Feb. 25, 1941 H. RODER 2,233,183

FREQUENCY MODULATION SYSTEM

Filed Nov. 12, 1938 2 Sheets-Sheet 1

Fig. 1.

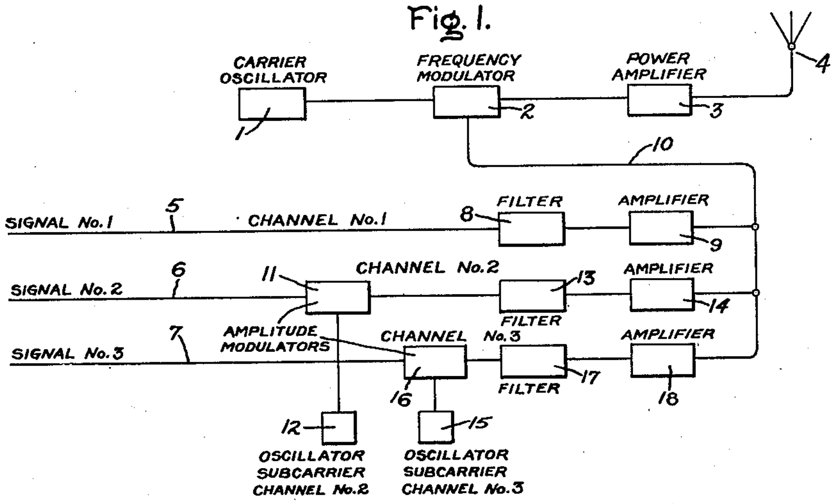

Fig. 2.

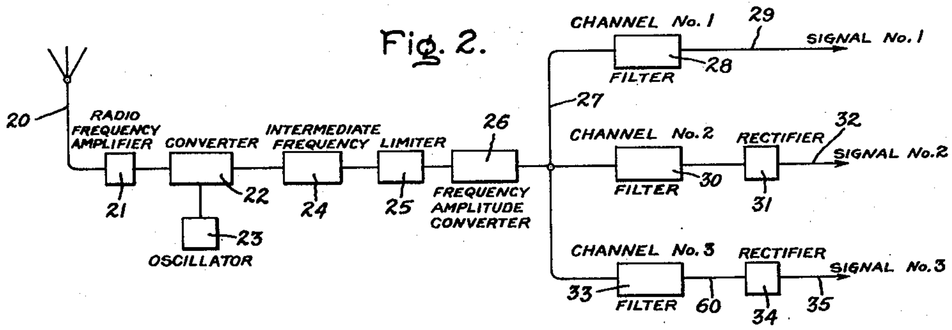

Fig. 3.

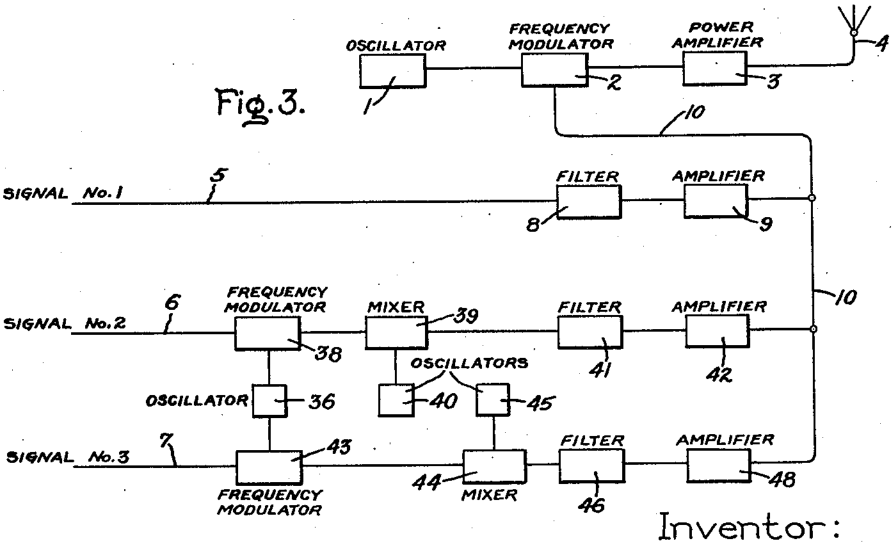

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Fig. 4.

