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Adaptive control and synchronization of a modified Chua's circuit system

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Abstract

This study addresses the adaptive control and synchronization of the modified Chua's circuit system with unknown system parameters. Adaptive control law is applied to suppress chaos to one of the three steady states. Adaptive control law is applied to achieve synchronization of two identical modified Chua's circuit systems. Routh–Hurwitz criteria and Lyapunov direct method are used to study the asymptotic stability of the steady states. Numerical simulations are included to show the effectiveness of the proposed control method.

1. Introduction

Research efforts have investigated the chaos control and chaos synchronization problems in many physical chaotic systems [1–13]. The control problem attempts to stabilize a chaotic attractor to either a periodic orbit or an equilibrium point. Recently, after the pioneering work of Ott et al. [1], several control strategies for stabilizing chaos have been proposed. There are two main approaches for controlling chaos, nonfeedback control [2,3] and feedback control [4,13].

The concept of chaos synchronization involves making two chaotic systems which oscillate in a synchronized manner. The idea of synchronizing two identical chaotic systems with different initial conditions was introduced by Pecora and Carrol [14].

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Several different approaches, including some conventional linear control techniques and advanced nonlinear control schemes, are already applied to the above problems. In these works, it is essential to know the values of system parameters. In practical situations, the values of these parameters are unknown. Therefore, the derivation of an adaptive controller for the control and synchronization of chaotic systems in the presence of unknown parameters is an important issue [15–18].

Chua's circuit has been studied extensively as a prototypical electronic system [19]. Chen and Dong [5,6] applied the linear feedback control for guiding the chaotic trajectory of the circuit to limit cycle. Hartley and Mossayebi [8] showed the control of modified Chua's circuit systems and demonstrated how to design a controller for tracking the capacitor voltage of the system by applying the input–output and state space techniques. Saito and Mitsubori [9] stabilized chaotic attractor to desired periodic orbit by proposing a simple control method. Hwang et al. [10] proposed a feedback control on a modified Chua's circuit to drag the chaotic trajectory to fixed points. The modified Chua's circuit is described by the following dynamical system:

$$\begin{aligned}
\dot{x} &= p\big(y - \tfrac{1}{7}(2x^3 - x)\big),\\
\dot{y} &= x - y + z,\\
\dot{z} &= -qy,
\end{aligned} \tag{1}$$

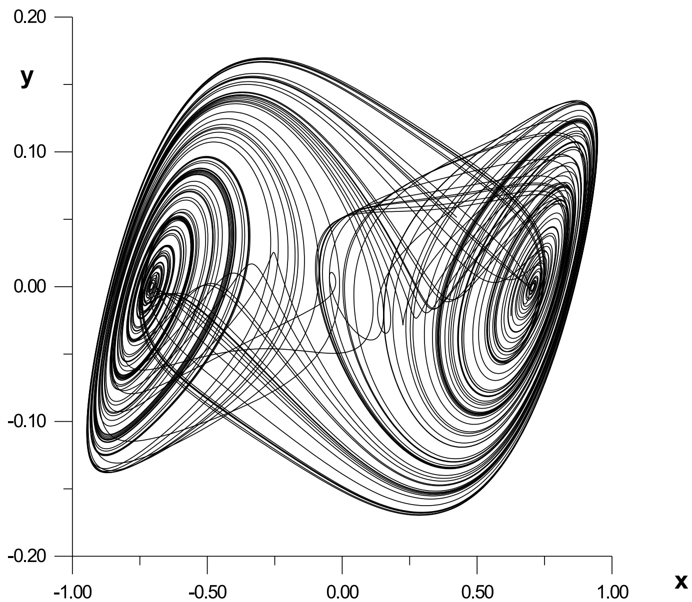

Fig. 1. The chaotic attractor of the modified Chua's circuit in the x–y plane.

where $p > 0$ and $q > 0$ are system parameters, x and y are the voltages across two capacitors and z is the current through the inductor. The system (1) has three equilibrium points $E_+ = (\sqrt{.5}, 0, -\sqrt{.5})$, $E_0 = (0,0,0)$ and $E_- = (-\sqrt{.5}, 0, \sqrt{.5})$. The chaotic attractor of the system (1) in the x–y plane is shown in Fig. 1.

The aim of this paper is to introduce a simple, smooth and adaptive controller for resolving the control and synchronization problems of the modified Chua's circuit systems. It is assumed that the two parameters p and q are unknown and only a single state variable is available for implementing the feedback controller. In Section 2, the stability of the equilibrium points E_+, E_0 and E_- is studied. In Section 3, feedback control and adaptive control are studied. In Section 4, adaptive synchronization of two identical modified Chua's circuits is studied. In Section 5, numerical simulation is presented. Section 6 is the conclusion.

