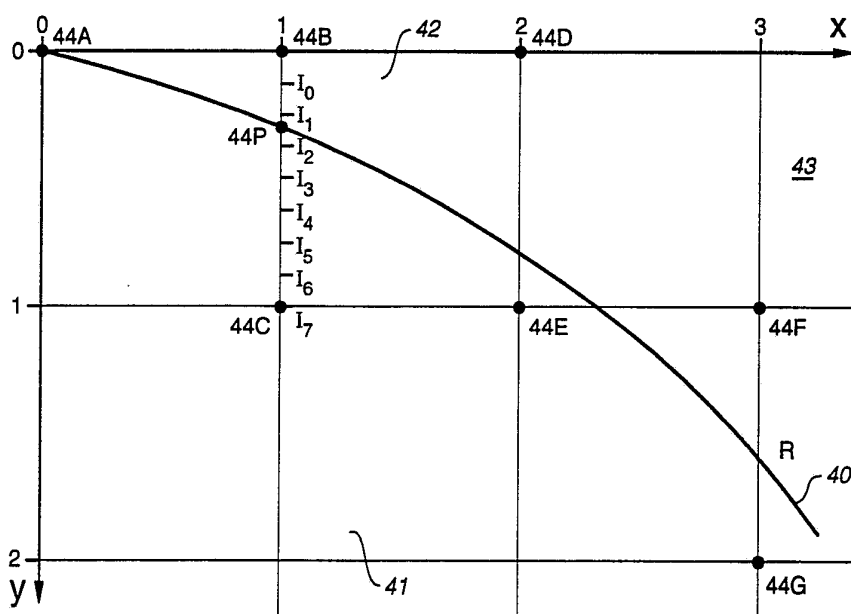




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(54) Title: AN ANTI-ALIASING METHOD FOR POLYNOMIAL CURVES USING INTEGER ARITHMETICS



(57) Abstract

A method of performing anti-aliasing on polynomial curves using only integer arithmetic. The anti-aliasing method includes the steps of: defining a polynomial equation of a curve (40), dividing grid units into a finite number of sub-intervals (I_0 - I_7), associating a mix ratio to each of the sub-intervals (I_0 - I_7) determining which sub-interval (I_2) the curve (40) bisects, and assigning a mix ratio to each picture element bordering the grid unit according to the mix ratio associated with the sub-interval (I_2) determined to be bisected by the curve (40).

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AN ANTI-ALIASING METHOD FOR POLYNOMIAL
CURVES USING INTEGER ARITHMETICS

BACKGROUND OF THE INVENTION

Field of the Invention

5 The present invention relates to methods for minimizing distortions caused by aliasing in computer-generated graphic display systems that employ cathode ray tube displays.

Background of the Invention

10 Cathode ray tube (CRT) display devices operate by scanning an electron beam rapidly back and forth across a screen to illuminate discrete picture-element locations, or "pixels", along the horizontal scan lines. The horizontal scan lines are organized by
15 synchronizing signals, with each frame containing a fixed number (e.g., 525) of visible horizontal scan lines and retrace lines. The net effect of a complete series of scans is to produce a snapshot-like "frame" that contains video data as to the state of each pixel
20 location on each scan line. (Figure 1A shows, for example, a 5x11 pixel grid that comprises a small portion of a complete frame of video information.) The frames are reproduced at a standard rate (e.g., sixty frames per second).

25 In practice, it is not possible to draw a perfectly continuous curve with a CRT device. As an example, Figure 1A shows a continuous non-horizontal curve 10 which is not aligned with either the columns or rows of pixels in the grid. When such a curve is
30 approximated on a CRT screen, discontinuities or

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"jaggies" can be observed along the curve. This phenomenon -- which is known as *aliasing* -- is shown in Figure 1B where the curve 10 is approximated as a series of staircase-like structures.

5 Various anti-aliasing techniques have been suggested. For example, as shown in Figure 1C, selected pixels can be only partially illuminated so that a jagged curve can be made to look smooth to the human eye. In this example, a starting pixel 12A is
10 fully illuminated because the curve 10 is exactly aligned with the starting pixel 12A. The curve 10 then goes between other pixels 12B and 12C but not in exact alignment with the center of either of the other pixels 12B or 12C. The distances from the curve 10 to the
15 centers of other pixels 12B and 12C equal $2/8$ grid units and $6/8$ grid units, respectively, in Figure 1C. Therefore, pixel 12B and 12C are illuminated to an intensity of 75 percent ($6/8$) and 25 percent ($2/8$), respectively, of their full scale intensity.
20 Similarly, the distances from the curve 10 to the centers of still other pixels 12D and 12E are both $4/8$ grid units, and consequently these pixels 12D and 12E are both illuminated at 50 percent of their full scale intensity. In this way, the anti-aliasing technique
25 blurs the edges of the curve 10 to obtain smooth edge graphics. The above distances are selected to be even fractions of grid units for convenience of illustration but likely involve less convenient numbers.

 When employing a color CRT display,
30 conventional anti-aliasing technique becomes even more complicated. As shown in Figure 2A, if a curve 20 with color A is drawn on a background of color B without anti-aliasing, the pixels 22 are assigned either color A (the curve color) or color B (the background color).

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Because of the discreet nature of the color pixels 22 on the screen, jaggies are inevitable on typical CRT screens. Using an anti-aliasing technique, as shown in Figure 2B, the pixels surrounding the curve 20 are assigned color values A_1, A_2, \dots etc. These color intensities A_1, A_2, \dots etc. are a mix of the curve color A with background color B according to the equation:

$$A_i = (1-p_i)*A + p_i*B$$

10 where p_i is the mix ratio determined by the distance between a pixel center and the curve 20 with the mixed colors; the curve 20 looks smooth to a human eye.

