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(54) Title of the Invention: **Source separation method**
Abstract Title: **Seismic source separation method**

(57) A method is described for separating the unknown contributions of two or more sources from a commonly acquired wavefield including the determination of a wavenumber dependent model which reconstructs the wavefield of the sources independently below a frequency set by the slowest physical propagation velocity and applying an inversion based on the model to the commonly acquired wavefield to separate the contributions. There is at least one parameter varying non-periodically between the sources from one activation to the following activation. The parameter is signal amplitude, signal spectrum, activation time or location. The model incorporates the non-periodic variation of the parameter. Each wavefield has a bounded support in a transform domain, e.g. a cone shaped region in the frequency wavenumber domain.

Fig. 1 (B)

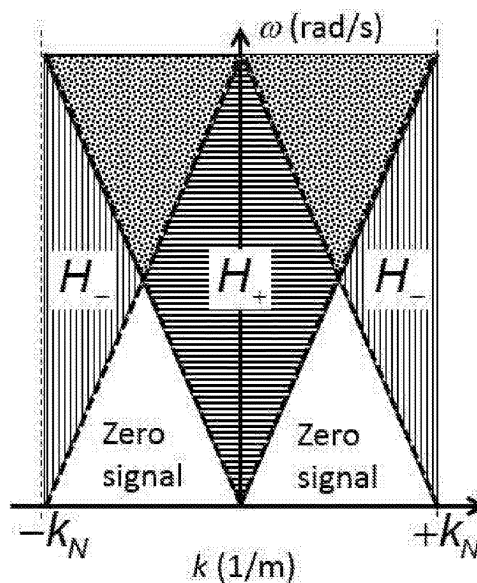
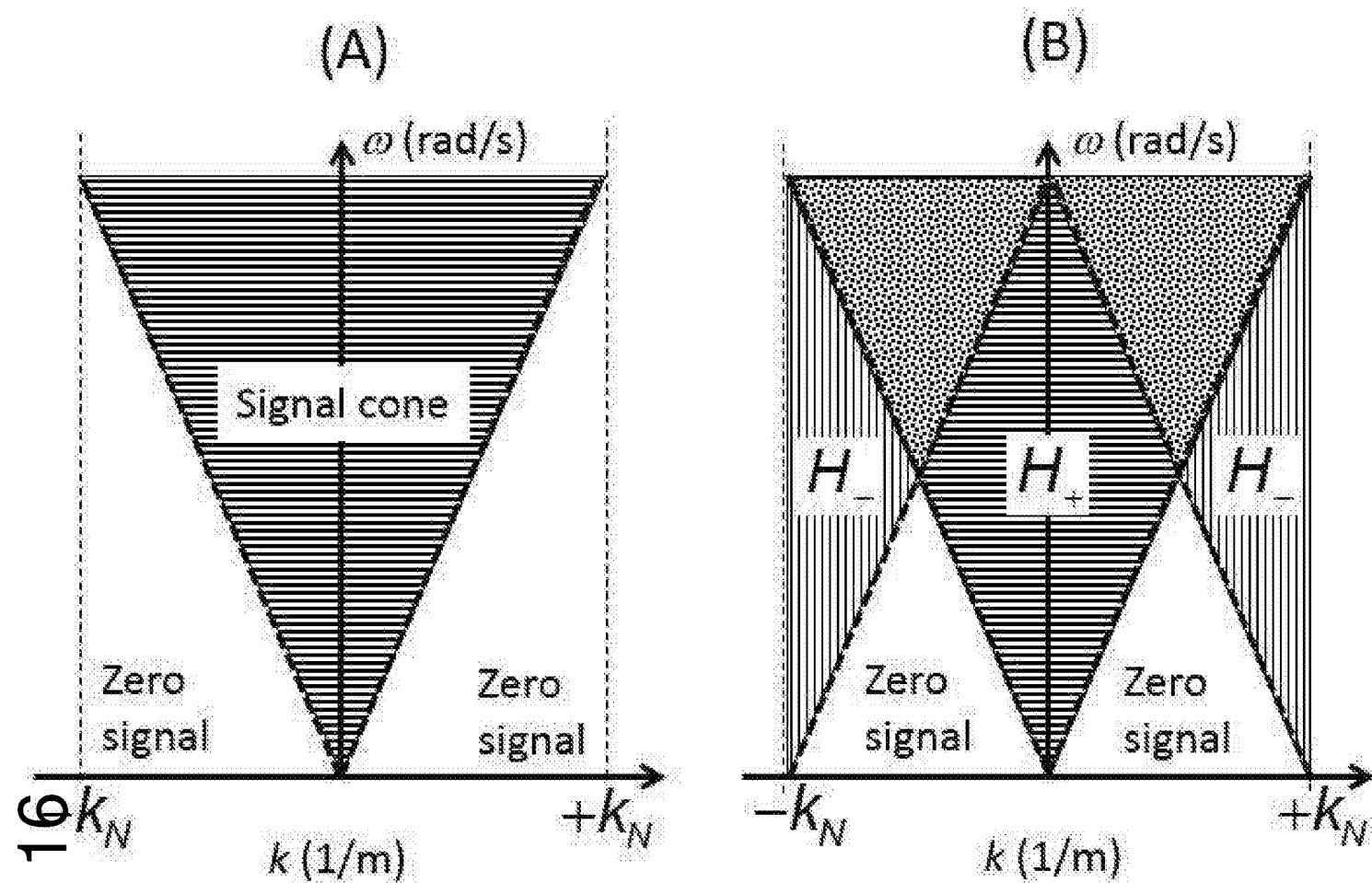