Remark 1. The system has a bounded, zero volume, globally attracting set [20]. Therefore, the state trajectories $x(t)$, $y(t)$, and $z(t)$ are globally bounded for all $t \geqslant 0$ and continuously differentiable with respect to time. Consequently, there exist three positive constants s_1, s_2 and s_3 such that $|x(t)| \leqslant s_1 < \infty$, $|y(t)| \leqslant s_2 < \infty$ and $|z(t)| \leqslant s_3 < \infty$ hold for all $t \geqslant 0$.

2. Stability of the equilibrium points

Proposition 1. *For $p = 10$ and $q = \frac{100}{7}$, the three equilibrium points E_+, E_0 and E_- of the system* (1) *are unstable.*

Proof. The Jacobian matrix of the system (1) about the equilibrium point $E = (\bar{x}, \bar{y}, \bar{z})$ is

$$J0 = \begin{bmatrix} \frac{p(-6\bar{x}^2+1)}{7} & p & 0 \\ 1 & -1 & 1 \\ 0 & -q & 0 \end{bmatrix}.$$

The characteristic equation of $J0$ is

$$\lambda^3 + \left(1 + \frac{p(6\bar{x}^2 - 1)}{7}\right)\lambda^2 + \left[q + p\left(\frac{6\bar{x}^2 - 8}{7}\right)\right]\lambda + \frac{pq(6\bar{x}^2 - 1)}{7} = 0.$$

Let

$$\beta_1 = 1 + \frac{p(6\bar{x}^2 - 1)}{7}, \quad \beta_2 = q + p\left(\frac{6\bar{x}^2 - 8}{7}\right) \quad \text{and} \quad \beta_3 = \frac{pq(6\bar{x}^2 - 1)}{7}.$$

For $p = 10$ and $q = \frac{100}{7}$: Firstly, if $\bar{x} = 0$, then $\beta_1 < 0$ and $\beta_3 < 0$. According to Routh–Hurwitz criteria the equilibrium point $(0, 0, 0)$ is unstable. Secondly, if $\bar{x} = \pm\sqrt{.5}$, then $\beta_1 > 0$, $\beta_2 > 0$ and $\beta_1\beta_2 < \beta_3$. According to Routh–Hurwitz criteria the equilibrium points $E_+ = (\sqrt{.5}, 0, -\sqrt{.5})$ and $E_- = (-\sqrt{.5}, 0, \sqrt{.5})$ are unstable.

Since the equilibrium points of system (1) are unstable, the control problem takes place. □

3. Adaptive control

Let us consider the controlled system of the system (1) has the form

$$\begin{aligned}
\dot{x} &= p\big(y - \tfrac{1}{7}(2x^3 - x)\big) + u_1,\\
\dot{y} &= x - y + z + u_2,\\
\dot{z} &= -qy + u_3,
\end{aligned} \tag{2}$$

where u_1, u_2 and u_3 are external control inputs which will drag the chaotic trajectory (x, y, z) of the modified Chua system (1) to $E = (\bar{x}, \bar{y}, \bar{z})$ one of the three steady states E_-, E_0 and E_+. Let the control law take the following form:

$$u_1 = -k(x - \bar{x}), \quad u_2 = u_3 = 0,$$

where k is a positive feedback gain.

3.1. Stabilizing the equilibrium point $E = (\bar{x}, \bar{y}, \bar{z})$

In order to suppress chaos to $E = (\bar{x}, \bar{y}, \bar{z})$, we introduce the external control law $u_1 = -k(x - \bar{x}), u_2 = u_3 = 0$ with x as the feedback variable into system (2). Hence the controlled system (2) has the following form:

$$\begin{aligned}
\dot{x} &= p\big(y - \tfrac{1}{7}(2x^3 - x)\big) - k(x - \bar{x}),\\
\dot{y} &= x - y + z,\\
\dot{z} &= -qy.
\end{aligned} \tag{3}$$

The controlled system (3) has the equilibrium point $E = (\bar{x}, \bar{y}, \bar{z})$. The system (3) can be stabilized to the steady state $E = (\bar{x}, \bar{y}, \bar{z})$ if $k \geqslant k^*$ is satisfied and the system parameters are constant and known.

Let us consider that $\xi_1 = x - \bar{x}$, $\xi_2 = y - \bar{y}$, $\xi_3 = z - \bar{z}$ and $\alpha = p(\frac{1}{7}(-6\bar{x}^2 + 1))$.