In the prior art, the basic technique to implement anti-aliasing involves finding distances 15 between pixel centers and a curve and then using the distances for determining mix ratios. Some more sophisticated techniques involve finding the overlap area of a curve with a certain width and a pixel area and then using the overlap area for determining mix 20 ratios. Most anti-aliasing methods use floating point arithmetics to find the distance, but such operations have several drawbacks. For example, such operations usually require expensive math co-processors and other specialized circuitry.

25 SUMMARY OF THE INVENTION

Generally speaking, the present invention presents a method of performing anti-aliasing on polynomial curves using only integer arithmetic. The method can be applied to polynomial curves including 30 lines (linear curves), conic curves (circles and ellipses) and cubic curves.

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In implementing the method of the present invention, it is assumed that a finite number of different mix ratios are available. For instance, the available mix ratios, p , might be:

5 $p = 0/8, 1/8, 2/8, 3/8, 4/8, 5/8, 6/8, \text{ and } 7/8.$

Further in implementing the method of the present invention, the determination of the exact distance between the intermediate point and a second pixel need not be determined but one needs only to
10 divide the interval between pixels (i.e., the grid unit) into a finite number of sub-intervals and to assign an intensity value or mix ratio to the pixel according to the sub-interval in which the curve crosses.

15 The present invention employs a method of anti-aliasing a curve on a CRT monitor including the steps of: defining a polynomial equation of a curve, dividing grid units into a finite number of sub-intervals, associating a mix ratio to each of the sub-
20 intervals, determining which sub-interval the curve bisects, and assigning a mix ratio to each picture element bordering the grid unit according to the mix ratio associated with the sub-interval determined to be bisected by the curve.

25 The method of the present invention has application in both monochrome and color monitors, and the mix ratio may be a simple gray level or a color mix.

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BRIEF DESCRIPTION OF THE DRAWINGS

The present invention can be further understood with reference to the following description in conjunction with the appended drawings, wherein like elements are provided with the same reference numerals. In the drawings:

Figure 1A shows a pixel grid;

Figure 1B shows a straight line 10 which is approximated as a series of staircase-like structures.

Figure 1C shows a prior art anti-aliasing technique;

Figure 2A shows aliasing on a color monitor;

Figure 2B shows the effect of anti-aliasing on a color monitor;

Figure 3 shows the color format of HiColorDAC;

Figure 4 is an example that illustrates the present invention; and

Figures 5A and 5B are a flow chart of the method of the present invention.

DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENTS

The following describes a method for generating a curve 40 displayed on a grid structure 43 of a CRT display screen. The curve is based upon a polynomial equation of its mathematical expression. In

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practice, the graphically-displayed curve may be less than a pixel width wide, but typically is more likely to be several pixel grid units wide. In the latter case, the below-described method addresses the edge of
5 the curve.

As shown in Figure 4, curve 40 is represented by a polynomial equation $F(x,y)=0$, whose coefficients are rational numbers. The region 42 above the curve 40 is represented by $F(x,y) < 0$, while the region 41 below
10 the curve 40 is represented by $F(x,y) > 0$. The curve 40 starts from a first pixel 44A with coordinate (0,0). It is assumed that the absolute value of the derivative of x over y is less than or equal to 1 (i.e., $|dy/dx| \leq 1$); otherwise, the x and y terms are exchanged.
15 Therefore, by moving one grid unit in an x direction, a point on the curve 40 will move by a fractional unit in the y direction to an intermediate point 44P between a second pixel 44B and a third pixel 44C.

To find the distance between the intermediate
20 point 44P and a second pixel 44B so that the mix ratio p for the second pixel 44B and third pixel 44C can be determined, it can be assumed initially that there are a finite number of different mix ratios, i.e., $p = 0/8, 1/8, 2/8, 3/8, 4/8, 5/8, 6/8, 7/8$. The mix ratio may
25 be a simple grayness level or a color mixture if a color monitor is utilized. Therefore, the method does not require the determination of the exact distance between the intermediate point 44P and a second pixel 44B, for example. The interval between the second and
30 third pixels 44B and 44C, for example, is divided into 8 sub-intervals, e.g., $I_0, I_1, I_2, I_3, I_4, I_5, I_6,$ and I_7 . The total number m of intervals I_m is arbitrary and is determined by the graphic system hardware capability,

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but is selected to be eight ($m = 8$) for purposes of illustration.

Then, a determination is made of the sub-interval I_i (where $I_i = (i/8, (i+1)/8)$ and $i \leq m$) to which the intermediate point belongs. This can be done by evaluating the polynomial equation $F(x,y) = 0$ using all possible intermediate points in a curve $F(0,0/8), F(0,1/8), F(0,2/8) \dots F(0,6/8), F(0,7/8), F(0,8/8)$ to find i such that

$$F(0,i/8) \leq 0 \text{ and } F(0,(i+1)/8) > 0.$$

which guarantees that $P \in I_i$. As a result, the mix ratio is determined to be $P_i = i/8$.

Because $F(x,y)$ is a polynomial with rational coefficients, a sufficiently large integer μ can be found such that the evaluation of

$$\mu F(x,y+i/8)$$

involves only integer operations for integers x, y and i . Therefore, the assumption is made that $F(x,y+i/8)$ is an integer for all integers x, y and i . Otherwise, $F(x,y)$ is multiplied by μ .