Fig. 1



28 07 16

g. 2

Blended data

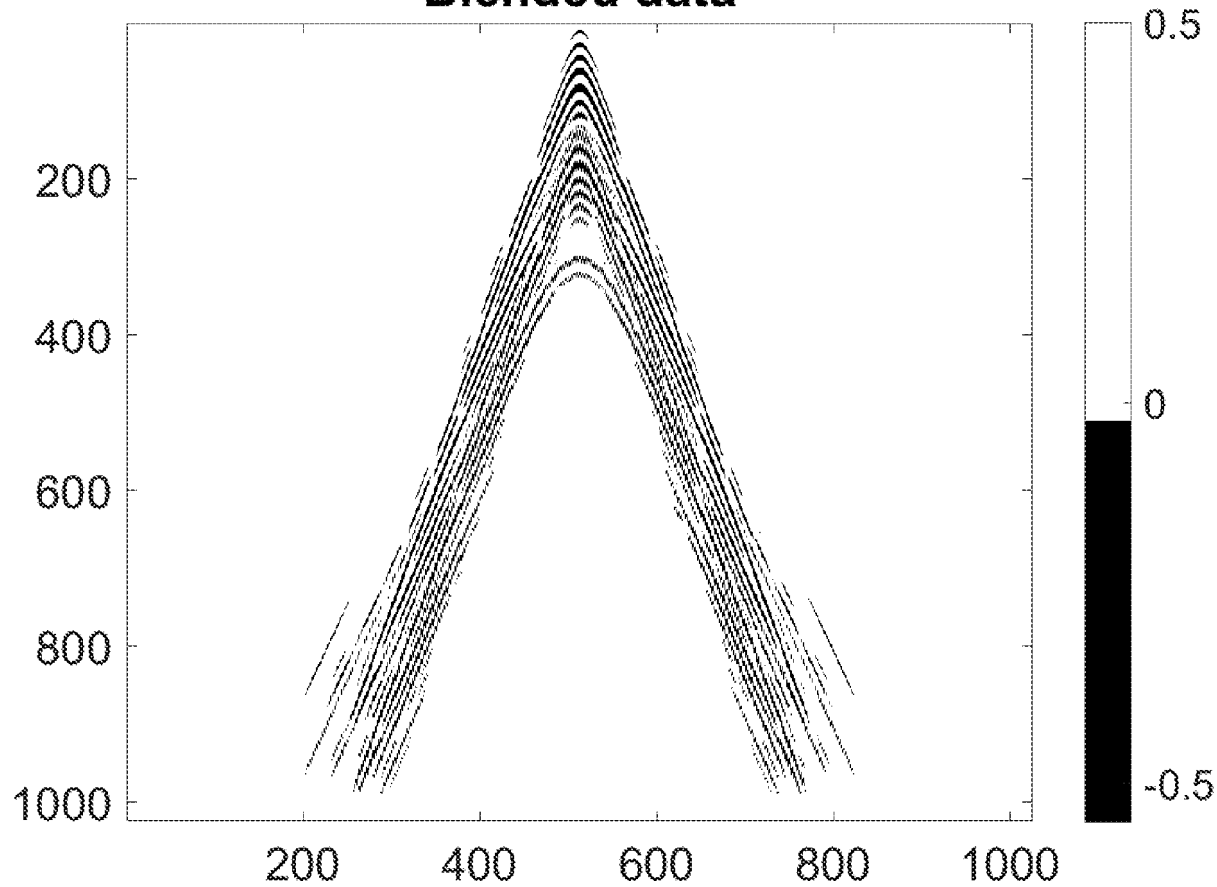
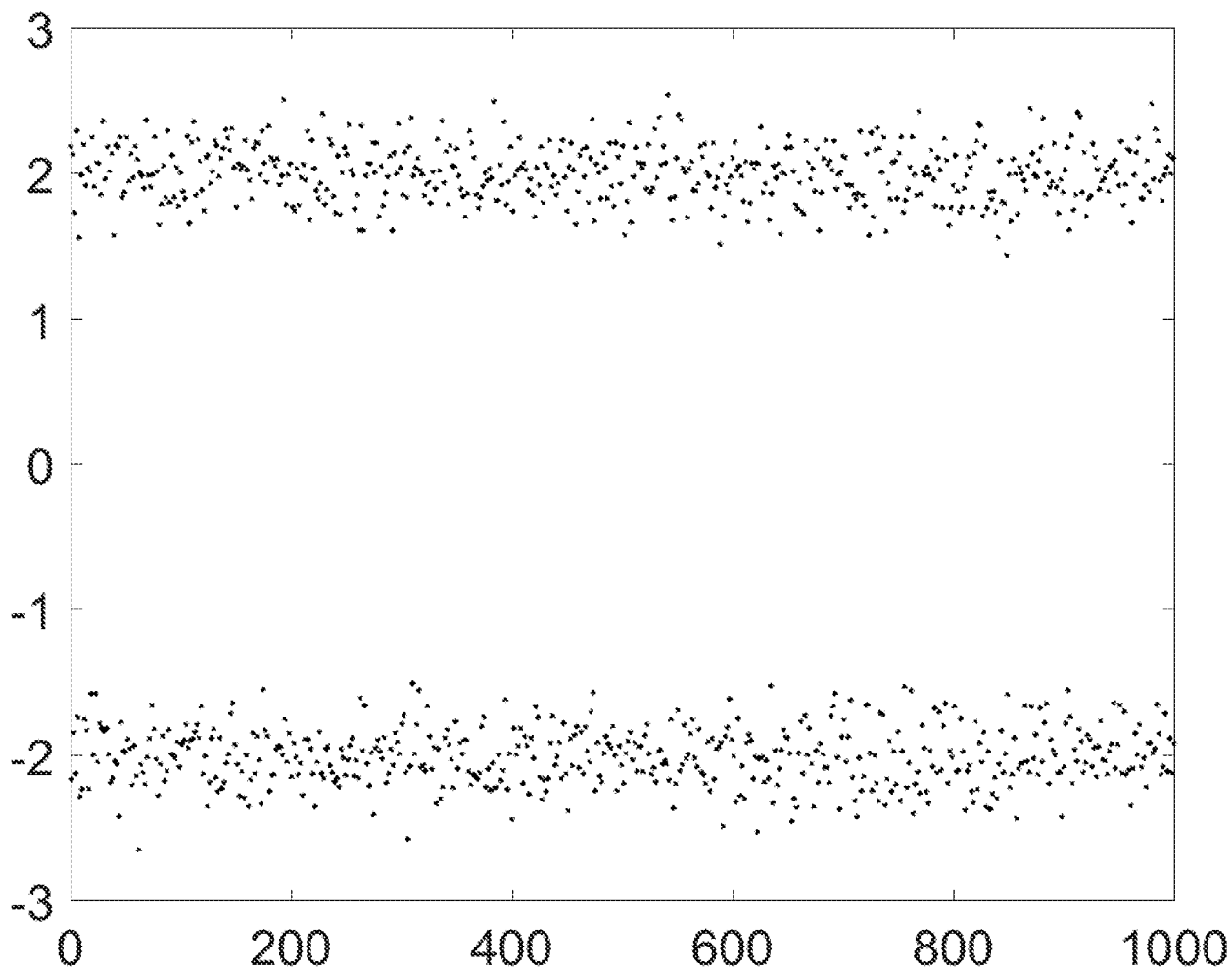