Proposition 2. *The equilibrium point $E = (\bar{x}, \bar{y}, \bar{z})$ of the system* (3) *is asymptotically stable for $k \geqslant k^* = p + \alpha$, where p, q are positive.*

Proof. The Jacobian matrix of the system (3) about the equilibrium point $E = (\bar{x}, \bar{y}, \bar{z})$ is

$$J = \begin{bmatrix} p(\frac{-6\bar{x}^2+1}{7}) - k & p & 0 \\ 1 & -1 & 1 \\ 0 & -q & 0 \end{bmatrix}. \tag{4}$$

The linearized system of (3) is given by

$$\begin{aligned} \dot{\xi}_1 &= (\alpha - k)\xi_1 + p\xi_2, \\ \dot{\xi}_2 &= \xi_1 - \xi_2 + \xi_3, \\ \dot{\xi}_3 &= -q\xi_2. \end{aligned} \tag{5}$$

We study the stability of the equilibrium point $(0,0,0)$ of the system (5). Consider the Lyapunov function $V(\xi_1, \xi_2, \xi_3)$ is

$$V(\xi_1, \xi_2, \xi_3), = \frac{1}{2}\left[\frac{q}{p}\xi_1^2 + q\xi_2^2 + \xi_3^2\right]. \tag{6}$$

The time derivative of V in the neighbourhood of $(0,0,0)$ is

$$\dot{V} = -q(\xi_2 - \xi_1)^2 - \frac{q\xi_1^2}{p}(k - \alpha - p).$$

It is clear that $\dot{V} < 0$ if $k \geqslant k^* = p + \alpha$. According to Lyapunov stability theory the equilibrium point $(0,0,0)$ is asymptotically stable. □

Remark 2. The chaotic attractor of the modified Chua circuit (x, y, z) is suppressed to a limit cycle around the equilibrium point $E_0 = (0,0,0)$ by taking $k = 9.9152$ in (3) (see Fig. 5).

The feedback control law derived thus far requires that the system parameters must be known a priori. However, in many real applications it can be difficult to determine exactly the values of the system parameters. Consequently, the feedback gain k cannot be appropriately chosen to guarantee the stability of the controlled system. For overestimating k an expensive and too conservative control effort is needed. To overcome these drawbacks, an adaptive control with single state variable feedback control is derived.

An adaptive control with x as the feedback variable is added into the first equation in system (2). In this case the control law is

$$u_1 = -g(x - \bar{x}), \quad u_2 = u_3 = 0, \tag{7}$$

where g, an estimate of g_1, is updated according to the following adaptive algorithm:

$$\dot{g} = \gamma(x - \bar{x})^2, \tag{8}$$

where γ is an adaption gain. Then the controlled system (2) has the following form:

$$\begin{aligned}
\dot{x} &= p\big(y - \tfrac{1}{7}(2x^3 - x)\big) - g(x - \bar{x}),\\
\dot{y} &= x - y + z,\\
\dot{z} &= -qy,\\
\dot{g} &= \gamma(x - \bar{x})^2.
\end{aligned} \tag{9}$$

Proposition 3. *For $g = g_1 \geqslant p + \alpha$ and p, q are positive, the equilibrium point $E = (\bar{x}, \bar{y}, \bar{z})$ of the system (9) is asymptotically stable.*

Proof. Let us consider the Lyapunov function as follows:

$$V = \frac{1}{2}\left[\frac{q}{p}(x - \bar{x})^2 + q(y - \bar{y})^2 + (z - \bar{z})^2 + \frac{q}{\gamma p}(g - g_1)^2\right]. \tag{10}$$

The time derivative of V in the neighbourhood of the equilibrium point $(\bar{x}, \bar{y}, \bar{z})$ of the system (9) is

$$\begin{aligned}
\dot{V} &= q(x - \bar{x})\left(y - \frac{1}{7}(2x^3 - x)\right) - \frac{qg}{p}(x - \bar{x})^2 + q(y - \bar{y})(x - y + z)\\
&\quad -qy(z - \bar{z}) + \frac{q}{p}(g - g_1)(x - \bar{x})^2.
\end{aligned}$$

Put $\eta_1 = x - \bar{x}$, $\eta_2 = y - \bar{y}$ and $\eta_3 = z - \bar{z}$. Since $(\bar{x}, \bar{y}, \bar{z})$ is an equilibrium point of the uncontrolled system (1), then $\dot{V}$ becomes

$$\begin{aligned}
\dot{V} &= q\eta_1\left[\eta_2 - \frac{1}{7}(2\eta_1^3 - \eta_1) - \frac{1}{7}(6\bar{x}^2\eta_1 + 6\bar{x}\eta_1^2)\right] + q\eta_2(\eta_1 - \eta_2 + \eta_3)\\
&\quad - q\eta_2\eta_3 + \frac{q}{p}(g - g_1)\eta_1^2\\
&= 2q\eta_1\eta_2 - \frac{2q}{7}\eta_1^4 - q\eta_1^2\left(\frac{g_1}{p} - \frac{1}{7} + \frac{6\bar{x}^2}{7}\right) - \frac{6q\bar{x}\eta_1^3}{7} - q\eta_2^2\\
&= -q(\eta_2 - \eta_1)^2 - q\eta_1^2\left[\frac{g_1}{p} - \left(1 + \frac{(-6\bar{x}^2 + 1)}{7}\right)\right] - \frac{6q\bar{x}\eta_1^3}{7}.
\end{aligned}$$