Furthermore, the increment of $F(x,y)$:

$$\Delta F_{\Delta x, \Delta y}(x,y) = F(x+\Delta x, y+\Delta y) - F(x,y)$$

is also a polynomial and is always one order lower than $F(x,y)$. Therefore, after $F(0,0)$ is calculated, to calculate $F(0,1/8)$, it is only necessary to calculate $\Delta F_{0,1/8}(0,0)$ and then add it to $F(0,0)$, and so on. In this way, computational burden can be further reduced

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because only a lower order polynomial needs to be evaluated.

After the mix ratio $p_i = i/8$ is found, a curve can be drawn with anti-aliasing. For example, if the
 5 curve $F(x,y)=0$ needs to be drawn with color A, the background colors on the $F(x,y)<0$ side and the $F(x,y)>0$ side of the curve are color B1 and color B2, respectively. Then, the pixel 44B will have the mixed color

$$10 \quad \text{pixel_color1} = p_i * \text{ColorB1} + (1-p_i) * \text{ColorA} \quad (1)$$

and the pixel 44C will have the mixed color

$$\text{pixel_color2} = (1-p_i) * \text{ColorB2} + p_i * \text{ColorA} \quad (2)$$

In general, color B1, color B2, and color A are all color vectors with three components of red,
 15 green, and blue:

$$\text{ColorB1} = (\text{ColorB1}_r, \text{ColorB1}_g, \text{ColorB1}_b)$$

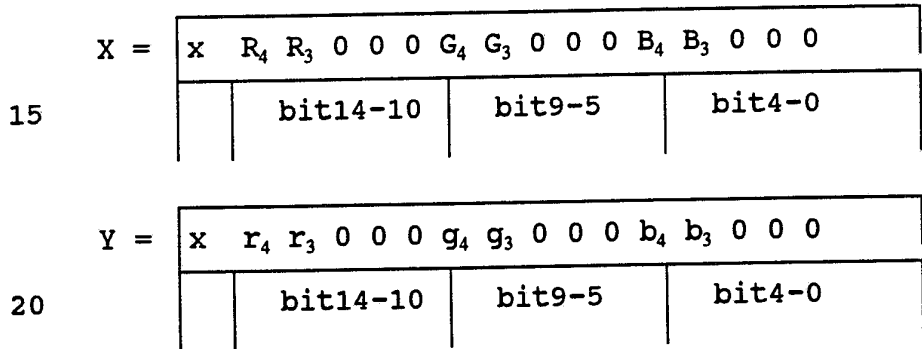
$$\text{ColorB2} = (\text{ColorB2}_r, \text{ColorB2}_g, \text{ColorB2}_b)$$

$$\text{ColorA} = (\text{ColorA}_r, \text{ColorA}_g, \text{ColorA}_b).$$

Therefore, the multiplications and additions in (1) or
 20 (2) must be performed on three components separately.

However, in the case when HiColor Palette is used, it is possible to perform the mixing color operation of (1) or (2) in only one step instead of three, as explained below.

As shown in Figure 3, HiColor Palette uses 2 bytes to directly represent a pixel color with 5-bit each for red, green, and blue components and bit-15 ignored. For easy manipulation of mixing colors, the two most significant bits of R, G, B components, i.e., bits R₄, R₃, G₄, G₃, B₄, B₃, are used to define a color. The three least significant bits are used to obtain different percentages of the defined color. Therefore, 64 different colors can be defined and 8 different mix percentages are available. For example, if color X and color Y are given as follows



then, a mixed color Z_i can be obtained by

$$Z_i = p_i X + (1-p_i) Y.$$

In this case, the red, green, and blue components do not need to be handled separately. Therefore, the mixing color operation can be completed in a single step.

In the special case of a linear curve (i.e., a line) and eight mix ratios, the foregoing equation can be expressed

$$Z_i = \frac{8-i}{8} * X + \frac{i}{8} * Y$$

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where $i = 0, 1, 2, \dots, 8$.

To find the mix ratio $P = p/8$, the following criterion function is used:

$$e(x,y) = 8 (x*dy - y*dx).$$

5 In this example, the coefficient 8 is used so that only integer operation is involved when the function is evaluated. Note that $e(x,y) = 0$ if a point (x,y) is on a curve 40, $e(x,y) > 0$ if a point (x,y) is on one side of the curve 40 and $e(x,y) < 0$ if a point (x,y) is on
10 another side of curve 40, as shown in Figure 4.

One grid unit is divided into eight intervals as shown in the adjacent pixels 44B and 44C in Figure 4. If the curve 40 passes between points $(0,1)$ and $(1,1/8)$, then the mix ratio is $p = 1/8$. If the curve
15 40 passes between points $(1,1/8)$ and $(1,2/8)$, then the mix ratio is $p = 2/8$. In general, if a curve 40 passes between points $(1,(p-1)/8)$ and $(1,p/8)$, then the mix ratio is $p=p/8$. Which interval through which the curve 40 will pass is determined by evaluating the criterion
20 function. If

$$e(1,(p-1)/8) \geq 0 \text{ and } e(1,p/8) < 0$$

then the curve 40 passes between $(1,(p-1)/8)$ and $(1,p/8)$.

Because $e(x,y)$ is a linear function, then

$$\begin{aligned} 25 \quad e(x+\Delta x, y+\Delta y) &= e(x,y) + e(\Delta x, \Delta y) \\ &= e(x,y) + e(0, 1/8) \\ &= e(x,y) - dx, \end{aligned}$$

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and evaluation of the function can be performed incrementally and computation time can be reduced.

After the proper mixture of color Z_i is found, gamma correction may be performed before the color values are sent to triple digital-to-analog converters (DACs, not shown) which drive the monitor. Gamma correction, which is conventional *per se*, is desirable because the luminous output of the phosphors of the CRT monitor screen has a nonlinear relation to the input value of the DAC. Gamma correction may be accomplished through conventional methods.

