stepsize:=6;
  for r from 1 to stepsize do
    print(r, Lc[1,r]);
  od:
  for r from stepsize+1 to p do
    stepdifference:=Lc[1,r]-Lc[1,r-stepsize]:
    if( stepdifference<>0 ) then print(r,Lc[1,r],stepdifference): fi:
  od:
  print("Linear coefficients completed.");
```

$$stepsize := 6$$

$$1, \frac{1909}{2}$$

$$2, 994$$

$$3, \frac{1925}{2}$$

$$4, 986$$

$$5, \frac{1925}{2}$$

$$6, 994$$

"Linear coefficients completed."

The quadratic terms. First print all quadratic coefficients up to the presumed period "stepsize".. Then analyze for period and print the difference (at step "stepsize") if they are not repeating.

```
> stepsize:=2;
  for r from 1 to stepsize do
    print(r, Lc[2,r]);
  od:
  for r from stepsize+1 to p do
    stepdifference:=Lc[2,r]-Lc[2,r-stepsize]:
    if( stepdifference<>0 ) then print(r,Lc[2,r],stepdifference): fi:
```

```
od:
print("Quadratic coefficients completed.");
```

$$stepsize := 2$$

$$1, \frac{-7493}{20}$$

$$2, \frac{-3769}{10}$$

"Quadratic coefficients completed."

Calculate the zeroth constituent of the **magilatin symmetry-type counting function** . Find its constant term.

```
> lzeroth:=expand(
sum(coeff(lgfstandnum,x,p*jj)*binomial(d+t/p-jj,d),jj=0..d+1) );
print(subs(t=0,lzeroth)):
```

$$lzeroth := -9 + \frac{17}{2} t - \frac{2029}{720} t^2 + \frac{97}{216} t^3 - \frac{3}{64} t^4 + \frac{1}{240} t^5$$

$$-9$$

Extract the constituents of the magilatin symmetry-type counting function.