Fig. 3



2007 16
20.5

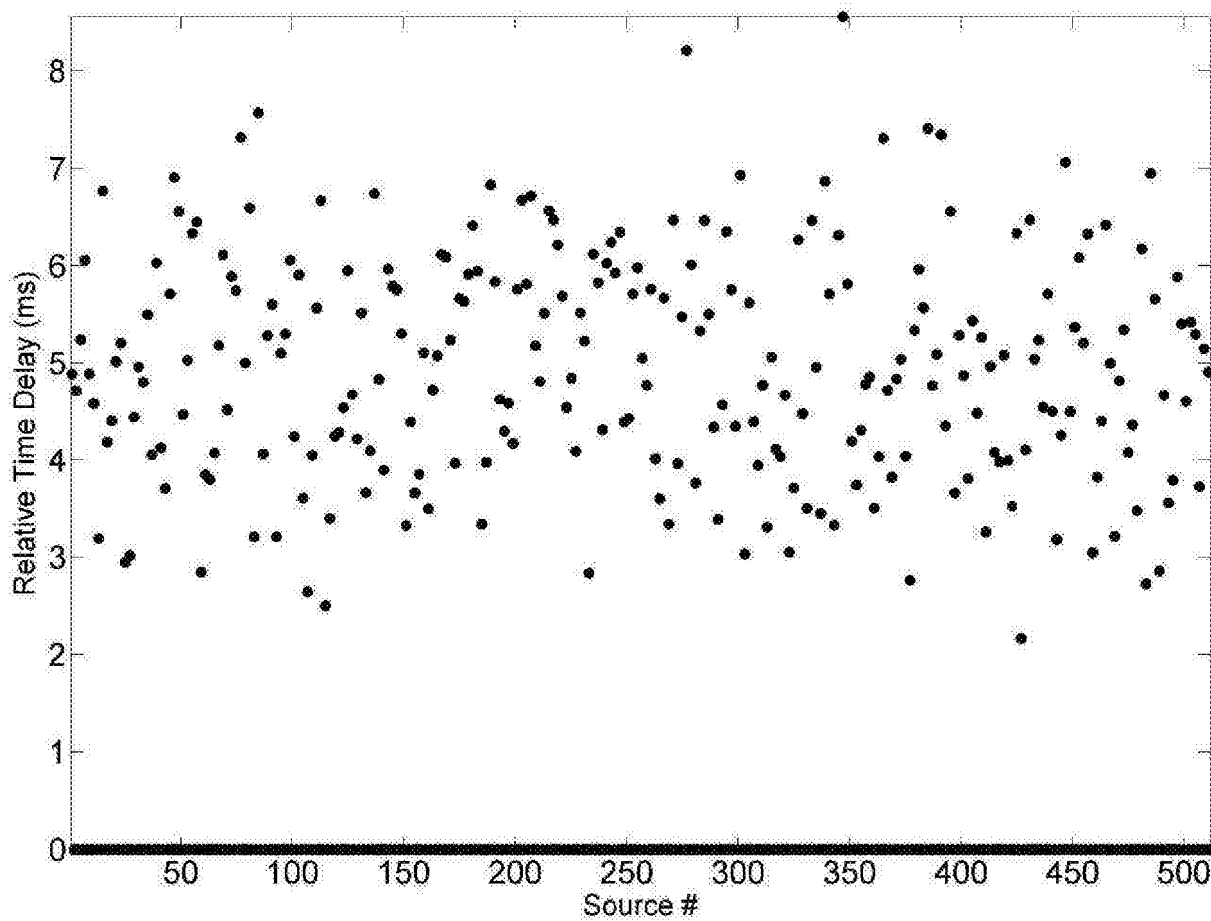


Fig. 4A

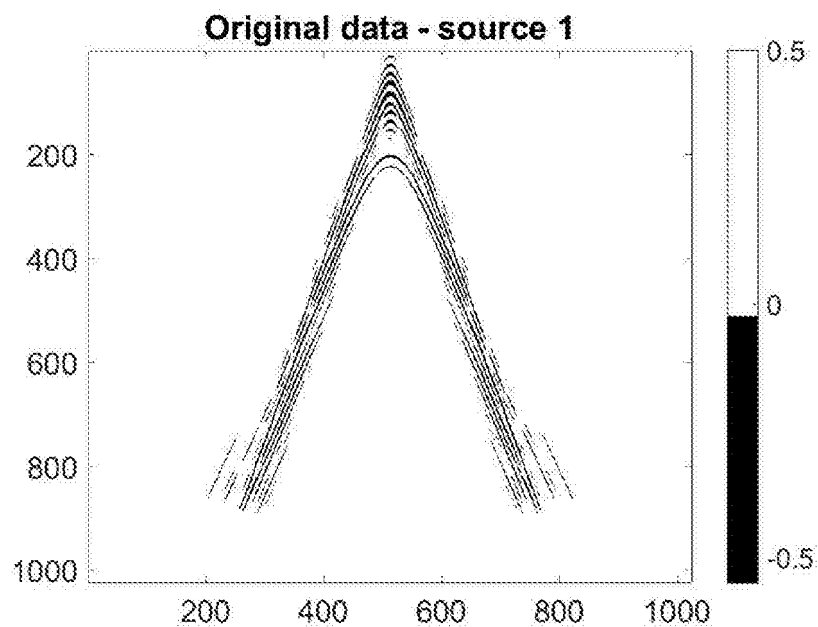


Fig. 4B

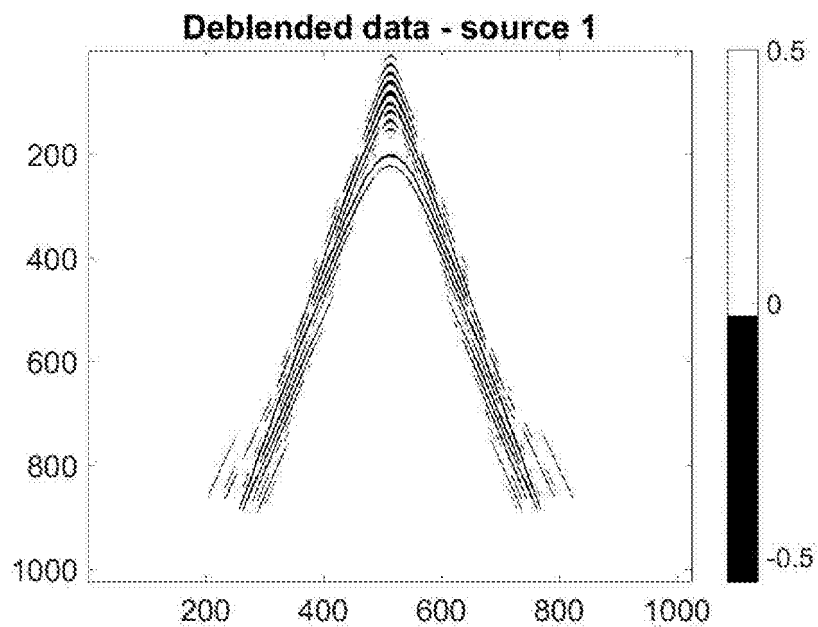
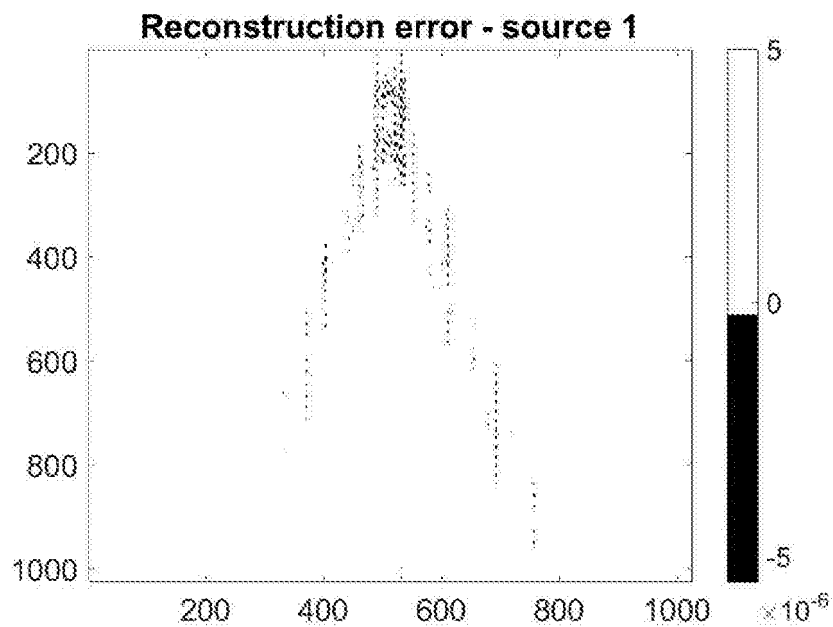
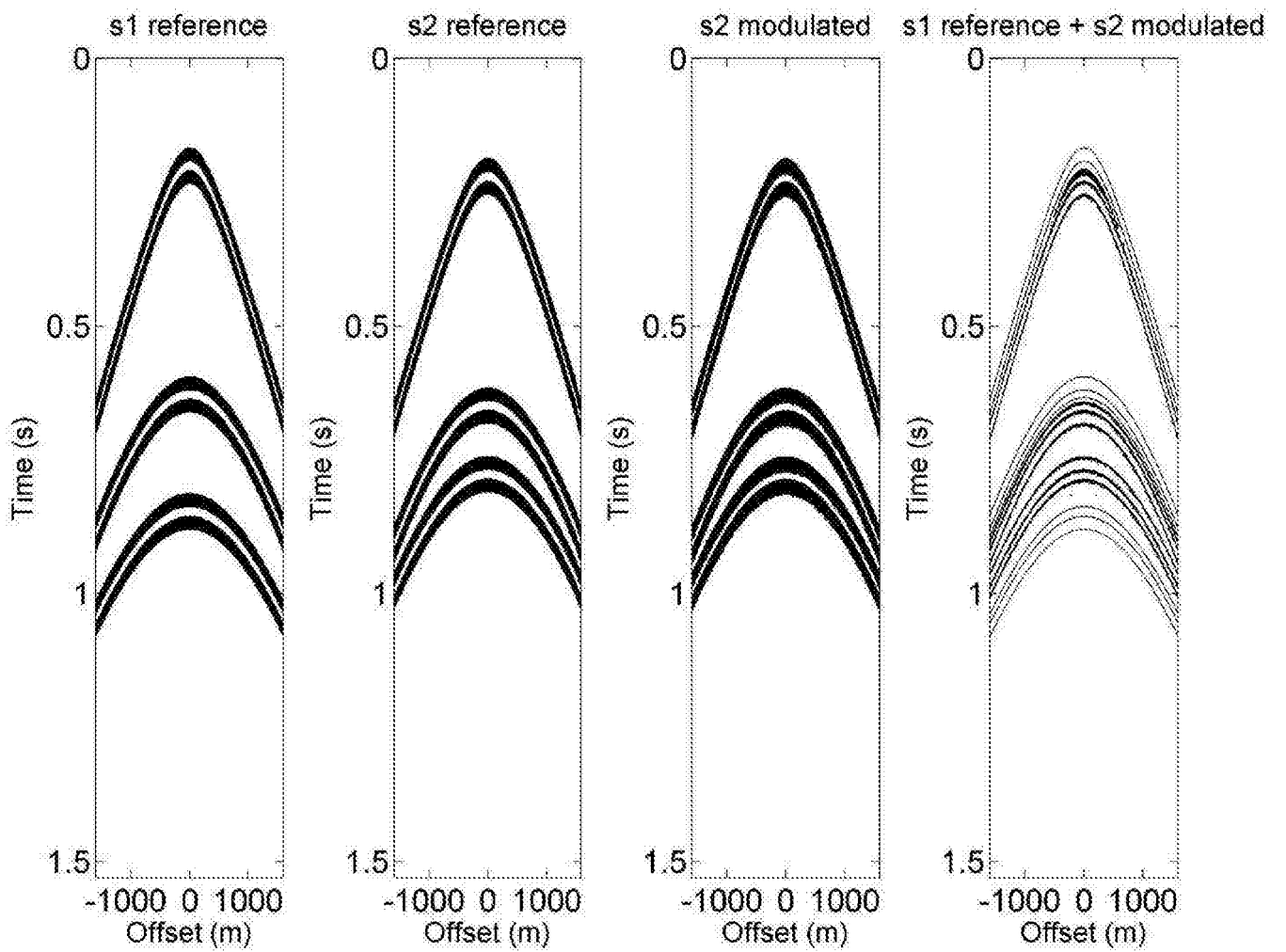