It is clear that for the positive parameters p, q and γ, if we choose $g = g_1 \geqslant p + \alpha$ and $|\eta_1|$ is sufficiently small, then $\dot{V}$ is negative definite and from (10) the Lyapunov function V is positive definite. From Lyapunov stability theorem it follows that the equilibrium point $x = \bar{x}$, $y = \bar{y}$, $z = \bar{z}$, $g = g_1$ of the system (9) is asymptotically stable. $\square$

4. Adaptive synchronization

We assume that we have two modified Chua's circuit systems and that the drived system with the subscript 1 drives the response system with the subscript 2. The systems are

$$\begin{aligned} \dot{x}_1 &= p\big(y_1 - \tfrac{1}{7}(2x_1^3 - x_1)\big), \\ \dot{y}_1 &= x_1 - y_1 + z_1, \\ \dot{z}_1 &= -qy_1 \end{aligned} \tag{11}$$

and

$$\begin{aligned} \dot{x}_2 &= p\big(y_2 - \tfrac{1}{7}(2x_2^3 - x_2)\big) + u, \\ \dot{y}_2 &= x_2 - y_2 + z_2, \\ \dot{z}_2 &= -qy_2. \end{aligned} \tag{12}$$

We have introduced the control input u into the first equation in the system (12). This input is to be determined for the purpose of synchronizing the two identical modified Chua's circuit systems with the same but unknown parameters p and q. Let us define the state errors between the response system (12) and the derive system (11) as follows:

$$e_x = x_2 - x_1, \quad e_y = y_2 - y_1, \quad e_z = z_2 - z_1. \tag{13}$$

Subtracting Eq. (11) from Eq. (12) and using the notation (13) yields

$$\begin{aligned} \dot{e}_x &= p\left(e_y - \frac{2}{7}(x_2^3 - x_1^3) + \frac{e_x}{7}\right) + u, \\ \dot{e}_y &= e_x - e_y + e_z, \\ \dot{e}_z &= -qe_y. \end{aligned} \tag{14}$$

Eq. (14) describes the error dynamics. It is clear that the synchronization problem is replaced by the equivalent problem of stabilizing the system (14) using a suitable choice of the control law u. The synchronization problem for the modified Chua circuit is to achieve the asymptotic stability of the zero solution of the error system (14) in the sense that

$$\|e(t)\| \to 0 \quad \text{as } t \to \infty,$$

where $e(t) = (e_x(t), e_y(t), e_z(t))$. For this purpose we consider the control law as follows:

$$u = -he_x, \tag{15}$$

where h is an estimated feedback gain which is updated according to the following adaption algorithm:

$$\dot{h} = \gamma e_x^2, \quad h(0) = 0. \tag{16}$$

Then the controlled resulting error system (14) can be expressed by the following dynamical system:

$$\begin{aligned}
\dot{e}_x &= p\left(e_y - \frac{2}{7}(x_2^3 - x_1^3) + \frac{e_x}{7}\right) - he_x, \\
\dot{e}_y &= e_x - e_y + e_z, \\
\dot{e}_z &= -qe_y, \\
\dot{h} &= \gamma e_x^2.
\end{aligned} \tag{17}$$

Proposition 4. *The zero solution of the error system* (17) *is asymptotic stable for* $h = h_1 \geqslant p(1 + \frac{2}{7}a)$, *where* $a = x_2^2 + x_1x_2 + x_1^2$.

Proof. Let us consider Lyapunov function is

$$V = \frac{1}{2}\left[\frac{q}{p}e_x^2 + qe_y^2 + e_z^2 + \frac{q}{\gamma p}(h - h_1)^2\right], \tag{18}$$

where h_1 is a positive constant. The time derivative of Eq. (18) in the neighbourhood of the zero solution of the system (17) is

$$\dot{V} = -q(e_y - e_x)^2 + \frac{q}{p}e_x^2(p - h_1) - \frac{2q}{7}e_x^2(x_2^2 + x_1x_2 + x_1^2).$$

It is clear that $x_2^2 + x_1x_2 + x_1^2$ is positive for all $x_i \in R$, $i = 1, 2$. Let $a = x_2^2 + x_1x_2 + x_1^2$. Then $\dot{V}$ can be rewritten in the following form:

$$\dot{V} = -q(e_y - e_x)^2 - \frac{q}{p}e_x^2\left(h_1 - p - \frac{2p}{7}a\right). \tag{19}$$

From (19) and for the positive parameters p, q and γ it is clear that if we take $h = h_1 \geqslant p(1 + \frac{2}{7}a)$, then $\dot{V}$ is negative definite and from (18) the Lyapunov function V is positive definite. From Lyapunov stability theorem it follows that the equilibrium points $e_x = 0$, $e_y = 0$, $e_z = 0$, $h = h_1$ of the system (17) is h, asymptotic stable. The proof is completed. □