```
> lconstituent[0]:=lzeroth:
for r from 1 to p do
  lconstituent[r]:=expand(sum(
coeff(lgfstandnum,x,p*jj+r)*binomial(d+(t-r)/p-jj,d),   jj=0..d)):
#  print(r):
#  print( lconstituent[r] ):
  print( factor(lconstituent[r]) ):
od;
```

$$lconstituent_1 := \frac{1163}{144} t - \frac{4013}{1440} t^2 + \frac{97}{216} t^3 - \frac{3}{64} t^4 + \frac{1}{240} t^5 - \frac{49213}{8640}$$

$$\frac{1}{8640} (t - 1) (36 t^4 - 369 t^3 + 3511 t^2 - 20567 t + 49213)$$

$$lconstituent_2 := \frac{17}{2} t - \frac{2029}{720} t^2 + \frac{97}{216} t^3 - \frac{3}{64} t^4 + \frac{1}{240} t^5 - \frac{235}{27}$$

$$\frac{1}{8640} (t - 2) (36 t^4 - 333 t^3 + 3214 t^2 - 17920 t + 37600)$$

$$lconstituent_3 := \frac{131}{16} t - \frac{4013}{1440} t^2 + \frac{97}{216} t^3 - \frac{3}{64} t^4 + \frac{1}{240} t^5 - \frac{2823}{320}$$

$$\frac{1}{8640} (t - 3) (36 t^4 - 297 t^3 + 2989 t^2 - 15111 t + 25407)$$

$$lconstituent_4 := \frac{151}{18} t - \frac{1144}{135} - \frac{2029}{720} t^2 + \frac{97}{216} t^3 - \frac{3}{64} t^4 + \frac{1}{240} t^5$$

$$\frac{151}{18} t - \frac{1144}{135} - \frac{2029}{720} t^2 + \frac{97}{216} t^3 - \frac{3}{64} t^4 + \frac{1}{240} t^5$$

$$lconstituent_5 := \frac{131}{16} t - \frac{12313}{1728} - \frac{4013}{1440} t^2 + \frac{97}{216} t^3 - \frac{3}{64} t^4 + \frac{1}{240} t^5$$

$$\frac{131}{16}t - \frac{12313}{1728} - \frac{4013}{1440}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5$$

$$lconstituent_6 := \frac{17}{2}t - \frac{2029}{720}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5 - \frac{41}{5}$$

$$\frac{17}{2}t - \frac{2029}{720}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5 - \frac{41}{5}$$

$$lconstituent_7 := \frac{1163}{144}t - \frac{4013}{1440}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5 - \frac{12953}{1728}$$

$$\frac{1163}{144}t - \frac{4013}{1440}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5 - \frac{12953}{1728}$$

$$lconstituent_8 := \frac{17}{2}t - \frac{2029}{720}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5 - \frac{1229}{135}$$

$$\frac{17}{2}t - \frac{2029}{720}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5 - \frac{1229}{135}$$

$$lconstituent_9 := -\frac{2503}{320} + \frac{131}{16}t - \frac{4013}{1440}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5$$

$$-\frac{2503}{320} + \frac{131}{16}t - \frac{4013}{1440}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5$$

$$lconstituent_{10} := \frac{151}{18}t - \frac{2029}{720}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5 - \frac{218}{27}$$

$$\frac{151}{18}t - \frac{2029}{720}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5 - \frac{218}{27}$$

$$lconstituent_{11} := \frac{131}{16}t - \frac{4013}{1440}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5 - \frac{63293}{8640}$$

$$\frac{131}{16}t - \frac{4013}{1440}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5 - \frac{63293}{8640}$$

$$lconstituent_{12} := -9 + \frac{17}{2}t - \frac{2029}{720}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5$$

$$-9 + \frac{17}{2}t - \frac{2029}{720}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5$$

$$lconstituent_{13} := \frac{1163}{144}t - \frac{4013}{1440}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5 - \frac{59581}{8640}$$

$$\frac{1163}{144}t - \frac{4013}{1440}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5 - \frac{59581}{8640}$$

$$lconstituent_{14} := \frac{17}{2}t - \frac{2029}{720}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5 - \frac{1229}{135}$$

$$\frac{17}{2}t - \frac{2029}{720}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5 - \frac{1229}{135}$$

$$lconstituent_{15} := \frac{131}{16}t - \frac{539}{64} - \frac{4013}{1440}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5$$

$$\frac{131}{16}t-\frac{539}{64}-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$lconstituent_{16}:=\frac{151}{18}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{982}{135}$$

$$\frac{151}{18}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{982}{135}$$

$$lconstituent_{17}:=\frac{131}{16}t-\frac{12313}{1728}-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$\frac{131}{16}t-\frac{12313}{1728}-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$lconstituent_{18}:=\frac{17}{2}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{47}{5}$$

$$\frac{17}{2}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{47}{5}$$

$$lconstituent_{19}:=\frac{1163}{144}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{68221}{8640}$$

$$\frac{1163}{144}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{68221}{8640}$$

$$lconstituent_{20}:=\frac{17}{2}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{235}{27}$$

$$\frac{1}{8640}(t-2)(36t^4-333t^3+3214t^2-17920t+37600)$$

$$lconstituent_{21}:=\frac{131}{16}t-\frac{2119}{320}-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$\frac{131}{16}t-\frac{2119}{320}-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$lconstituent_{22}:=\frac{151}{18}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{218}{27}$$

$$\frac{151}{18}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{218}{27}$$

$$lconstituent_{23}:=\frac{131}{16}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{73661}{8640}$$

$$\frac{131}{16}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{73661}{8640}$$

$$lconstituent_{24}:=\frac{17}{2}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{47}{5}$$

$$\frac{17}{2}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{47}{5}$$

$$lconstituent_{25}:=\frac{1163}{144}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{11225}{1728}$$

$$\frac{1163}{144}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{11225}{1728}$$

$$lconstituent_{26} := -\frac{1067}{135}+\frac{17}{2}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$-\frac{1067}{135}+\frac{17}{2}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$lconstituent_{27} := \frac{131}{16}t-\frac{539}{64}-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$\frac{131}{16}t-\frac{539}{64}-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$lconstituent_{28} := \frac{151}{18}t-\frac{1144}{135}-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$\frac{151}{18}t-\frac{1144}{135}-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$lconstituent_{29} := \frac{131}{16}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{65021}{8640}$$