Fig. 4C

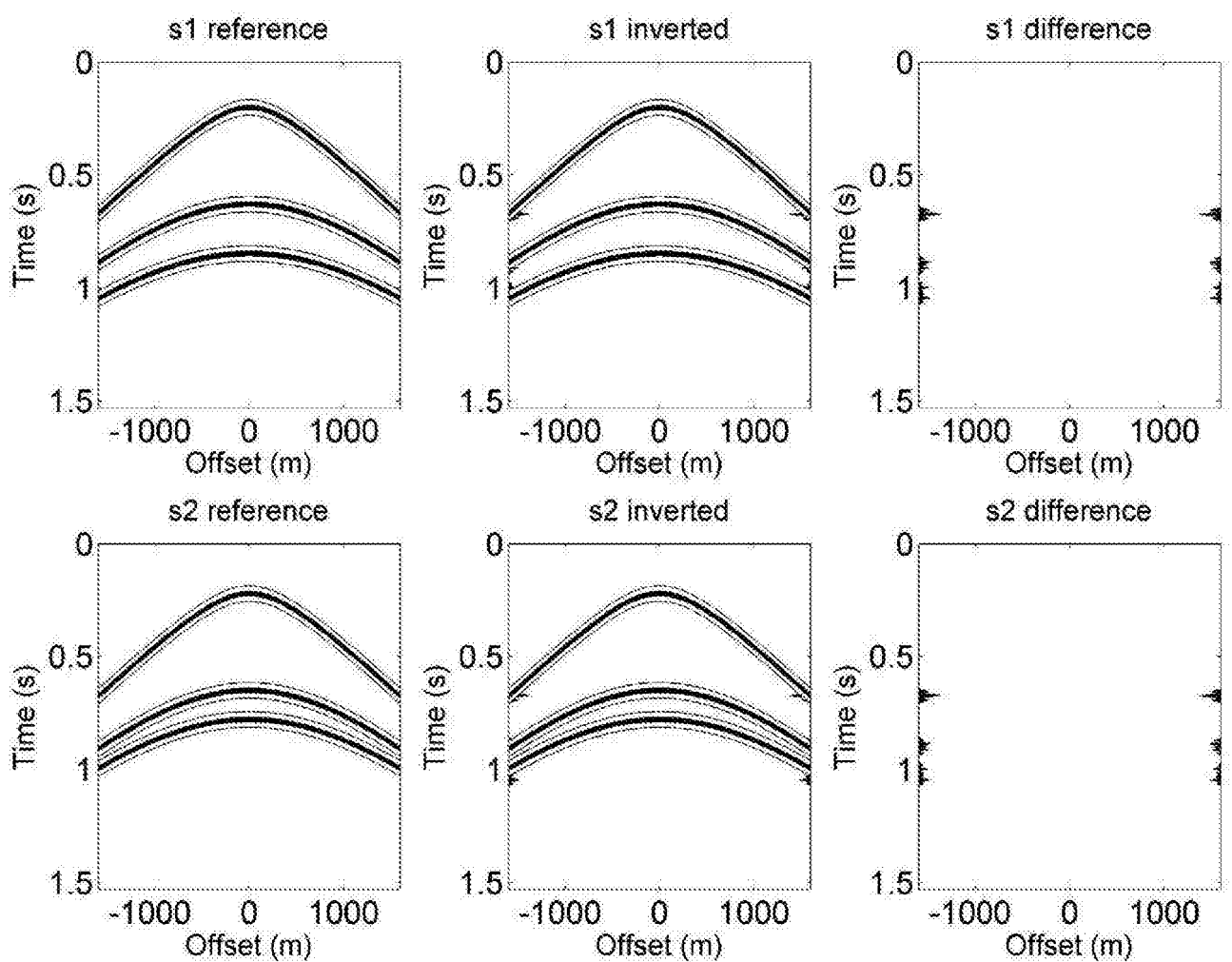


28 07 16

Fig. 6A



28-07-16
gg. 6B



Source Separation Method

Field of the invention

[0001] The present invention relates to methods for separating contributions from two or more different sources in a common set of measured signals representing a wavefield, particularly of seismic sources and of sets of recorded and/or processed seismic signals.

Description of related art

[0002] A common and long-standing problem in physical wavefield experimentation is how to separate recorded signals from two or more simultaneously emitting sources. In particular, for more than a decade, the simultaneous source problem has (arguably) been the most pertinent problem to solve to efficiently acquire data for 3D reflection seismic imaging of complex Earth subsurface structures.

[0003] Modern digital data processing of wavefields (or signals) uses a discretized version of the original wavefield, say g , that is obtained by sampling g on a discrete set. The Nyquist-Shannon sampling theorem shows how g can be recovered from its samples; for an infinite number of equidistant samples and given sample rate k_s , perfect reconstruction is guaranteed provided that the underlying signal was bandlimited to $|k| \leq k_N = k_s/2$ (Shannon, 1949; Nyquist, 1928), where k_N is the so-called Nyquist wavenumber. The Nyquist-Shannon sampling theorem is equally relevant both to signals generated from a single source being recorded on multiple receivers (receiver-side sampling) as well as signals generated from multiple sources and recorded at a single receiver (source-side sampling).

[0004] Assume that the wavefield g is measured at a specific recording location for a source that is excited at different source positions along an essentially straight line. The sampling theorem then dictates how the source locations must be sampled for a given frequency of the source and phase velocity of the wavefield. One aspect of the sampling problem addressed in the current invention is as follows. Instead of using one source, we want to use two (or more) sources to for instance increase the rate at which data can be acquired. The second source is triggered simultaneously or close in time with the first source while moving along another arbitrarily oriented line to excite the wavefield h . At the recording location the wavefields interfere and the sum of the two wavefields, $f = g + h$, is now measured. There is no known published exact solution to perfectly separate the wavefields g and h that were produced from each source from the combined measurement f (e.g., see Ikelle, 2010; Abma et al., 2015; Kumar et al., 2015; Mueller et al., 2015).

[0005] It may therefore be seen as an object of the invention, to present new and/or improved methods for generating source-separated data. It may also be seen as an object of the present invention to extend such methods into fields, where they have not been applied before, such as estimation and removal of seismic interference.

Brief summary of the invention

[0006] A method for separating wavefields generated by two or more source contributing to a common set of measured or recorded signals are provided suited for seismic applications and other purposes, substantially as shown in and/or described in connection with at least one of the figures, and as set forth more completely in the claims.

[0007] Advantages, aspects and novel features of the present invention, as well as details of an illustrated embodiment

thereof, may be more fully understood from the following description and drawings.

Brief Description of the Drawings

[0008] In the following description reference is made to the attached figures, in which:

Figs. 1A,B illustrate how in a conventional marine seismic survey all signal energy of sources typically sits inside a "signal cone" bounded by the propagation velocity of the recording medium and how this energy can be split in a transform domain by applying a modulation to a second source;

Fig. 2 shows jointly recorded wavefield data from two sources measured at a stationary receiver;

Fig. 3 shows time delays as applied to the second source in the data of Fig. 2;

Figs. 4 A - 4 C show the original and the reconstructed wavefield of the first source in the data of Fig. 2 and the reconstruction error when reconstructing the wavefield of the source after applying the separation method;

Fig. 5 shows the relative time delays between two sources as used in another example;

Figs. 6A,B illustrate the construction and separate reconstruction of the wavefields of two sources in the example of Fig. 5;

Detailed Description

[0009] The following examples may be better understood using a theoretical overview as presented below.

[0010] The slowest observable (apparent) velocity of a signal along a line of recordings in any kind of wave experimentation is identical to the slowest physical propagation velocity in the medium where the recordings are made. As a result, after a spatial and temporal Fourier transform, large parts of the frequency-wavenumber (ωk) spectrum inside the Nyquist frequency and wavenumber tend to be empty. In particular, for marine reflection seismic data (Robertsson et al., 2015), the slowest observable velocity of arrivals corresponds to the propagation velocity in water (around 1500m/s).

[0011] Fig. 1(A) illustrates how all signal energy when represented in or transformed into the frequency-wavenumber (ωk) domain sits inside a "signal cone" centered at $k = 0$ and bounded by the propagation velocity of the recording medium.