By following the flow chart of Figures 5A and 5B, a polynomial curve $F(x,y) = 0$ can be drawn from point (x_1,y_1) to point (x_2,y_2) with anti-aliasing. The method includes a number of assumptions which, for all points (x,y) along the curve, are:

- (1) $|dy/dx| \leq 1$ (Step 61). Otherwise, divide the curve into several sub-curves and exchange x and y (Step 62).
- (2) A total of m different mix ratios are available.
- (3) $F(x,y+i/m)$ is the integer for integers of x , y , i (Step 63). Otherwise, $F(x,y)$ is multiplied by a sufficiently large integer (Step 64).
- (4) $F(x_1+s_x,y_1) < 0$, where $s_x = \text{signum}(x_2-x_1)$ (Step 65). Otherwise, $F(x,y)$ is multiplied by -1 (Step 66).
- (5) The background colors on the $F(x,y) < 0$ side and the $F(x,y) > 0$ side of the curve are ColorB1 and ColorB2, respectively. The curve color is ColorA. ColorB1, ColorB2, and ColorA are all color vectors with three components: red, green, and blue.
- (6) $F(x_1,y_1) = 0$.

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The method also involves a drawing procedure, which includes:

Procedure 1.

5 $sx = \text{signum}(x_2 - x_1)$, $sy = \text{signum}(y_2 - y_1)$,
 $x = x_1$, $y = y_1$ (Step 67),
 $\text{pixel_color1} = \text{ColorA}$ (Step 68),
 perform gamma correction on pixel_color1
 (Step 69).

Procedure 2.

10 plot pixel (x, y) with pixel_color1 (Step
 70).

Procedure 3.

15 if $F(x + sx, y + sy) \leq 0$ (Step 71) then
 $x = x + sx$, $y = y + sy$, $i = 0$ (Step 73)
 else
 $x = x + sx$ (Step 72).

Procedure 4.

 while $F(x, y + (i + 1)/m) \leq 0$ (Step 74) do
 $i = i + 1$ (Step 75).

20 Procedure 5.

$\text{pixel_color1} = (i/m) * \text{ColorB1} +$
 $(1 - i/m) * \text{ColorA}$ (Step 76),
 $\text{pixel_color2} = (1 - i/m) * \text{ColorB2} +$
 $(i/m) * \text{ColorA}$ (Step 77),
25 perform gamma correction on pixel_color1 and
 pixel_color2 (Step 78),
 plot pixel (x, y) with pixel_color1 (Step
 79),
 plot pixel $(x, y + 1)$ with pixel_color2 (Step
30 80).

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Procedure 6.

if x=x2 (Step 81) then
stop (Step 82)
else
5 go to Procedure 3 (Step 71).

The foregoing has described the principles, preferred embodiments and modes of operation of the present invention. For example, the colors may be expressed in other color space models instead of the
10 RGB model. Accordingly, the present invention should not be construed as being limited to the particular embodiments discussed. Instead, the above-described embodiments should be regarded as being illustrative rather than restrictive, and it should be appreciated
15 that variations may be made in those embodiments by workers skilled in the art without departing from the scope of the present invention as defined by the following claims.

WHAT IS CLAIMED IS:

1. A method of displaying a curve having an illumination state corresponding to a particular color or gray scale value on a display monitor having pixels spaced one grid unit apart in both the horizontal and vertical axes, each pixel capable of displaying one of a plurality of illumination states, the curve being substantially approximated by a polynomial equation of the form $F(x,y) = 0$ where pairs of x and y values map onto points of the display monitor, comprising the steps of:

dividing at least one grid unit defined by the distance between a first pixel having a first pixel illumination state, and an adjacent second pixel having a second pixel illumination state, into an integer number of sub-units, the distance between adjacent sub-units defining a sub-interval;

determining which sub-interval the curve bisects;

generating a signal for displaying a first mixed illumination state on the first pixel and a second mixed illumination state on the second pixel, the first and second mixed illumination states being combinations of the curve illumination state with, respectively, the first and second pixel illumination states, and the ratio of the curve illumination state to respective first and second pixel illumination states being defined by the sub-interval bisected by the curve.

2. A method according to Claim 1, wherein the polynomial equation is multiplied by a sufficiently large factor such that results of the polynomial equation are integers for each of the sub-intervals.

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3. A method according to Claim 2, wherein the factor may be 1.

4. A method according to Claim 1, wherein the plurality of illumination states is a grayness level.

5 5. A method according to Claim 1, wherein the plurality of illumination states is a color mixture.

6. A method according to Claim 1, wherein the display monitor is a monochrome monitor.

7. A method according to Claim 1, wherein the
10 display monitor is a color monitor.

8. A method according to Claim 1, wherein the determining step includes evaluating the polynomial equation with the sub-unit coordinates of each sub-interval.

15 9. A method according to Claim 1, wherein the curve illumination state, first pixel illumination state, and second pixel illumination state each comprise a color vector with red, green and blue components.

20 10. A method according to Claim 1, wherein the curve illumination state, first pixel illumination state, and second pixel illumination state each comprise digital bits corresponding to color components.

25 11. A method according to Claim 1, wherein the curve is a curved line.

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12. A method according to Claim 1, wherein the curve is the boundary of a polygon area.