$$\frac{131}{16}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{65021}{8640}$$

$$lconstituent_{30} := -9+\frac{17}{2}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$-9+\frac{17}{2}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$lconstituent_{31} := \frac{1163}{144}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{57853}{8640}$$

$$\frac{1163}{144}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{57853}{8640}$$

$$lconstituent_{32} := \frac{17}{2}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{235}{27}$$

$$\frac{1}{8640}(t-2)\,(36\,t^4-333\,t^3+3214\,t^2-17920\,t+37600)$$

$$lconstituent_{33} := -\frac{2503}{320}+\frac{131}{16}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$-\frac{2503}{320}+\frac{131}{16}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$lconstituent_{34} := \frac{151}{18}t-\frac{1144}{135}-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$\frac{151}{18}t-\frac{1144}{135}-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$lconstituent_{35} := \frac{131}{16}t-\frac{14041}{1728}-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$\frac{131}{16}t-\frac{14041}{1728}-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$lconstituent_{36}:=\frac{17}{2}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{41}{5}$$

$$\frac{17}{2}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{41}{5}$$

$$lconstituent_{37}:=\frac{1163}{144}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{11225}{1728}$$

$$\frac{1163}{144}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{11225}{1728}$$

$$lconstituent_{38}:=\frac{17}{2}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{1229}{135}$$

$$\frac{17}{2}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{1229}{135}$$

$$lconstituent_{39}:=\frac{131}{16}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{2823}{320}$$

$$\frac{1}{8640}(t-3)(36t^4-297t^3+2989t^2-15111t+25407)$$

$$lconstituent_{40}:=\frac{151}{18}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{218}{27}$$

$$\frac{151}{18}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{218}{27}$$

$$lconstituent_{41}:=\frac{131}{16}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{54653}{8640}$$

$$\frac{131}{16}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{54653}{8640}$$

$$lconstituent_{42}:=-9+\frac{17}{2}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$-9+\frac{17}{2}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$lconstituent_{43}:=\frac{1163}{144}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{68221}{8640}$$

$$\frac{1163}{144}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{68221}{8640}$$

$$lconstituent_{44}:=\frac{17}{2}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{1229}{135}$$

$$\frac{17}{2}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{1229}{135}$$

$$lconstituent_{45}:=\frac{131}{16}t-\frac{475}{64}-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$\frac{131}{16}t-\frac{475}{64}-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$lconstituent_{46} := \frac{151}{18}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{982}{135}$$

$$\frac{151}{18}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{982}{135}$$

$$lconstituent_{47} := \frac{131}{16}t-\frac{14041}{1728}-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$\frac{131}{16}t-\frac{14041}{1728}-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$lconstituent_{48} := \frac{17}{2}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{47}{5}$$

$$\frac{17}{2}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{47}{5}$$

$$lconstituent_{49} := \frac{1163}{144}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{59581}{8640}$$

$$\frac{1163}{144}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{59581}{8640}$$

$$lconstituent_{50} := \frac{17}{2}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{235}{27}$$

$$\frac{1}{8640}(t-2)(36t^4-333t^3+3214t^2-17920t+37600)$$

$$lconstituent_{51} := -\frac{2439}{320}+\frac{131}{16}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$-\frac{2439}{320}+\frac{131}{16}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5$$

$$lconstituent_{52} := \frac{151}{18}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{218}{27}$$

$$\frac{151}{18}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{218}{27}$$

$$lconstituent_{53} := \frac{131}{16}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{65021}{8640}$$

$$\frac{131}{16}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{65021}{8640}$$

$$lconstituent_{54} := \frac{17}{2}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{47}{5}$$

$$\frac{17}{2}t-\frac{2029}{720}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{47}{5}$$

$$lconstituent_{55} := \frac{1163}{144}t-\frac{4013}{1440}t^2+\frac{97}{216}t^3-\frac{3}{64}t^4+\frac{1}{240}t^5-\frac{12953}{1728}$$

$$\frac{1163}{144}t - \frac{4013}{1440}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5 - \frac{12953}{1728}$$

$$lconstituent_{56} := -\frac{1067}{135} + \frac{17}{2}t - \frac{2029}{720}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5$$

$$-\frac{1067}{135} + \frac{17}{2}t - \frac{2029}{720}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5$$

$$lconstituent_{57} := \frac{131}{16}t - \frac{475}{64} - \frac{4013}{1440}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5$$

$$\frac{131}{16}t - \frac{475}{64} - \frac{4013}{1440}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5$$

$$lconstituent_{58} := \frac{151}{18}t - \frac{1144}{135} - \frac{2029}{720}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5$$

$$\frac{151}{18}t - \frac{1144}{135} - \frac{2029}{720}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5$$

$$lconstituent_{59} := \frac{131}{16}t - \frac{4013}{1440}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5 - \frac{73661}{8640}$$

$$\frac{131}{16}t - \frac{4013}{1440}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5 - \frac{73661}{8640}$$

$$lconstituent_{60} := -9 + \frac{17}{2}t - \frac{2029}{720}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5$$

$$-9 + \frac{17}{2}t - \frac{2029}{720}t^2 + \frac{97}{216}t^3 - \frac{3}{64}t^4 + \frac{1}{240}t^5$$

Extract the coefficients of the constituents.