[0012] In a wavefield experiment it may be that a source is excited sequentially for multiple source locations along a line while recording the reflected wavefield on at least one receiver. The source may be characterized by its temporal signature. In the conventional way of acquiring signals representing a wavefield the source may be excited using the same signature from source location to source location, denoted by integer n . Next, consider the alternative way of acquiring such a line of data using a periodic sequence of source signatures: every second source may have a constant signature and every other second source may have a signature which can for example be a scaled or filtered function of the first source signature. Let this scaling or convolution filter be denoted by $a(t)$, with frequency-domain transform $A(\omega)$. Analyzed in the frequency domain, using for example a receiver gather (one receiver station measuring the response from a sequence of sources) recorded in this way, can be constructed

from the following modulating function $m(n)$ applied to a conventionally sampled and recorded set of wavefield signals:

$$m(n) = \frac{1}{2}[1 + (-1)^n] + \frac{1}{2}A[1 - (-1)^n],$$

which can also be written as

$$m(n) = \frac{1}{2}[1 + e^{i\pi n}] + \frac{1}{2}A[1 - e^{i\pi n}]. \quad (0.1)$$

[0013] By applying the function m in Eq. 0.1 as a modulating function to data $f(n)$ before taking a discrete Fourier transform in space (over n), $F(k) = \mathcal{F}(f(n))$, the following result can be obtained:

$$\mathcal{F}(f(n)m(n)) = \frac{1+A}{2}F(k) + \frac{1-A}{2}F(k - k_N), \quad (0.2)$$

which follows from a standard Fourier transform result (wavenumber shift) (Bracewell, 1999).

[0014] Eq. 0.2 shows that the recorded data f will be mapped into two places in the spectral domain as illustrated in Fig. 1(B) and as quantified in Tab. I for different choices of $A(\omega)$.

$A(\omega)$	$H_- = (1 - A)/2$	$H_+ = (1 + A)/2$
1	0	1
-1	1	0
0	1/2	1/2
$\frac{1}{2}$	1/4	3/4
$e^{i\omega T}$	$(1 - e^{i\omega T})/2$	$(1 + e^{i\omega T})/2$
$1 + e^{i\omega T}$	$-e^{i\omega T}/2$	$1 + e^{i\omega T}/2$

TAB. I. Mapping of signal to cone centered at $k = 0$ (H_+) and cone centered at $k = k_N$ (H_-) for different choices of $A(\omega)$ for signal separation or signal apparition in Eq. (0.2).

[0015] Part of the data will remain at the signal cone centered around $k = 0$ (denoted by H_+ in Fig. 1(b)) and part of the data will be mapped to a signal cone centered around k_N (denoted by H_-). It can be observed that by only knowing one of these parts of the data it is possible to predict the other.

[0016] A particular application of interest that can be solved by using the result in Eq. (0.2) is that of simultaneous source separation. Assume that a first source with constant signature is moved along an essentially straight line with uniform sampling of the source locations where it generates the wavefield g . Along another essentially straight line a second source is also moved with uniform sampling. Its signature is varied for every second source location according to the simple deterministic modulating sequence $m(n)$, generating the wavefield h . The summed, interfering data $f = g + h$ are recorded at a receiver location.

[0017] In the frequency-wavenumber domain, where the recorded data are denoted by $F = G + H$, the H -part is partitioned into two components H_+ and H_- with $H = H_+ + H_-$ where the H_- -component is nearly "ghostly apparent" and isolated around the Nyquist-wavenumber [Fig. 1(B)], whereas G and H_+ are overlapping wavefields around $k = 0$. Furthermore, H_- is a known, scaled function of H . The scaling depends on the chosen $A(\omega)$ function (Tab. I), and can be deterministically removed, thereby producing the full appearance of the transformed wavefield H . When H is found, then $G = F - H$ yielding the separate wavefields g and h in the time-space domain.

[0018] The concept may be extended to the simultaneous acquisition of more than two source lines by choosing different modulation functions for each source.

[0019] Acquiring a source line where the first two source locations have the same signature, followed by another two with the same signature but modified from the previous two by the function $A(\omega)$ and then repeating the pattern again until the full source line has been acquired, will generate additional signal cones centered around $\pm k_N/2$.

[0020] This process may be referred to as "wavefield apparition" or "signal apparition" in the meaning of "the act of becoming visible". In the spectral domain, the wavefield caused by the periodic source sequence is nearly "ghostly apparent" and isolated.

[0021] Fig. 1(B) also illustrates a possible limitation of signal apparition. The H_+ and H_- parts are separated within the respective lozenge-shaped regions in Fig. 1(B). In the triangle-shaped parts they interfere and may no longer be separately predicted without further assumptions. In the example shown in Fig. 1(B), it can therefore be noted that the maximum unaliased frequency for a certain spatial sampling is reduced by a factor of two after applying signal apparition. Assuming that data are adequately sampled, the method nevertheless enables full separation of data recorded in wavefield experimentation where two source lines are acquired simultaneously.

[0022] As is evident from Tab. I, the special case $A = 1$ corresponds to regular acquisition and thus produces no signal apparition. Obviously, it is advantageous to choose A significantly different from unity so that signal apparition becomes significant and above noise levels. The case where $A = -1$ (acquisition of data where the source signature flips polarity between source locations) may appear to be the optimal choice as it fully shifts all energy from $k = 0$ to k_N (Bracewell, 1999). Although this is a valid choice for modeling, it is not practical for many applications (e.g., for marine air gun sources, see Robertsson et al., 2015) as it requires the ability to flip polarity of the source signal. The case where $A = 0$ (source excited every second time only)

may be a straightforward way to acquire simultaneous source data but has the limitation of reduced sub-surface illumination. A particularly attractive choice of $A(\omega)$ for wave experimentation seems to let every second source be excited a time shift T later compared to neighbouring recordings, that is, select $A = e^{i\omega T}$.

[0023] The above description assumes a simple modulating sequence $m(n)$, and thus generating the wavefield h . Applications in which such a simple modulating sequence can be applied may however be limited in practice. In practice it is difficult to obtain perfectly periodic time shifts from a measurement setup. It is for example common practice for seismic vessels to shoot or trigger their sources at predetermined (essentially equidistant) positions, and due to practical variations (vessel velocity etc.) it will be difficult to realize shots at both predetermined locations and times.

[0024] In the following, there are therefore described further methods accommodating applications in which the modulation sequence includes variations or deviations, signal dither, or, in extreme circumstances, gives rise to aperiodic source signals. We refer herein to modulations sequences including any such variations as non-periodic.

[0025] First are presented methods for treating simultaneous source data using quasi-periodic time delays in the shots as well as position variations. Quasi-periodic time delays can be understood as delays with periodic carrying signal overlaid with a non-periodic (for instance random) pattern. The resulting combined variation can therefore be considered to be non-periodic. In the case where the recorded data is band-limited, and sufficiently densely sampled, this may provide a way to conduct separation or deblending of two or more source contributions for moderately sized non-periodic variations in the time shift.

[0026] Let us start by recapitulating some one-dimensional properties of the Fourier transform. We will use the notation

$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i x \xi} dx$$

for the Fourier transform in one variable, and consequently $\hat{f}(\omega, \xi)$ for the Fourier transform of two dimensional function $f(t, x)$ with a time (t) and spatial (x) dependence.

[0027] The Poisson sum formula

$$\sum_{k=-\infty}^{\infty} f(k) = \sum_{k=-\infty}^{\infty} \hat{f}(k)$$

can be modified (by using the Fourier modulation rule) as

$$\sum_{k=-\infty}^{\infty} f(k) e^{-2\pi i \xi k} = \sum_{k=-\infty}^{\infty} \mathcal{F}(f(\cdot) e^{-2\pi i \xi \cdot})(k) = \sum_{k=-\infty}^{\infty} \hat{f}(\xi + k).$$

[0028] By the standard properties of the Fourier transform it is straightforward to show that

$$\sum_{k=-\infty}^{\infty} f(k \Delta_x + x_0) e^{-2\pi i (k \Delta_x + x_0) \xi} = \frac{1}{\Delta_x} \sum_{k=-\infty}^{\infty} \hat{f}\left(\xi + \frac{k}{\Delta_x}\right) e^{-2\pi i x_0 \frac{k}{\Delta_x}}$$

hold for the spatial sampling parameter Δ_x and some fixed spatial shift x_0 . Suppose that $f_1 = f_1(t, x)$ and $f_2 = f_2(t, x)$ are two spatially bandlimited functions. By rescaling of units we can thus assume that they satisfy

$$\hat{f}_1(\omega, \xi) = \hat{f}_2(\omega, \xi) = 0, \quad \text{if } |\xi| > \frac{1}{4}. \quad (1.1)$$

[0029] The recorded data is now modelled as a sum of the two functions, but where one of them has been subjected to a

periodic time shift. Denote the recorded data by $d = d(t, x)$, this blending can thus be described by

$$d(t, k) = f_1(t, k) + f_2(t - \Delta_t (-1)^k, k) = \mathcal{F}^*(\hat{f}_1)(t, k) + \mathcal{F}^*(\hat{f}_2(\omega, \xi)e^{-2\pi i(-1)^k \Delta_t \omega})(t, k),$$

where we assume that the data is spatially sampled at equally spaced points $x = k$.