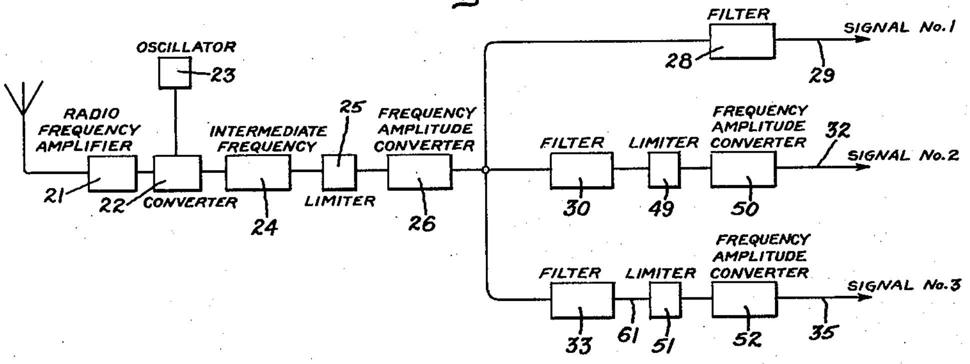


Fig. 5.

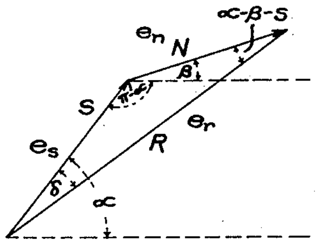


Fig. 6.

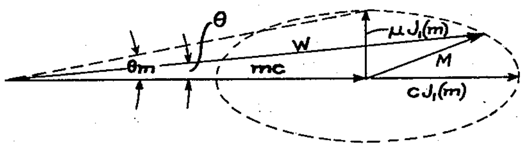
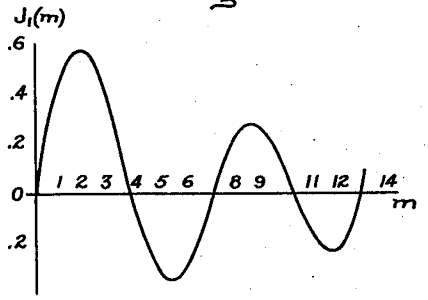


Fig. 7.



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Patented Feb. 25, 1941 2,233,183

UNITED STATES PATENT OFFICE

2,233,183

FREQUENCY MODULATION SYSTEM

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Application November 12, 1938, Serial No. 240,144

3 Claims. (Cl. 250—17)

My invention relates to frequency modulation systems and more particularly to such systems in which two or more signals are transmitted on the same carrier frequency. One of its objects is to obtain optimum noise elimination in such a system.

One of the great advantages resulting from the use of frequency modulation in the transmission of radio signals is the reduction of reception of undesired noise currents. This advantage of frequency modulation over amplitude modulation is especially pronounced if the shift in the carrier frequency be of the order of at least five times the highest frequency to be transmitted.

It has been proposed to transmit two or more signals by modulation of a single carrier wave. This has been done by modulating a main carrier wave and one or more subcarrier waves each with a signal to be transmitted, the modulation of the main carrier being frequency modulation, and that of the subcarriers being either amplitude or frequency modulation. The main carrier is then additionally frequency modulated with each of the modulated subcarriers. I have found that to obtain the maximum noise elimination on the subchannels so formed the frequency of the various subcarrier waves must have a definite relationship to the shift in frequency of the main carrier produced thereby.

The novel features which I believe to be characteristic of my invention are set forth with particularity in the appended claims. My invention itself, however, both as to its organization and method of operation together with further objects and advantages thereof may best be understood by reference to the following description taken in connection with the accompanying drawings in which Figs. 1 and 2 represent a frequency modulation transmission system; Figs. 3 and 4 represent a modification thereof, and Figs. 5, 6 and 7 represent certain diagrams pertinent to the operation of the system shown in Figs. 1 to 4.