```
> for r from 1 to p do
   for coeffdeg from 0 to d do
    lc[coeffdeg,r]:=coeff(lconstituent[r],t,coeffdeg):
    #print( r, lc[coeffdeg,r] ):
   od:
  od:
```

Print and analyze the constituent coefficients for periods. First the higher coefficients, which are constant. Print the first coefficient, then any that don't repeat the preceding value (there are none).

```
> for coeffdeg from 3 to d do
    print("degree", coeffdeg, "coeff", lc[coeffdeg,1]):
    for r from 2 to p do
      stepdifference:=lc[coeffdeg,r]-lc[coeffdeg,r-1]:
      if( stepdifference<>0 ) then
      print(r,lc[coeffdeg,r],stepdifference):
      fi:
    od:
  od:
```

$$\text{"degree"}, 3, \text{"coeff"}, \frac{97}{216}$$

$$\text{"degree"}, 4, \text{"coeff"}, \frac{-3}{64}$$

$$"degree", 5, "coeff", \frac{1}{240}$$

Next, the constant terms, whose period is expected to be 60. Print all constant terms up to the presumed period "stepsize". Print the difference (at step "stepsize") if they are not repeating.

```
> stepsize:=30;
  for r from 1 to stepsize do
    print(r, lc[0,r]);
  od:
  for r from stepsize+1 to p do
    stepdifference:=lc[0,r]-lc[0,r-stepsize]:
    if( stepdifference<>0 ) then print(r,lc[0,r],stepdifference): fi:
  od:
  print("Constant terms completed.");
```

$$stepsize := 30$$

$$1, \frac{-49213}{8640}$$

$$2, \frac{-235}{27}$$

$$3, \frac{-2823}{320}$$

$$4, \frac{-1144}{135}$$

$$5, \frac{-12313}{1728}$$

$$6, \frac{-41}{5}$$

$$7, \frac{-12953}{1728}$$

$$8, \frac{-1229}{135}$$

$$9, \frac{-2503}{320}$$

$$10, \frac{-218}{27}$$

$$11, \frac{-63293}{8640}$$

$$12, -9$$

$$13, \frac{-59581}{8640}$$

$$14, \frac{-1229}{135}$$

$$15, \frac{-539}{64}$$

$$16, \frac{-982}{135}$$

$$17, \frac{-12313}{1728}$$

$$18, \frac{-47}{5}$$

$$19, \frac{-68221}{8640}$$

$$20, \frac{-235}{27}$$

$$21, \frac{-2119}{320}$$

$$22, \frac{-218}{27}$$

$$23, \frac{-73661}{8640}$$

$$24, \frac{-47}{5}$$

$$25, \frac{-11225}{1728}$$

$$26, \frac{-1067}{135}$$

$$27, \frac{-539}{64}$$

$$28, \frac{-1144}{135}$$

$$29, \frac{-65021}{8640}$$

$$30, -9$$

$$31, \frac{-57853}{8640}, -1$$

$$33, \frac{-2503}{320}, 1$$

$$35, \frac{-14041}{1728}, -1$$

$$37, \frac{-11225}{1728}, 1$$

$$39, \frac{-2823}{320}, -1$$

$$41, \frac{-54653}{8640}, 1$$

$$43, \frac{-68221}{8640}, -1$$

$$45, \frac{-475}{64}, 1$$

$$47, \frac{-14041}{1728}, -1$$

$$49, \frac{-59581}{8640}, 1$$

$$51, \frac{-2439}{320}, -1$$

$$53, \frac{-65021}{8640}, 1$$

$$55, \frac{-12953}{1728}, -1$$

$$57, \frac{-475}{64}, 1$$

$$59, \frac{-73661}{8640}, -1$$

"Constant terms completed."

Now, the linear terms. First print all linear coefficients up to the presumed period "stepsize". Then analyze for period and print the difference (at step "stepsize") if they are not repeating.

```
> stepsize:=6;
  for r from 1 to stepsize do
    print(r, lc[1,r]);
  od:
  for r from stepsize+1 to p do
    stepdifference:=lc[1,r]-lc[1,r-stepsize]:
    if( stepdifference<>0 ) then print(r,lc[1,r],stepdifference): fi:
  od:
  print("Linear coefficients completed.");
```

$$stepsize := 6$$

$$1, \frac{1163}{144}$$

$$2, \frac{17}{2}$$

$$3, \frac{131}{16}$$

$$4, \frac{151}{18}$$

$$5, \frac{131}{16}$$

$$6, \frac{17}{2}$$

"Linear coefficients completed."

The quadratic terms. First print all quadratic coefficients up to the presumed period "stepsize". Then analyze for period and print the difference (at step "stepsize") if they are not repeating.

```
> stepsize:=2;
  for r from 1 to stepsize do
    print(r, lc[2,r]);
  od:
  for r from stepsize+1 to p do
    stepdifference:=lc[2,r]-lc[2,r-stepsize]:
    if( stepdifference<>0 ) then print(r,lc[2,r],stepdifference): fi:
  od:
  print("Quadratic coefficients completed.");
```

$$stepsize := 2$$

$$1, \frac{-4013}{1440}$$

$$2, \frac{-2029}{720}$$

"Quadratic coefficients completed."

```
>
```