5. Numerical simulation

We will show a series of numerical experiments to demonstrate the effectiveness of the proposed control scheme. Fourth-order Runge–Kutta method is used to integrate the differential equations with time step 0.01. The parameters p, q and γ are chosen $p = 10$, $q = \frac{100}{7}$ and $\gamma = 1$ in all simulations to ensure the existence of chaos in the absence of control. The initial states are taken $x = .65, y = 0, z = 0$ in the controlling chaos problem, however, $x_1 = .65$, $y_1 = 0$ and $z_1 = 0$ of the derive system and $x_2 = .2$, $y_2 = .1$ and $z_2 = .1$ of the response system are chosen in the synchronization problem.

5.1. Chaos control to equilibrium points

5.1.1. Feedback control

The equilibrium point $E_0 = (0, 0, 0)$ of the system (2) is stabilized for $k_1 = 12$. Fig. 2 shows that the chaos is suppressed to the equilibrium point E_0 with time. The control is activated at $t = 20$. The equilibrium point $E_+ = (\sqrt{.5}, 0, -\sqrt{.5})$ of the system (2) is stabilized for $k_1 = 8$. Fig. 3 shows that the chaotic trajectory can be stabilized to the equilibrium point E_+ with time. The control is activated at $t = 20$. Fig. 4 shows the stabilization of the equilibrium point $E_- = (-\sqrt{.5}, 0, \sqrt{.5})$ of the system (2) where $k_1 = 8$. The control is

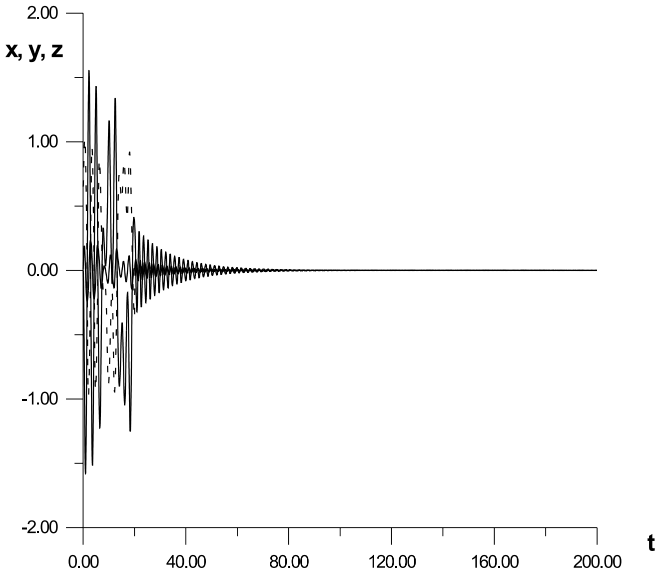

Fig. 2. The stabilization of the equilibrium point E_0 of the system (2). The control law $u_1 = -12x$, $u_2 = u_3 = 0$ is activated at $t = 20$.

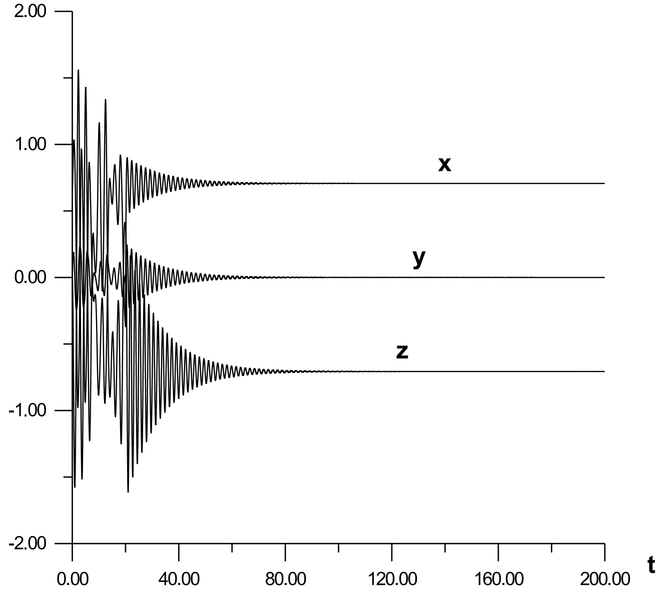

Fig. 3. The stabilization of the equilibrium point E_+ of the system (2). The control law $u_1 = -8(x - \sqrt{.5}), u_2 = u_3 = 0$ is activated at $t = 20$.