13. In an apparatus having storage means for storing a color A and a color B in a HiColor format, the HiColor format comprising the storage of 5 bits representing a red component of color, 5 bits representing a green component of color, and 5 bits representing a blue component of color, a method for mixing color A with color B to produce a third color C according to the formula $C = pA + (1-p)B$, where $0 \leq p \leq 1$, comprising the steps of:

a) storing a HiColor representation of color A in which the three least significant bits of the red component, the three least significant bits of the green component, and the three least significant bits of the blue component are all zero;

b) storing a HiColor representation of color B in which the three least significant bits of the red component, the three least significant bits of the green component, and the three least significant bits of the blue component are all zero; and

c) producing a third HiColor representation corresponding to color C according to the formula $C = pA_{\text{stored}} + (1-p)B_{\text{stored}}$, where A_{stored} and B_{stored} are, respectively, the stored representations of colors A and B, and wherein each of A_{stored} and B_{stored} is manipulated as a single value.

14. In an apparatus having a storage means for storing a color A and a color B in a true-color format, the true-color format comprising the storage of x bits representing a red component of color, y bits representing a green component of color, and z bits representing a blue component of color, x, y and z each being integers, a method for mixing color A with color

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B to produce a third color C according to the formula $C = pA + (1-p)B$, where $0 \leq p \leq 1$, comprising the steps of:

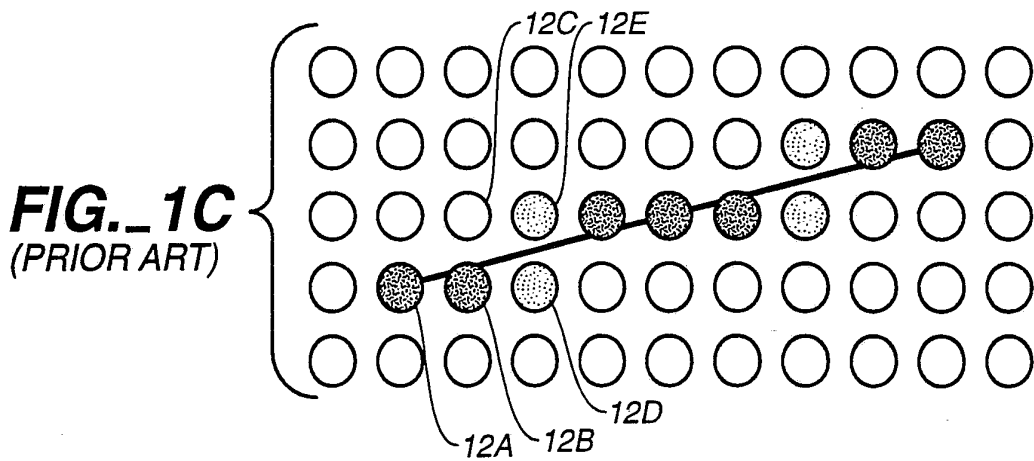
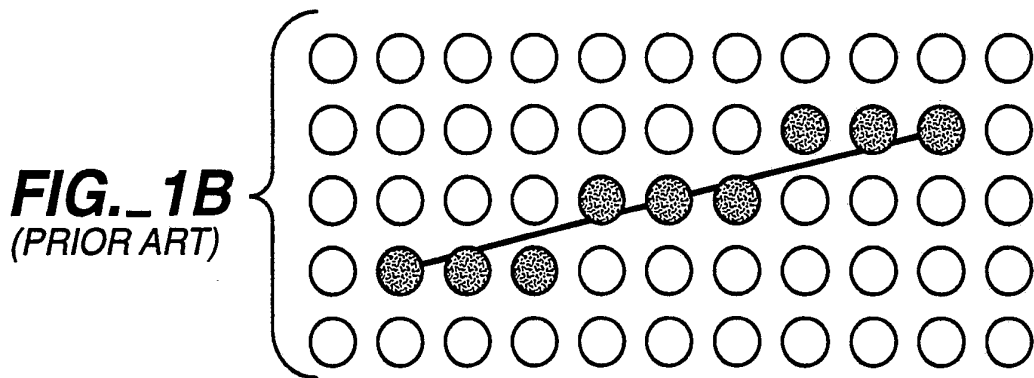
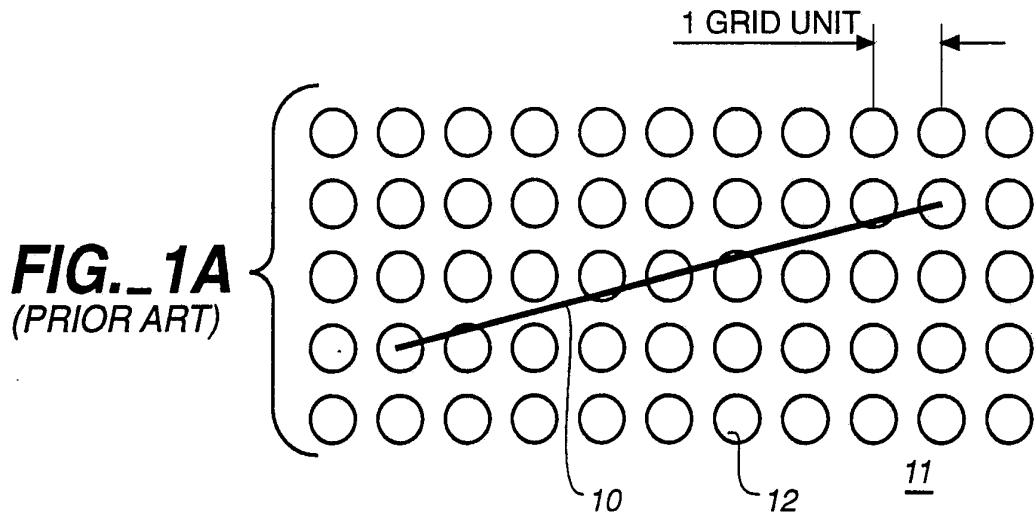
- 5 a) storing a true-color representation of color A in which the w least significant bits of the red component, the w least significant bits of the green component, and the w least significant bits of the blue component are all zero, where $w \leq x$, $w \leq y$, and $w \leq z$;
- 10 b) storing a true-color representation of color B in which the w least significant bits of the red component, the w least significant bits of the green component, and the w least significant bits of the blue component are all zero, where
- 15 $w \leq x$, $w \leq y$, and $w \leq z$; and
- c) producing a third true-color representation corresponding to color C according to the formula $C = pA_{\text{stored}} + (1-p)B_{\text{stored}}$, where A_{stored} and B_{stored} are, respectively, the stored representations of colors
- 20 A and B, and wherein each of A_{stored} and B_{stored} is manipulated as a single value.