Referring to Fig. 1 of the drawings I have indicated at 1 therein a carrier wave oscillator the output of which is supplied to the input of a frequency modulator. The output from this frequency modulator, which is frequency modulated in accordance with certain desired signals may be supplied to a power amplifier 3 and supplied to an antenna 4. The various signals which are to be transmitted are supplied to the system over channels 5, 6, and 7. We will assume that these signals are of the various voice or music frequencies. Signal No. 1 is supplied over circuit 5, filter 8 and amplifier 9 of channel No. 1 and conductor 10 to the input of the frequency modulator 2 whereby it is caused to produce a frequency modulation of the carrier wave radiated by the antenna.

Signal No. 2 is supplied over circuit 6 to an amplitude modulator 11 in which it is caused to modulate the output from a carrier wave oscillator 12 of channel No. 2. This modulator 11 may be assumed to produce amplitude modulations in oscillations of the frequency of the oscillator 12 and to supply such oscillations through a filter 13 and amplifier 14 to the circuit 10 and hence to the input of the frequency modulator 2. Hence the frequency of the carrier wave of oscillator 2 is modulated by the carrier wave generated by oscillator 12, the amplitude of which is modulated in accordance with signal No. 2.

Signal No. 3 is supplied through a channel No. 3, similar to that through which signal No. 2 is supplied and which comprises an oscillator 15, amplitude modulator 16, filter 17 and amplifier 18 whereby the frequency of the carrier wave of oscillator 1 is further modulated by oscillations from oscillator 15, the amplitude of which are modulated in accordance with signal No. 3. Filters 8, 13 and 17 are, of course, designed to transmit only the frequencies of their respective channels.

Fig. 2 shows a receiver adapted for use in connection with the transmitter of Fig. 1. It comprises the antenna 20 which supplies the intercepted oscillations to a radio frequency amplifier 21, the output of which is supplied to a superheterodyne converter 22 in which it is caused to beat with oscillations produced by the local superheterodyne oscillator 23 to produce an intermediate frequency which is supplied through an intermediate frequency amplifier 24 and amplitude limiter 25 to a frequency amplitude converter 26 whereby the frequency variations of the received carrier wave are converted to amplitude variations thereof. In this way all of the currents supplied through circuit 10 of Fig. 1 to modulator 2 thereof are reproduced upon the conductor 27 of Fig. 2. Signal No. 1 is then supplied through band pass filter 28 to output circuit 29 of channel No. 1. Currents of the frequency of oscillator 12 together with its modulation side bands are supplied through band pass filter 30, which is similar to filter 13 of Fig. 1, to a detector or rectifier 31 whereby signal No. 2 is reproduced on output circuit 32 of channel No. 2. Oscillations of the frequency of oscillator **15** together with its modulation side bands are also supplied through band pass filter **33** to a detector, or rectifier, **34** whereby signal No. 3 is reproduced on the output circuit **35**.

Before discussing the relationships between the various carrier wave frequencies of the system of Figs. 1 and 2 to produce optimum noise elimination in the various channels I shall first describe the system of Figs. 3 and 4.

The system of Figs. 3 and 4 is similar to the system of Figs. 1 and 2 with the exception that the channels for signals No. 2 and No. 3 involve frequency modulation of their respective subcarriers rather than amplitude modulation thereof. Referring to Fig. 3 the output from oscillator **1** is supplied to the carrier wave input of frequency modulator **2** the output of which is supplied through a power amplifier **3** to the antenna **4**.

The channel of signal No. 1 is identical with that for signal No. 1 of Fig. 1 and comprises the band pass filter **8**, amplifier **9**, and circuit **10** leading to the signal input of the frequency modulator **2**.

The carrier wave for the channels for signals No. 2 and No. 3 is supplied by an oscillator **36** the output of which is supplied to a frequency modulator **38** whereby its frequency is modulated in accordance with signal No. 2. The output from this frequency modulator is reduced in frequency by means of a mixer **39** in which it is caused to beat with the output of an oscillator **40** thereby to produce a lower frequency carrier wave. This lower frequency and its side bands are supplied through a band pass filter **41**, an amplifier **42** to circuit **10**.