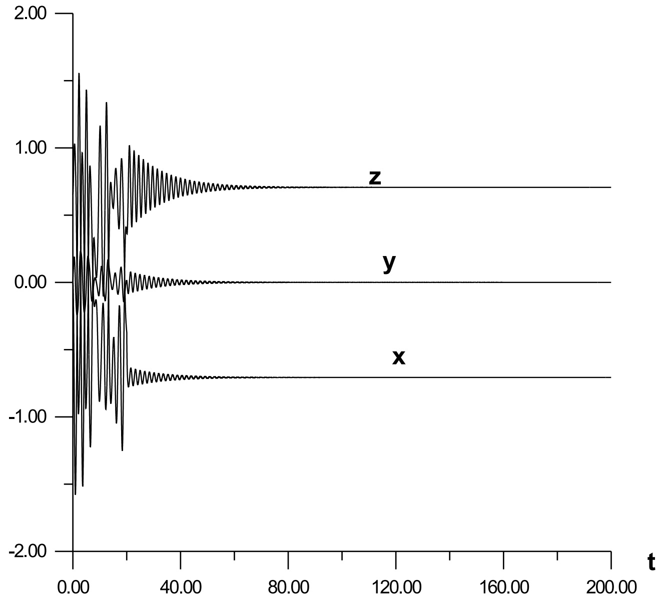

Fig. 4. The stabilization of the equilibrium point E_- of the system (2). The control law $u_1 = -8(x + \sqrt{.5}), u_2 = u_3 = 0$ is activated at $t = 20$.

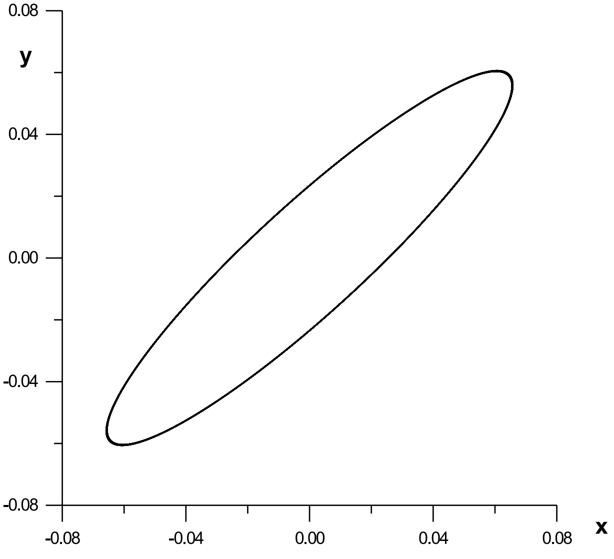

Fig. 5. The chaotic attractor of the system (1) is suppressed to limit cycle contains the equilibrium point E_0. The control law $u_1 = -9.9152x, u_2 = u_3 = 0$.

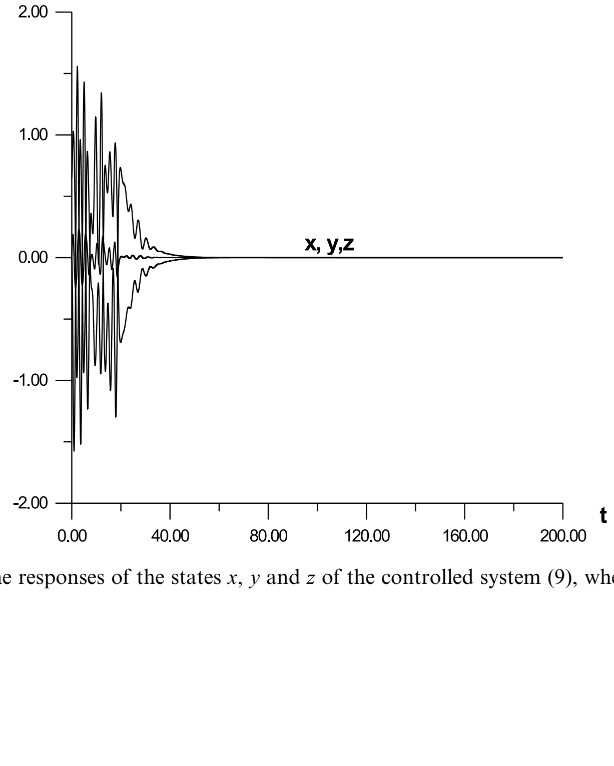

Fig. 6. The time responses of the states x, y and z of the controlled system (9), where $\bar{x} = 0$, $\bar{y} = 0$ and $\bar{z} = 0$.