15. A method according to Claim 13, wherein step
- a) further comprises if the three least significant bits of color A's red, green, and blue components are
- 25 not zero, then substituting for color A the closest color to color A in which the three least significant bits of the red, green, and blue components are zero, and wherein further step b) further comprises if the three least significant bits of color B's red, green,
- 30 and blue components are not zero, then substituting for color B the closest color to color B in which the three least significant bits of the red, green, and blue components are zero.

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16. A method according to Claim 14, wherein step
a) further comprises if the w least significant bits of
color A's red, green, and blue components are not zero,
then substituting for color A the closest color to
5 color A in which the w least significant bits of the
red, green, and blue components are zero, and wherein
further step b) further comprises if the w least
significant bits of color B's red, green, and blue
components are not zero, then substituting for color B
10 the closest color to color B in which the w least
significant bits of the red, green, and blue components
are zero.

17. A method according to Claim 1 in which the
step of determining further comprises the steps of:
15 evaluating the polynomial equation at points
represented by adjacent sub-units to produce a first
value and a second value; and
determining the bisected sub-interval to be
the one for which the two values are of opposite signs.



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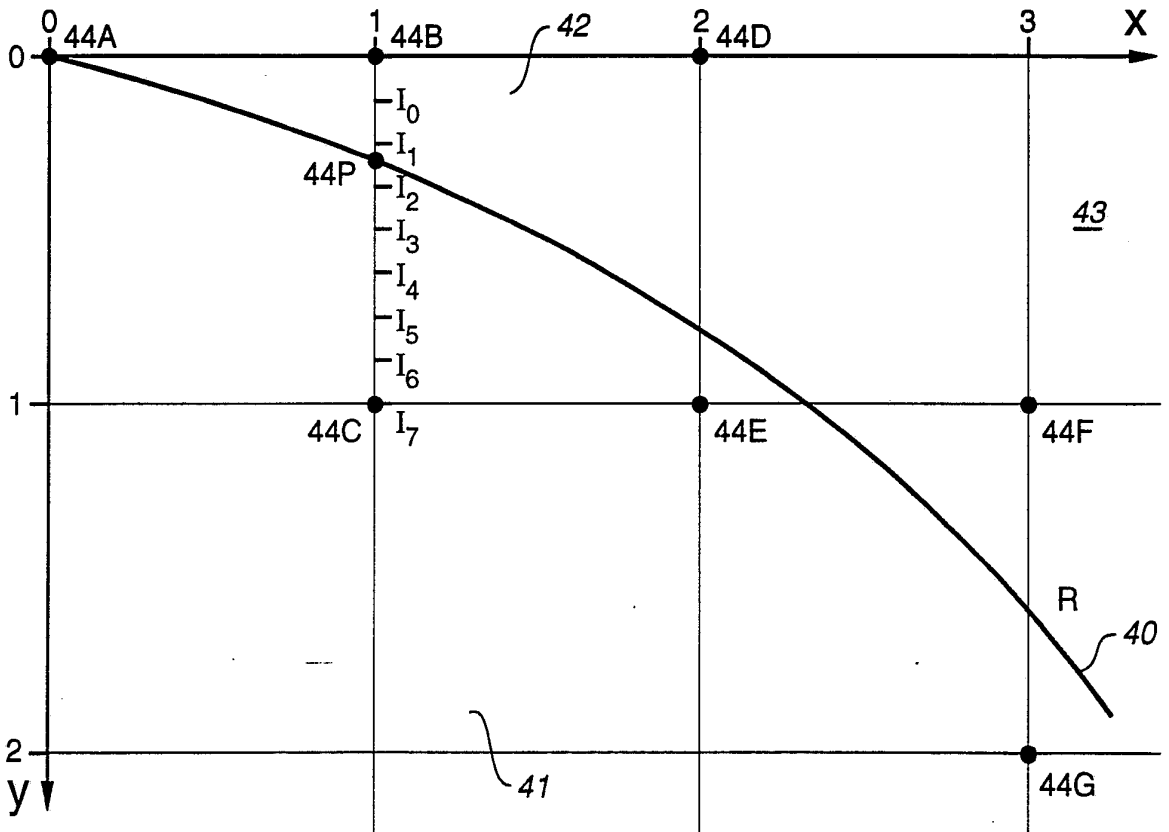
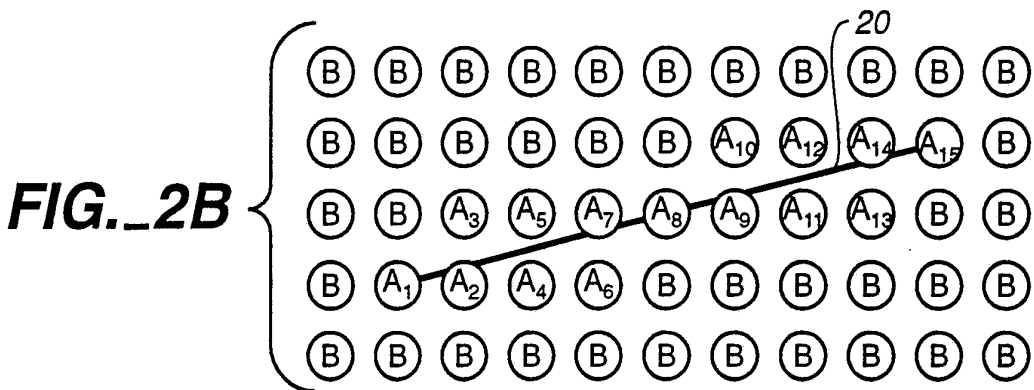
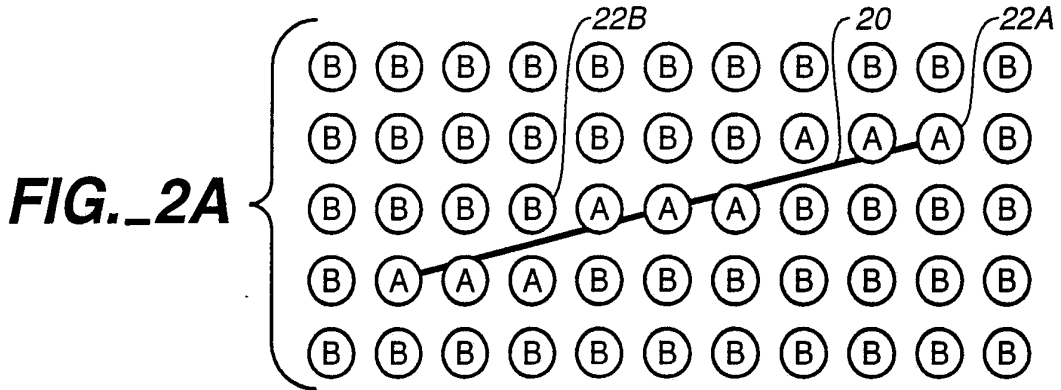
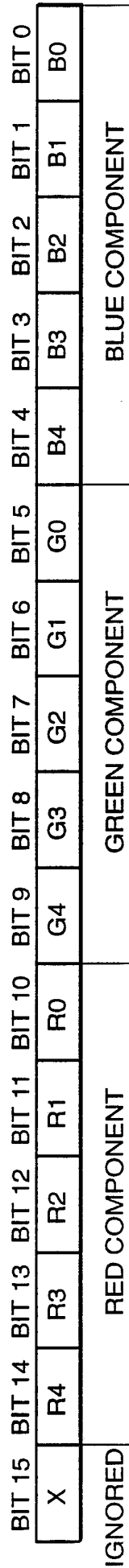