In a more general version more general filtering operations than time shifts can be applied. Let a_k be filter operators (acting on the time variable) where the k dependence is such it only depends on if k is odd or even, i.e., that $a_k = a_{k(\bmod 2)}$.

$$\begin{aligned} d(t, k) &= f_1(t, k) + a_k f_2(t - \Delta_t (-1)^k, k) \\ &= \mathcal{F}^*(\hat{f}_1)(t, k) + \mathcal{F}^*(\hat{f}_2(\omega, \xi)e^{-2\pi i(-1)^k \Delta_t \omega})(t, k). \end{aligned}$$

[0030] Applying the result described above, it then follows that a modulated sum of d over all even points ($2k$) in combination with a one-dimensional Fourier transform in the time variable yield the relation

$$\begin{aligned} &\int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} d(t, 2k) e^{-2\pi i((2k)\xi + t\omega)} dt \\ &= \frac{1}{2} \sum_{k=-\infty}^{\infty} \hat{f}_1\left(\omega, \xi + \frac{k}{2}\right) + \hat{a}_0(\omega) \hat{f}_2\left(\omega, \xi + \frac{k}{2}\right) e^{-2\pi i \Delta_t \omega}. \end{aligned}$$

[0031] A similar treatment of the odd terms ($2k + 1$) gives

$$\begin{aligned} &\int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} d(t, 2k + 1) e^{-2\pi i((2k+1)\xi + t\omega)} dt \\ &= \frac{1}{2} \sum_{k=-\infty}^{\infty} \left(\hat{f}_1\left(\omega, \xi + \frac{k}{2}\right) + \hat{a}_1(\omega) \hat{f}_2\left(\omega, \xi + \frac{k}{2}\right) e^{2\pi i \Delta_t \omega} \right) (-1)^k. \end{aligned}$$

[0032] We now define $D_1(\omega, \xi)$ as the sum that includes both odd and even terms. It thus holds that

$$D_1(\omega, \xi) = \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} d(t, k) e^{-2\pi i(k\xi + t\omega)} dt = \left(\sum_{k=-\infty}^{\infty} \hat{f}_1(\omega, \xi + k) \right) + \left(\sum_{k=-\infty}^{\infty} \hat{f}_2\left(\omega, \xi + \frac{k}{2}\right) \frac{1}{2} (\hat{a}_0(\omega) e^{-2\pi i \Delta_t \omega} + (-1)^k \hat{a}_1(\omega) e^{2\pi i \Delta_t \omega}) \right). \quad (1.2)$$

[0033] Because of the support assumption (1.1), we see that if $|\xi + 1/2| < 1/4$, then most of the terms above vanish, and it is therefore possible to obtain the values of $\hat{f}_2(\xi)$ from

$$D_1(\omega, \xi) = (\hat{a}_0(\omega) e^{-2\pi i \Delta_t \omega} + (-1)^k \hat{a}_1(\omega) e^{2\pi i \Delta_t \omega}) \hat{f}_2\left(\omega, \xi - \frac{1}{2}\right).$$

[0034] The values of \hat{f}_1 can be obtained in a similar fashion by considering instead

$$D_2(\omega, \xi) = \int_{-\infty}^{\infty} \sum_{k=-\infty}^k d(t + \Delta_t (-1)^k, k) e^{-2\pi i(k\xi + t\omega)} dt,$$

and

$$D_2(\omega, \xi) = (\hat{a}_0(\omega) e^{-2\pi i \Delta_t \omega} + (-1)^k \hat{a}_1(\omega) e^{2\pi i \Delta_t \omega}) \hat{f}_1\left(\omega, \xi - \frac{1}{2}\right).$$

[0035] The role of the filtering operations \hat{a}_0 and \hat{a}_1 are thus merely multiplicative, and for the sake of simplifying the presentation we therefore satisfy with considering the case where $\hat{a}_0 = \hat{a}_1 = 1$ for which it holds that

$$D_1\left(\omega + \frac{1}{2}, \xi\right) = -i \sin(2\pi \Delta_t \omega) \hat{f}_2(\omega, \xi).$$

and

$$D_2\left(\omega + \frac{1}{2}, \xi\right) = -i\sin(2\pi \Delta_t \omega) \hat{f}_1(\omega, \xi).$$

[0036] Like in the method above this procedure uses recovering the respective function at the cones that shifted away from the origin. While the values of \hat{f}_1 and \hat{f}_2 can be obtained in a very direct manner, the procedure is heavily relying on the periodicity of the alternating shifts in sampling, and may therefore be sensitive to perturbations of these time shifts.

[0037] It should be noted that a similar result can be obtained by modulating the amplitude instead of the time. In the most general case the modulation can be a filter with frequency dependent amplitude and phase.

[0038] Another way to recover \hat{f}_1 and \hat{f}_2 is to consider the case where $|\xi| < \frac{1}{4}$, for which (1.2) simplifies to

$$\int_{-\infty}^{\infty} \sum_{-\infty}^k d(t, k) e^{-2\pi i(k\xi + t\omega)} dt = \hat{f}_1(\omega, \xi) + \hat{f}_2(\omega, \xi) \cos(2\pi \Delta_t \omega),$$

and similarly for the shifted version

$$\int_{-\infty}^{\infty} \sum_{-\infty}^k d(t + \Delta_t (-1)^k, k) e^{-2\pi i(k\xi + t\omega)} dt = \hat{f}_1(\omega, \xi) \cos(2\pi \Delta_t \omega) + \hat{f}_2(\omega, \xi).$$

[0039] This implies that for each pair of values (ω, ξ) (with $|\xi| < 1/4$ and $\omega \Delta_t \notin \mathbb{Z}$), $\hat{f}_1(\omega, \xi)$ and $\hat{f}_2(\omega, \xi)$ can be obtained by solving the linear system

$$\begin{pmatrix} 1 & \cos(2\pi \Delta_t \omega) \\ \cos(2\pi \Delta_t \omega) & 1 \end{pmatrix} \begin{pmatrix} \hat{f}_1(\omega, \xi) \\ \hat{f}_2(\omega, \xi) \end{pmatrix} = \begin{pmatrix} D_1(\omega, \xi) \\ D_2(\omega, \xi) \end{pmatrix}. \quad (1.3)$$

or

$$\hat{f}_1(\omega, \xi) = \frac{D_1(\omega, \xi) - \cos(2\pi \Delta_t \omega) D_2(\omega, \xi)}{\sin^2(2\pi \Delta_t \omega)},$$

$$\hat{f}_2(\omega, \xi) = \frac{D_2(\omega, \xi) - \cos(2\pi \Delta_t \omega) D_1(\omega, \xi)}{\sin^2(2\pi \Delta_t \omega)}.$$

[0040] In this way the function can thus be recovered by considering the data in the central cone (e.g. as shown in Fig 1(B)) alone. The different formulations could have different advantages, for instance in the presence of noise.

[0041] Let us now consider a more general sampling setup. Let $\mathbf{t} = \langle \mathbf{t}_k \rangle$ and $\mathbf{s} = \langle \mathbf{s}_k \rangle$ be two sequences and assume that data is modelled by

$$[0042] \quad d(t, k) = f_1(t, s_k) + a_k f_2(t + \mathbf{t}_k, s_k). \quad (1.4)$$

[0043] In the case where

$$\mathbf{t}_k = \Delta_t (-1)^k \quad \text{and} \quad \mathbf{s}_k = k, \quad \text{and} \quad a_k = 1, \quad (1.5)$$

the setup of the previous section is obtained. Let us introduce the operators $\mathcal{F}_{\mathbf{t}, \mathbf{s}}^*$, and $\mathcal{F}_{\mathbf{t}, \mathbf{s}}$ by

$$\mathcal{F}_{\mathbf{t}, \mathbf{s}}^* \hat{f}(t, s_k) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{a}_k(\omega) \hat{f} e^{2\pi i(\xi s_k + \omega(t + \mathbf{t}_k))} d\omega d\xi,$$

and

$$\mathcal{F}_{\mathbf{t}, \mathbf{s}} d(t, k) = \hat{a}_k(\omega) \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} d(t + \mathbf{t}_k, k) e^{2\pi i(\xi s_k + \omega t)} dt,$$

respectively. Note that (1.4) can now be written as

$$d(t, k) = \mathcal{F}_{0, s}^* \hat{f}_1(t, s_k) + \mathcal{F}_{\mathbf{t}, \mathbf{s}}^* \hat{f}_2(t, s_k).$$

[0044] Note also that for instance $\mathcal{F}_{-\mathbf{t}, \mathbf{s}} \mathcal{F}_{\mathbf{t}, \mathbf{s}}^* \hat{f} = \mathcal{F}_{0, s} \mathcal{F}_{0, s}^* \hat{f}$ since the two time shifts cancel out.