In the same way the output of oscillator **36** is supplied to a frequency modulator **43** whereby it is frequency modulated in accordance with signal No. 3. The output from this modulator is then reduced in frequency by mixer **44** in which it is caused to beat with the oscillations from an oscillator **45**. The reduced carrier wave, together with its side bands is then supplied through band pass filter **46** and amplifier **48** to circuit **10**. In this way these carrier waves, modulated in frequency by their respective signals No. 2 and No. 3, are caused to frequency modulate the output from oscillator No. 1.

Fig. 4 shows a receiver for use in connection with the system of Fig. 3. It is identical with the receiver of Fig. 2 with the exception that the rectifier **31** of Fig. 2 is replaced by an amplitude limiter **49** the output of which is supplied to a frequency amplitude converter **50** whereby the frequency modulated oscillations of the frequency of the output of mixer **39** are converted to amplitude variations and supplied to circuit **32**. These circuits of course represent signal No. 2. In the same way the rectifier **34** of Fig. 2 is replaced by amplitude limiter **51** the output of which is supplied to frequency amplitude converter **52** whereby signal No. 3 is reproduced and supplied to circuit **35**.

To consider the problem of elimination of noise in the channels through which signals No. 2 and No. 3 are transmitted let us represent the instantaneous frequency of the carrier wave emitted by the transmitter of Fig. 1 by the following equation.

$$\omega=\omega_0+A\sin a_1t+B(1+k_2\sin a_2t)\cos bt+C(1+k_3\sin a_3t)\cos ct \qquad (1)$$

in which $\omega=2\pi f$ where f is the instantaneous frequency emitted by the system of Fig. 1

$\omega_0=2\pi f_0$, where f_0 is the frequency of oscillator 1

A, B and C are the individual frequency shifts produced by signal No. 1 and subcarriers on which signals No. 2 and No. 3 are modulated respectively;

a_1, a_2 and a_3 are the respective signal frequencies;

k_2 and k_3 are the percentage of modulation of the two subcarriers respectively;

b and c are the frequencies of the subcarriers respectively in radians per second; and t represents time.

The instantaneous frequency of currents emitted by the transmitter of Fig. 3 may be represented by the following equation.

$$\omega=\omega_0+A\cos a_1t+B\cos(bt+P_2\sin a_2t)+C\cos(ct+P_3\sin a_3t) \qquad (2)$$

in which the terms ω, ω_0, A, B, C, a_1, a_2, a_3, b, c, and t are the same as above, and P_2 and P_3 represent the phase shifts impressed by signals No. 2 and No. 3 upon their respective subcarriers.

Now let us consider the degree to which, for example, channel No. 3 is susceptible to interference produced by undesired signals, stray electromotive forces, static, and the like. To do so let us assume that channels No. 1 and No. 2 are idle, there being no signal in channel No. 1 and no signal or subcarrier in channel No. 2. Let us also assume that the subcarrier in channel No. 3 is unmodulated. These assumptions may be made for either of the above systems and the following considerations apply to both of these systems.

The frequency emitted by the transmitter of either Fig. 1 or Fig. 3 then becomes

$$\omega=\omega_0+C\cos ct \qquad (3)$$

since all other terms of both Equations 1 and 2 drop out under the above assumed conditions.

The voltage produced in the antenna of the receiver may be expressed by the following equation:

$$e_s=S\sin\phi \qquad (4)$$

in which S is the amplitude and

$$\phi=\int\omega dt \qquad (5)$$

Substituting the value of ω given by Equation 3 into Equation 5 and integrating that equation we get

$$\phi=\omega_0t+\frac{C}{c}\sin ct$$

Substituting this value of ϕ into Equation 4 we get

$$e_s=S\sin\left(\omega_0t+\frac{C}{c}\sin ct\right) \qquad (6)$$

This equation thus represents the current produced in the antenna of the receiver during existence of the above conditions.

Now let us assume that a disturbing electromotive force e_n also affects the receiving antenna whose amplitude is N volts and whose frequency differs from the received carrier frequency by

$$\frac{\mu}{2\pi}$$

kilocycles. This interfering electromotive force may be expressed as follows:

$$e_n=N\sin[(\omega_0+\mu)t] \qquad (7)$$

The total electromotive force, which we will represent by e_r, induced in the antenna is then the vector sum of e_s and e_n.