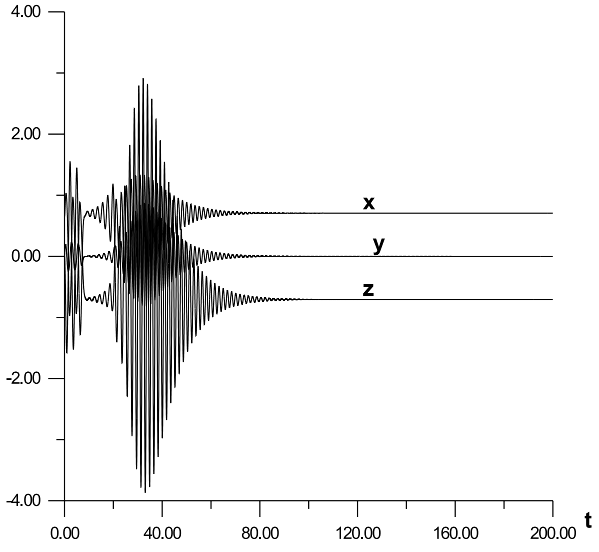

Fig. 7. The time responses of the states x, y and z of the controlled system (9), where $\bar{x} = \sqrt{0.5}$, $\bar{y} = 0$ and $\bar{z} = -\sqrt{0.5}$.

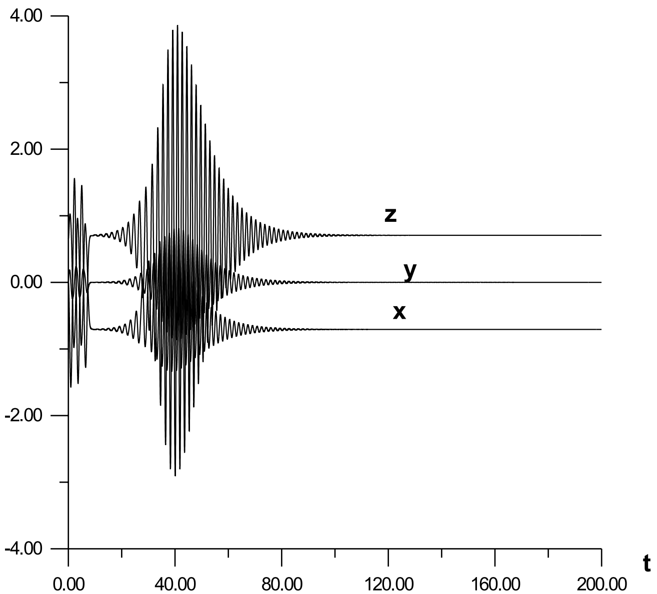

Fig. 8. The time responses of the states x, y and z of the controlled system (9), where $\bar{x} = -\sqrt{0.5}$, $\bar{y} = 0$ and $\bar{z} = \sqrt{0.5}$.

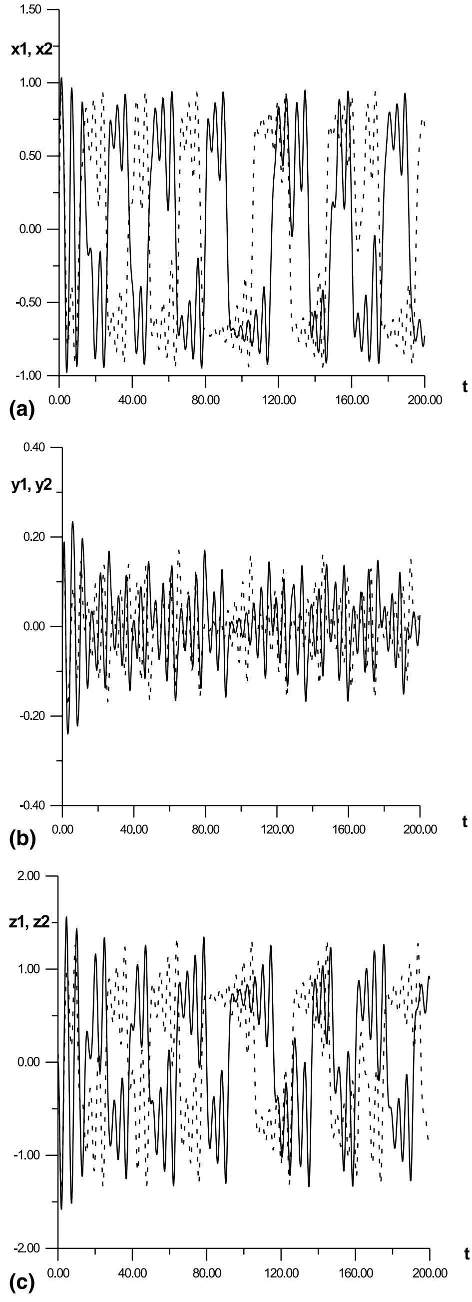

Fig. 9. Solution of the coupled modified Chua systems (11) and (12) with adaptive control deactivated: (a) signals x_1 and x_2; (b) signals y_1 and y_2; (c) signals z_1 and z_2.