[0045] Given a support constraint similar to (1.1), we follow the reconstruction procedure in a similar fashion as in the previous section. The analogous system of equations becomes

$$\begin{cases} \mathcal{F}_{0,s}\mathcal{F}_{0,s}^*\hat{f}_1 + \mathcal{F}_{0,s}\mathcal{F}_{t,s}^*\hat{f}_2 = \mathcal{F}_{0,s}d, \\ \mathcal{F}_{0,s}\mathcal{F}_{-t,s}^*\hat{f}_1 + \mathcal{F}_{0,s}\mathcal{F}_{0,s}^*\hat{f}_2 = \mathcal{F}_{-t,s}d. \end{cases} \quad (1.6)$$

[0046] Using a standard discretization in the frequency domain (and time), the operators may be realized using standard FFT in combination with shift operators in the case of equally spaced sampling in the spatial variable, or by using algorithms for unequally spaced FFT in the general case. The linear system (1.6) can for instance be solved by using an iterative solver. If (1.5) is approximately satisfied, the solution (1.3) may be used as a preconditioner. This means that a solution should be obtained in one iteration if (1.5) is satisfied and in case it is almost satisfied, only a few iterations should be required. The formulation above can be used in the case of irregular sampling in time; in space; or for both of the at the same time. Perturbations that are completely irregular (not following the tensor structure indicated here) can also be dealt with using the same framework.

[0047] For computational reasons it is important to be able to use fast solvers when applicable. By a fast solver we mean a method by which we can solve the problem of computing either forward or inverse operators in a method with low time complexity, for instance that of FFT.

[0048] The method set out above can be applied to various cases in which a separation of sources may be considered as advantageous. In the following the application of the problem of seismic interference cancellation is demonstrated.

[0049] Seismic interference (SI) occurs when two or more different seismic crews are acquiring seismic data in vicinity of each other. SI can be a major source of noise that can be difficult to remove particularly if it is arriving in the cross-line direction as the moveout of the signal is very similar to that of the interfering signal which can also be strong compared to deeper reflections that it may overlies. If

the dominant azimuth of seismic interference is known, it is possible to shift the acquired data so that it appears as far as possible from the seismic interference noise in ωk (e.g., noise centered at $k = 0$ and signal at k_{Nyquist}). The application requires real-time knowledge of exact firing times of the interfering vessel.

[0050] Knowledge of the exact firing time is of course a major limitation as a competing seismic crew likely will not want to share this information. It may also be somewhat irregular if the crew shoots on position and not time. However, using for instance the quasi-periodic apparition method presented above we can not only solve the simultaneous source problem (as described above) but also that of seismic interference since we do not need to know the exact firing times of the interfering crew beforehand. As long as we utilize a periodic or quasi-periodic acquisition sequence ourselves we will record a data set where the interfering signal can be apparated and removed from the acquired data. Once the data have been recorded we can detect the exact arrival time of the seismic interference in the data (this can be as simple as a windowed autocorrelation/crosscorrelation process).

[0051] The interfering data will likely be shot on position and therefore have a slight variation in arrival time from shot to shot. However, all we have to do is to include these estimated perturbations in arrival time of the seismic interference to apparate (i.e., shift in wavenumber) the seismic interference away from the acquired data. As a side note we note that the interfering survey is likely also seeing interference from the first crew using the apparition firing sequence. The method applied gives the opportunity to generate a separate set of data for the interfering survey.

[0052] Synthetic data were computed using a finite-difference solution of the wave equation for a typical two-dimensional (2D) North Sea subsurface velocity and density model consisting of sub horizontal sediment layers overlying rotated fault blocks. The data consist of the waveforms recorded at a

single stationary receiver (located on the seafloor) for shots along a line. The data are shown in Fig. 2. The time shifts t that have been applied to the second source are shown in Fig. 3, and the spatial sampling s is equidistant. An iterative scheme (preconditioned conjugate gradient) has been applied to the formulation using a preconditioner to obtain separated reconstructions of the data originating from each of the two sources. The data and the corresponding reconstruction and its error for the first source are shown in Fig. 4, while the counterparts for the second source (not shown) can be either derived directly from the above method or by subtracting the wavefield of the first source from the blended data. In this case 20 iterations of the iterative scheme were used. In this case the data were assumed to satisfy the bandlimit condition (1.1), and it is clear from the Fig. 4 that the reconstruction errors in this case can be made very small.

[0053] In the following variant of the method as described above is focusing on the cases of quasi-periodic and or pseudo-random timeshifts. Note, however, that the method can also be applied to the case of quasi-periodic or pseudo-random amplitude perturbations or to such perturbations of shot positions or of frequency-dependent amplitude and phase perturbations or arbitrary combinations thereof. This variant of the method, in its most general form, applies to any known (set of) modulation function(s).

[0054] Let $d(n)$ represent some data acquired as a function of a spatial coordinate x , i.e., at discrete locations x_n . The well-known convolution theorem states that the convolution in space of the data $d(n)$ with some spatial filter $f(n)$ can be computed by multiplication of the (discrete) Fourier transforms of the data $D(k) = \mathcal{F}(d(n))$ and of the spatial filter, $F(k) = \mathcal{F}(f(n))$, followed by inverse (discrete) Fourier transform, i.e., $f(n) * d(n) = \mathcal{F}^{-1}(F(k) \cdot D(k))$. Here we exploit the lesser-known dual of the convolution theorem, which states that multiplication in the space domain of $d(n)$ with a so-called modulation function/operator $m(n)$, corresponds to cyclic convolution of the (discrete) Fourier transform of the data,

$D(k)$, with the (discrete) Fourier transform of the modulation operator $M(k) = \mathcal{F}(m(n))$, followed by inverse (discrete) Fourier transform. Furthermore, we exploit the fact that cyclic convolution can be expressed conveniently as a matrix multiplication with a Toeplitz matrix.

[0055] Moreover, such a matrix multiplication can be rapidly evaluated by means of fast solvers such as the FFT, since the discrete Fourier matrices diagonalize cyclic matrices. This property is not true for the inverse of Toeplitz matrices. However, both Toeplitz matrices and their inverses have displacement rank 2, which allows for rapid evaluation of the application of these operators. The time complexity for these operations are bounded by the complexity for FFT.

[0056] Without loss of generality, let's assume that the data have been acquired in the following manner: a first source, s_1 , sailing with a constant ground speed, fires with regular time intervals, and a second source, s_2 , sailing also with a constant ground speed (and along the same line), fires alternately at the same time as s_1 and at Δt_n after s_1 , where Δt_n is drawn, e.g., from a normal distribution, $N(\mu, \sigma^2)$ with mean, μ , and variance, σ^2 . Since s_1 and s_2 are fired at the same time or shortly after each other, a recording of subsurface reflected waves will comprise a superposition of waves due to these two sources.

[0057] As shown below, we can formulate the separation of such data as the (least-squares) solution of a system of equations in the frequency-wavenumber domain. Let the N shots be enumerated by n , and let T_n denote the relative timing of the n -th shots fired by s_1 and s_2 . Thus, according to the paragraph above, $T_n = 0$ for odd shots $n = 2 \cdot i + 1$ and $T_n = \Delta t_n$ for even shots $n = 2 \cdot i + 2$ ($i = 0, 1, 2, \dots$ for $n \leq N$), . In the temporal frequency domain, the effect of the timeshift is multiplication with $e^{\pm i\omega T_n}$ where the sign depends on the choice of the definition of the temporal Fourier transform. Thus, for a particular frequency ω' and shot n' the time delay acts as a complex modulation $m(n') = e^{\pm i\omega' T_{n'}}$. As before, let $M(k) = \mathcal{F}(m(n))$ denote

the spatial (discrete) Fourier transform of the modulation function $m(n)$. A cyclic convolution matrix C can be formed by taking as the columns of C , circularly shifted versions of M , with the circular shift increasing by one each column, and having as many columns as number of points in M .

[0058] Since the timeshifts are formulated as relative to the s_1 times, the effective modulation function for the part of the simultaneous data due to s_1 is constant equal to 1 for each s_1 shot and each frequency. Thus, the transform of the implied modulation function for s_1 is a (discrete) delta function in wavenumber (at zero wavenumber) and the corresponding cyclic convolution matrix the identity matrix, I .