These quantities are vectorially represented in Fig. 5 where the vector e_s is drawn at an angle α to the horizontal and the vector e_n is drawn at an angle β to the horizontal, and the vector e_r represents the sum of these two vectors, where

$$\alpha=\frac{C}{c}\sin ct \text{ of Equation 6} \qquad (8)$$

and

$$\beta=\mu t \text{ of Equation 7} \qquad (9)$$

From the geometry of this vector diagram we find the relation

$$\frac{\sin\delta}{\sin(\alpha-\beta-\delta)}=\frac{N}{S} \qquad (10)$$

where δ represents the angle between vectors e_s and e_r.

The noise currents may be assumed to have an intensity not greater than one half of the intensity of the received signal current. A received noise current of such intensity, in any ordinary amplitude modulation system would render the received signal practically unintelligible and would produce some noticeable, although not necessarily objectionable, disturbance in a frequency modulation system. It is sufficient for the present purposes to assume that the interfering current is of intensity less than half of the intensity of the desired signal current.

From Fig. 5 we then see that the angle δ can never be greater than 30° since

$$\frac{N}{S}$$

is never greater than ½.

For angles of δ less than 30 degrees we may let

$$\sin\delta=\delta$$

and

$$\cos\delta=1$$

Substituting these quantities in Equation 10, putting

$$\frac{N}{S}=\frac{1}{2}$$

and solving for δ we find

$$\delta=\tfrac{1}{2}\sin(\alpha-\beta)\left[\frac{1}{1+\frac{1}{2}\cos(\alpha-\beta)}\right] \qquad (11)$$

The quantity $\frac{1}{2}\cos(\alpha-\beta)$ is always equal to or less than ½. Therefore, the bracketed term in Equation 11 may be expanded into a power series, and Equation 11 written as follows:

$$\delta=\tfrac{1}{2}\sin(\alpha-\beta)\left[1-\tfrac{1}{2}\cos(\alpha-\beta)+\tfrac{1}{4}\cos^2(\alpha-\beta)+\ldots\right]$$
$$=\tfrac{1}{2}\sin(\alpha-\beta)-\tfrac{1}{8}\sin 2(\alpha-\beta)+\ldots\ldots\ldots \qquad (12)$$

The resulting signal e_r has an instantaneous phase angle with respect to the horizontal line in Fig. 5 as an arbitrary reference, which is equal to $\alpha-\delta$. From Equations 8 and 12 this angle may be expressed as follows:

$$\alpha-\delta=\frac{C}{c}\sin ct+\tfrac{1}{2}\sin(\beta-\alpha)-\tfrac{1}{8}\sin 2(\beta-\alpha)+\ldots \qquad (13)$$

From 8 and 9

$$\beta-\alpha=\mu t-\frac{C}{c}\sin ct$$
$$=\mu t+\frac{C}{c}\sin(-ct)$$

Substituting this quantity into Equation 13 we find

$$\alpha-\delta=\frac{C}{c}\sin ct+\tfrac{1}{2}\sin\left[\mu t+\frac{C}{c}\sin(-ct)\right]-\tfrac{1}{8}\sin\left[2\mu t+\frac{2C}{c}\sin(-ct)\right]+\ldots \qquad (14)$$

Now using the Jacobi expansion formulas for the sine terms and denoting by

$$J_n\left(\frac{C}{c}\right)$$

the Bessel functions of order n and argument

$$\frac{C}{c}$$

and letting

$$\frac{C}{c}=m$$

we get (see for example, Proceedings of the Institute of Radio Engineers for December 1931, pages 2149 and 2150):

$$\alpha-\delta=m\sin ct+\tfrac{1}{2}[J_0(m)\sin\mu t-2J_1(m)\sin ct\cos\mu t$$
$$+2J_2(m)\cos 2ct\sin\mu t\ \ldots]$$
$$-\tfrac{1}{8}[J_0(2m)\sin 2\mu t-2J_1(2m)\sin ct\cos 2\mu t)$$
$$+2J_2(2m)\cos 2ct\sin 2\mu t\ \ldots] \qquad (15)$$

From this expression it is seen that the angle $\alpha-\delta$ varies at a rate dependent on a number of different frequencies c, μ, $c\pm\mu$, $2c\pm\mu$, 2μ, $c\pm2\mu$, $2c\pm2\mu$, etc.