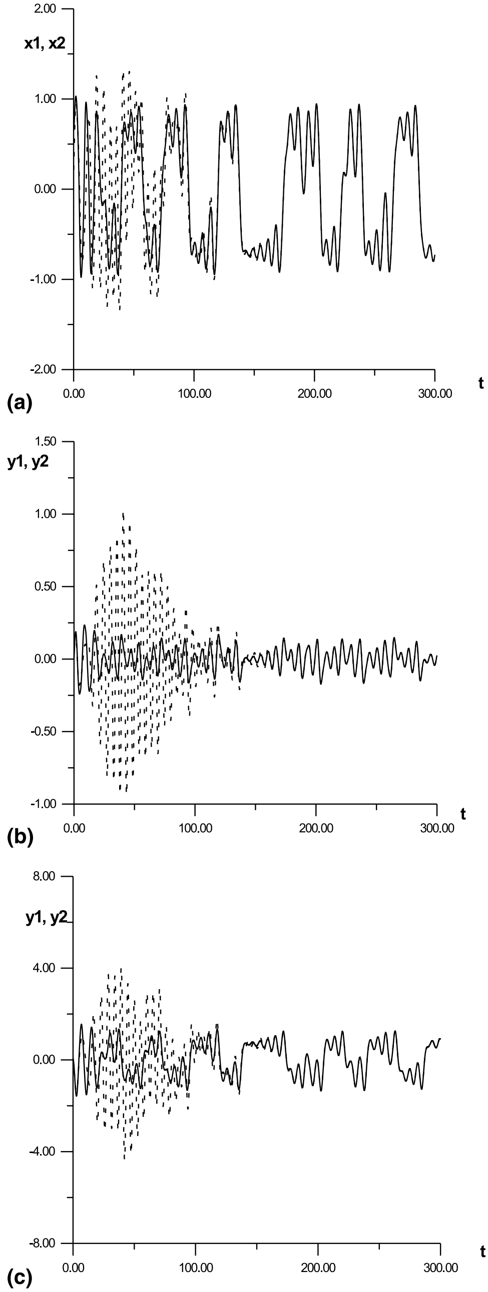

Fig. 10. Solution of the coupled modified Chua systems (11) and (12) with adaptive control activated: (a) signals x_1 and x_2; (b) signals y_1 and y_2; (c) signals z_1 and z_2.

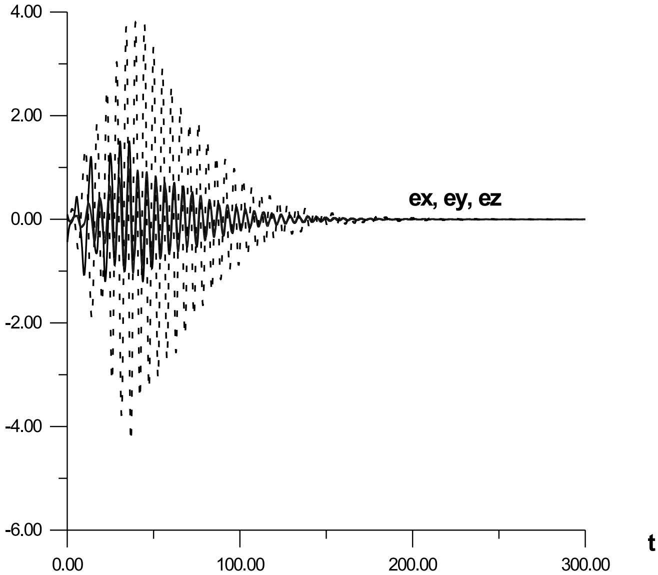

Fig. 11. The time responses for the state adaptive synchronization errors e_x, e_y and e_z of system (17). The adaptive control law is $u = -he_x$ with $\dot{h} = e_x^2$.

activated at $t = 20$. Fig. 5 shows that chaos is suppressed to limit cycle that contains the equilibrium point E_0 for $k_1 = 9.9152$.

5.1.2. Adaptive control

Figs. 6–8 show the time response for the states x, y and z of the controlled system (2) after applying adaptive feedback control and the successful results under the application of these adaptive control schemes.

5.2. Adaptive synchronization of two identical Chua circuits

Fourth-order Runge–Kutta method is used to solve the system of differential equations (11), (12) associated with feedback control law (15) and the adaption algorithm (16) with time step size 0.01. The results of the simulation of the two identical modified Chua circuits without adaptive control are shown in Figs. 9(a)–(c). Figs. 10(a)–(c) show the states evolution of the derive system (11) and the response system (12) after applying the feedback control (15) associated with the adaption algorithm (16). The initial conditions of the error system (17) are taken $e_x(0) = -0.45$, $e_y(0) = 0.1$ and $e_z(0) = 0.1$. Fig. 11 shows the evolutions of state synchronization errors and the history of the estimated feedback gain using feedback control law (15) associated with the adaption algorithm (16).

6. Conclusion

This work demonstrates that chaos in modified Chua's circuit can be easily controlled using adaptive control techniques.

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