[0059] Consider without loss of generality that the number of simultaneous source recorded traces N is even. Let Δx denote the spatial (shot) sampling interval. Then the spatial Nyquist wavenumber, K_N , is $K_N = 1/(2\Delta x)$, and the wavenumber resolution, Δk , after a discrete Fourier transform over space is $\Delta k = 2K_N/N$. The discrete wavenumbers (after "FFT-shifting" the zero-wavenumber component to the center of the spectrum by swapping halves of the obtained discrete Fourier transform values) are $K(k) = -K_N + (k-1)\Delta k$ ($k = 1, 2, \dots, N$). In this case, the index corresponding to the zero wavenumber, k_0 , is $\frac{N}{2} + 1$.

[0060] Let the unknowns be the data in the frequency wavenumber domain due to sources s_1 and s_2 , i.e. $D_1(k)$ and $D_2(k)$, respectively, without relative timeshifts. Let k_{min} and k_{max} denote the indices of the discrete wavenumbers closest to the minimum and maximum wavenumbers K_{min} and K_{max} to be inverted, but smaller and larger, respectively. The data between $-K_N$ and K_{min} and K_{max} and K_N are assumed to be zero (i.e., the support in the wavenumber domain of $D_1(k)$ and $D_2(k)$ is confined to K_{min} and K_{max}).

[0061] Thus, the (column) vector of unknowns can be denoted $D = [D_1(k_{min}:k_{max})^T D_2(k_{min}:k_{max})^T]^T$ where T denotes transpose and the colon denotes all or part of a range. Note, however, that because of the modulation function applied to s_1 data, the

range of observed wavenumbers is not restricted to from K_{min} to K_{max} . However, since the unknowns are restricted to that range, the forward modelling operator should be similarly restricted along the columns (i.e., on the model side) but not along the rows (i.e., on the measurement side). Thus, the forward modelling operator matrix G can be formed:

$$G = [I(:, k_{min}:k_{max}) C(:, k_{min}:k_{max})]$$

Thus, if we denote $D_{tot}(k)$ the FFT-shifted (discrete) Fourier transform of the observed aperiodic, (near-)simultaneous source data $d_{tot}(m)$ we have the following system of equations:

$$GD = D_{tot}$$

And we can compute, e.g., the least-squares solution D^{LS}

$$D^{LS} = (G^H G + \lambda^2 I)^{-1} G^H D_{tot},$$

Where the stabilisation λ^2 is usually chosen to be a percentage (e.g. 0.1%) of the maximum of $G^H G$ and H denotes complex-conjugate (Hermitian) transpose.

[0062] As in the example above, this variant can be applied successfully to the separation of a dataset to which two sources have been contributing to with one of the sources being modulated in such a manner.

[0063] In Fig. 5 and Fig. 6, the methodology described above is illustrated. In Fig. 5, the relative time delays between source 1 and source 2 are shown. Note that every other trace, the relative time delay is zero. In Fig. 6A, the reference data for source 1 and source 2 are shown (first and second panel from the left). In the third and fourth panel from the left, the modulated s_2 data and the simultaneous source data, i.e., s_1 reference + s_2 modulated are shown. The latter represents the input data d_{tot} for the method described above. The resulting least-squares reconstruction of the data for s_1 and s_2 are

shown in Fig. 6B. As can be seen, the two sources have been separated correctly.

[0064] Note that the exactly the same methods can be used to separate or approximately separate sources for which the relative time delay varies non-periodically for every trace (i.e., every shot). Such variations, if intentionally, can be termed pseudo-random variations.

[0065] Note that the description above has focussed on the separation of at least two simultaneous or near-simultaneous sources. But the methods described herein can also be applied to wavefields where one source is physical and a second source is virtual as induced by the presence of a free surface which is the subjected of a separate patent application. In such a case the current invention allows for the separation of recorded wavefields into ghost and ghost-free wavefields. Further details to such an application can be found in the patent application by the same applicant entitled "Method for deghosting and redatuming operator estimation" and filed in the United Kingdom at the same priority date as this application.

[0066] Throughout the description of the present invention we have made use of Fourier transforms to transform the data separating and isolating wavefields. It will be appreciated by those skilled in the art that other transforms such as Radon transforms, tau-p transforms, etc., can also be used for the same purpose. Furthermore, those skilled in the art may also prefer to implement the methods presented directly in the space and/or time domain. In such cases the transforms presented herein are replaced by their respective representations or mathematical equivalents in such domains, which can take either explicit or approximated/truncated forms and applied to the wavefield data representation in the respective domain.

[0067] A bounded support in a domain is the well-known mathematical generalisation of the better-known concept of

being bandlimited (such as in equation (1.1)). Examples of limited support are presented above.

[0068] While various embodiments of the present invention have been described above, it should be understood that they have been presented by way of example only, and not of limitation. For example it should be noted that where filtering steps are carried out in the frequency-wavenumber space, filters with the equivalent effect can also be implemented in many other domains such as tau-p or space-time.

[0069] Further, it should be understood that the various features, aspects and functionality described in one or more of the individual embodiments are not limited in their applicability to the particular embodiment with which they are described, but instead can be applied, alone or in various combinations, to one or more of the other embodiments of the invention.

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Claims

1. Wavefield acquisition and/or processing method including the steps of
 - (a) Obtaining wavefield recordings based on the simultaneous or near-simultaneous activation of at least two sources having at least one parameter selected from one or more of a group consisting of source signal amplitude, source signal spectrum, source activation time and source location at activation time, varying non-periodically between the sources from one activation to the following activation along an activation line describing the trajectory of the source in space or a temporal sequence of activations of a line of sources. ;
 - (b) Modelling the obtained wavefield using a model that incorporates the non-periodic variation in the at least one parameter as a sum of wavefields, generated by the at least two sources individually, each wavefield having bounded support in a transform domain. ;
 - (c) By means of the model inverting the obtained wavefield recordings to separate a contribution of at least one of the at least two sources to the obtained wavefield recordings.
2. The method of claim 1 where the bounded support consists of cone-shaped regions in the frequency wavenumber domain.
3. The method of claim 2, wherein using the model involves modelling the obtained wavefield recordings by means of unequally spaced discrete Fourier transform operators.

4. The method of any of the preceding claims, where the inverting step is carried out by solving a system of equations.
5. The method of claim 4, where the system of equations is solved iteratively.
6. The method of claim 5, where the system of equations is solved iteratively including use of a preconditioner.
7. The method of claim 6, where the preconditioner includes a least-squares solution for periodic timing, signal amplitude or signal spectrum variations and/or regular activation locations.
8. The method of claim 1, wherein the variations include source activation time, signal amplitude and or signal spectrum and the use of a model that involves the application of a cyclic convolution matrix and or its corresponding inverse using the transform of a modulation function associated with the parameter variations.
9. The method of claim 8, wherein the inverting involves computing a least-squares solution of a system of equations.
10. The method of any of the preceding claims, wherein at least one of the at least two sources is not part of the same survey and where the activation times of those sources are approximately known.
11. The method of claim 10, wherein activation times are determined from the acquired wavefield recordings.
12. The method of claim 10, wherein activation times are determined from correlations of acquired wavefield recordings.

13. The method of any of the preceding claims, where the at least one parameter is varying in a non-periodic controlled manner.

14. The method of claims 1 to 12, where the at least one parameter varying in a non-periodic uncontrolled manner.

15. The method of any of the preceding claims, where the at least one parameter varying in a non-periodic manner is composed of a periodic part overlain by non-periodic fluctuations.

16. The method of any of the preceding claims, wherein as part of the separation step the data are regularized onto a regular sampling grid in time and or space.

17. The method of any of the preceding claims in which the transformed domain is selected from one of a Fourier transform domain, a Radon transform domain, a Gabor time-frequency domain, and a tau-p transform domain.

18. The method of claims 17 in which the corresponding transforms are replaced by their respective representations or mathematical equivalents in corresponding transformed domains in the space and or time domain.

19. The method of any of the preceding claims in which the computations in the modelling and or inversion steps are done using fast solvers.



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Claims searched: 1-19

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Patents Act 1977: Search Report under Section 17

Documents considered to be relevant:

Category	Relevant to claims	Identity of document and passage or figure of particular relevance
A	-	WO2015/026451 A2 (EXXONMOBIL UPSTREAM RES)
A	-	GB2404442 A (PGS AMERICAS INC)
A	-	GEOPHYSICS, VOL. 79, NO. 5 (SEPTEMBER-OCTOBER 2014), Chen et al, "Iterative deblending of simultaneous-source seismic data using seislet-domain shaping regularization", Pages V179-V189

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G01V	0001/36	01/01/2006