The band pass filter **33** of the receiver of Figs. 2 and 4 above passes only the subcarrier and its frequency spectrum which includes its upper and lower modulation side bands. Only the frequencies c, $c\pm\mu$, and $c\pm2\mu$ are included in this spectrum and the terms of Equation 15 containing the other of the above mentioned frequencies may be ignored.

This expression then becomes

$$\alpha-\delta=m\sin ct-J_1(m)\sin ct\cos\mu t+\tfrac{1}{4}J_1(2m)\sin ct\cos 2\mu t \qquad (16)$$

This expression indicates the phase angle of the vector e_r as a function of time. In order to find the frequency variation of the vector e_r we differentiate this expression with respect to time and we get

$$\frac{d}{dt}(\alpha-\delta)=mc\cos ct$$
$$-J_1(m)[c\cos\mu t\cos ct-\mu\sin\mu t\sin ct]$$
$$+\tfrac{1}{4}J_1(2m)[c\cos 2\mu t\cos ct-2\mu\sin 2\mu t\sin ct] \qquad (17)$$

The first term of the right hand side of this equation represents the subcarrier of frequency c, the second term represents a modulation vector of this carrier having the frequency μ, and the third term represents a modulation vector having the frequency 2μ.

The first term of Equation 17 may be represented by a horizontal vector mc as shown in Fig. 6.

The term $cJ_1(m)\cos\mu t\cos ct$ may also be represented by a horizontal vector whose maximum length is $cJ_1(m)$ but which during one cycle of frequency μ varies in length as the $\cos\mu t$. This vector is represented in Fig. 6 and denoted by the legend $cJ_1(m)$.

The term $\mu J_1(m)\sin\mu t\sin ct$ can be represented by a vertical vector whose maximum length is $\mu J_1(m)$ but which, during one cycle of frequency μ, varies in length as the sin μt. This vector is denoted by $\mu J_1(m)$ in Fig. 6.

The vector sum of the quantities $(cJ_1(m) \cos \mu t)$ and $(\mu J_1(m) \sin \mu t)$ yields a new vector M shown in Fig. 6 whose end point transcribes an ellipse as shown by the dotted line in Fig. 6. The major axis of this ellipse has a length equal to $2\,cJ_1(m)$ while the minor axis has a length of $2\mu J_1(m)$.

Now, of course, since Equation 17 is a frequency equation the vectors of Fig. 6 are frequency vectors. However, since the frequency amplitude converter performs a linear translation of frequency variations to voltage variations this diagram, Fig. 6, may be taken to represent voltage vectors as well, these voltages being those which appear at the point 60 in Fig. 2, or at point 61 in Fig. 4.

The vector resulting from the summation of carrier vector mc and modulation vector M is represented by the vector W in Fig. 6.

Now, let us first consider the receiver shown in Fig. 2. Referring to Fig. 6 vector W varies in length between a maximum of $(mc+cJ_1(m))$ and a minimum of $(mc-cJ_1(m))$ and therefore is amplitude modulated by

$$\frac{cJ_1(m)}{mc}\times 100 \text{ per cent} \qquad (18)$$

In other words the unmodulated subcarrier c which is produced by oscillator 15 of Fig. 1 is received at point 60 of Fig. 2 with an amplitude modulation impressed on it of frequency μ and of a modulation depth given by Expression 18 produced by the received interfering electromotive forces if the interfering electromotive forces have an intensity equal to half of the intensity of the desired signal electromotive force.

Now, let us consider how the receiver of Fig. 4 responds to the voltage represented by vector W which appears at point 61 in Fig. 4. The limiter, of course, removes any amplitude variations of vector W. The phase variations, however, of the vector W, constitute the variable frequency from which the signal is reproduced through action of the frequency amplitude converter. Let us call the instantaneous angle between vector W and the vector mc, θ.

If the approximate maximum phase displacement between vector W and vector mc be designated θ_m then for small phase displacements

$$\theta_m=\frac{\mu J_1(m)}{mc} \qquad (19)$$

The phase variation occurs at a rate μ and is approximately sinusoidal. Thus

$$\theta=\frac{\mu J_1(m)}{mc}\sin \mu t \qquad (20)$$

The resulting frequency variation becomes, upon differentiation of Equation 20.

$$\frac{d\theta}{dt}=\frac{\mu^2 J_1(m)}{mc}\cos \mu t \qquad (21)$$

Referring now to Equation 2 we note that P_3 represents the phase shift impressed by signal No. 3 upon its respective subcarrier and that P_3a_3 is the frequency shift produced on the frequency of that subcarrier by the desired signal. The ratio

$$\frac{\frac{\mu^2 J_1(m)}{mc}}{P_3a_1}\cdot 100\% \qquad (22)$$

therefore expresses the relation between the intensity of the undesired disturbance and the desired signal.

Now we note from Expressions 18 and 22 that the interference, in either case, is proportional to the quantity $J_1(m)$. Fig. 7 shows the function $J_1(m)$ plotted against the quantity m the curve having been plotted, of course, from tables of Bessel functions. It will be observed from this curve that the quantity m has a number of values for which the quantity $J_1(m)$ is zero. The quantity m is the ratio

$$\left(\frac{C}{c}\right)$$

of the shift in frequency of the main carrier to the subcarrier frequency. Therefore, by making this ratio such that the quantity $J_1(m)$ is zero the interference will be minimized. The first three values of m for which the quantity $J_1(m)$ is zero are:

$$m=3.83$$
$$m=7.015$$
$$m=10.17$$

Accordingly, if the subcarrier frequency be for example, 50 kilocycles, then interference will be minimized if the shift of the main carrier frequency be $(50)(3.83)=191$ kilocycles, or $(50)(7.015)=351$ kilocycles; or $(50)(10.17)=508$ kilocycles. For practical reasons, however, the first of these values, 191 kilocycles, would usually be chosen although the other values may be employed to advantage in minimizing noise.

In the above consideration of Equation 17 the last term thereof was ignored. This term contributes an interference component having the frequency 2μ but whose amplitude is only in the order of one-quarter of the amplitude of the component contributed by the second term. In general the noise contributed by this term is small in comparison with that contributed by the second term, and this term may safely be disregarded. The magnitude of this term may be further indicated from Bessel's functions from which we find that if m be made equal to 3.83 as is above indicated to be necessary for minimum interference, then $J_1(2m)=.173$ which when substituted in this last term and multiplied by ¼ renders that term entirely negligible.

It will be seen from the steepness of the curve of Fig. 7 near the points where it crosses the axis m that for minimization of interference in accordance with the present invention the values of m above given should be rather closely adhered to and while some deviations therefrom may be necessary from practical consideration these deviations should not depart more than ten percent from the values given.

What I claim as new and desire to secure by Letters Patent of the United States is:

1. The method of signaling which comprises modulating the frequency of a main carrier wave at a rate dependent upon the frequency of a subcarrier wave, the range of frequencies over which the frequency of the main carrier is varied being greater than the frequency of the subcarrier substantially by any one of three multiplication factors as follows 3.83; 7.015; and 10.17.

2. The method of signaling which comprises modulating the frequency of a main carrier wave at a rate dependent upon the frequency of a subcarrier wave the range of frequencies over which the frequency of the main carrier is varied having a ratio to the frequency of the subcarrier for which $J_1(m)$ is substantially equal to zero where $J_1(m)$ is the Bessel function, order 1, ar-

gument m, where m is the ratio of the range of variation in frequency of the main carrier to the frequency of the subcarrier.

3. In a frequency modulation system, the method of minimizing reception of extraneous noise currents which comprises modulating the frequency of a main carrier wave at a rate dependent upon the frequency of a subcarrier wave, the range of frequencies over which the frequency of the main carrier is varied having a ratio to the frequency of the subcarrier not different by more than ten percent from one of the quantities 3.83, 7.015, 10.17 and any of the higher values of m for which the Bessel function, order 1, argument m is zero, where m is the ratio of the range of variation in frequency of the main carrier to the frequency of the subcarrier.

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