FIG. 4



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FIG.-3

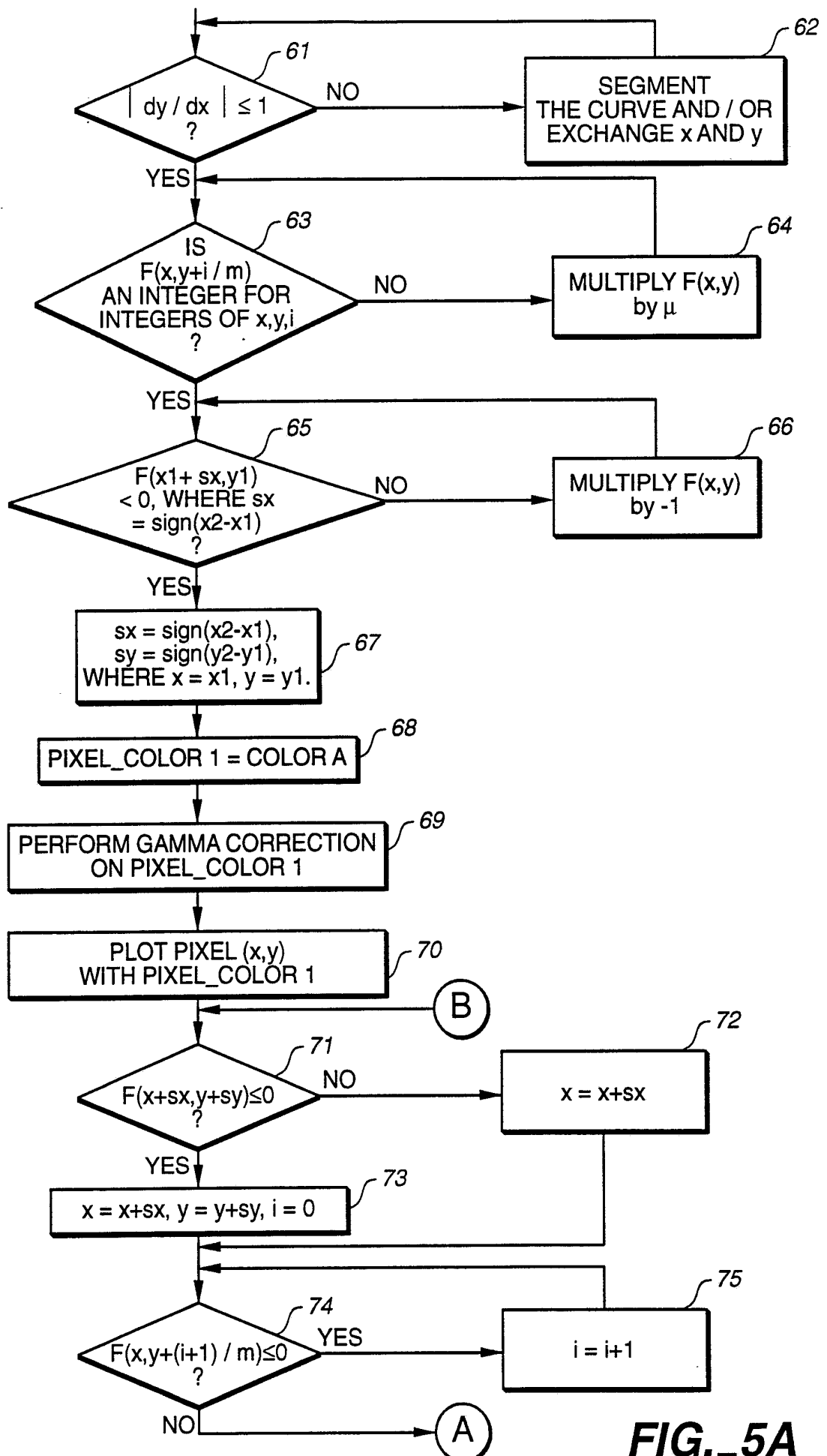


FIG. 5A

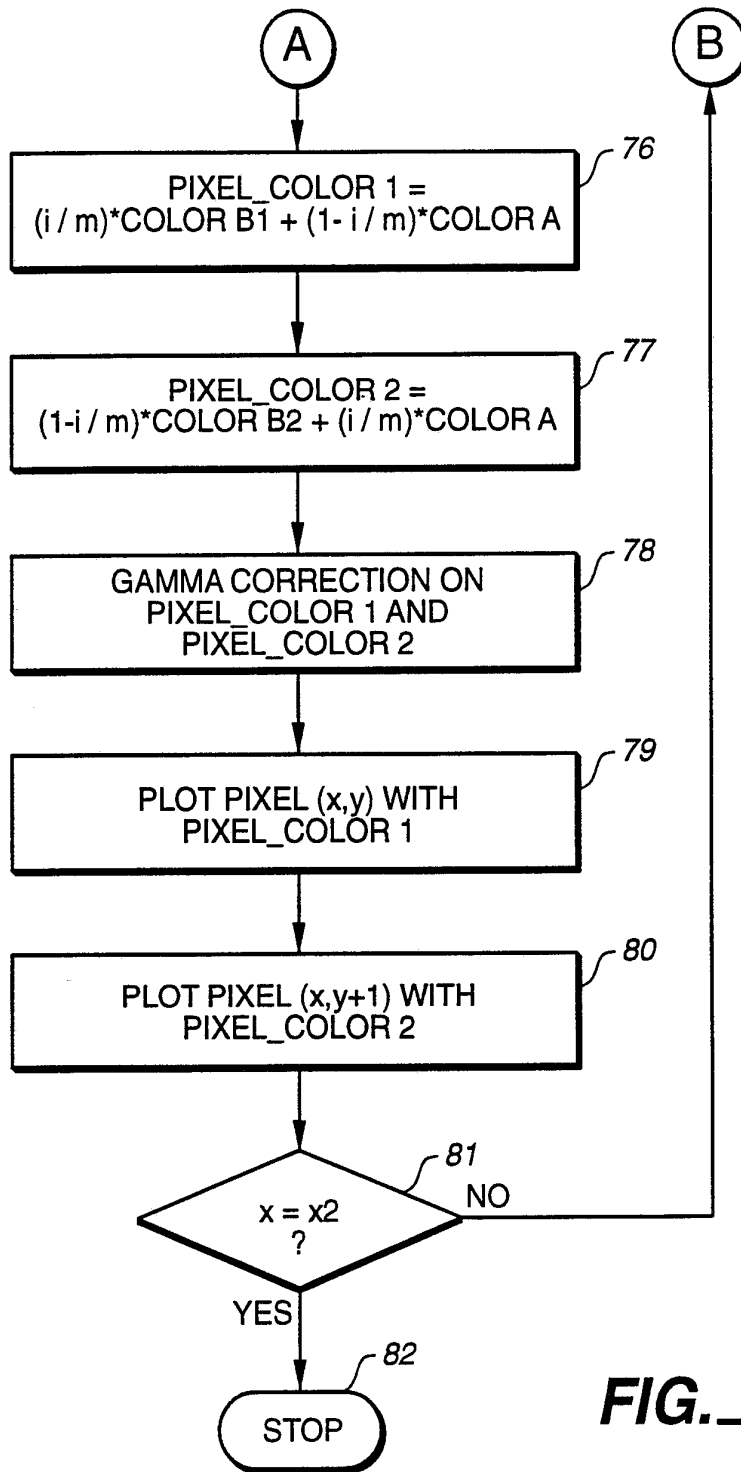


FIG. 5B

INTERNATIONAL SEARCH REPORT

PCT/US92/11342

A. CLASSIFICATION OF SUBJECT MATTER

IPC(5) :G06F 15/62

US CL :395/128,131

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)

U.S. : 395/125,126,127,128,129,130,131,132,162,166

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
Y	US, A, 4,811,245 (BUNKER ET AL.) 07 March 1989, See columns 8-9.	1-12 & 17
Y	US, A, 4,752,893 (GUTTAG ET AL.) 21 June 1988, See columns 2-3.	13-16

Further documents are listed in the continuation of Box C.

See patent family annex.

* Special categories of cited documents:	"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention
"A" document defining the general state of the art which is not considered to be part of particular relevance	"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone
"E" earlier document published on or after the international filing date	"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art
"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)	"&" document member of the same patent family
"O" document referring to an oral disclosure, use, exhibition or other means	
"P" document published prior to the international filing date but later than the priority date claimed	

Date of the actual completion of the international search

18 FEBRUARY 1993

Date of mailing of the international search report

05 MAR 1993

Name and mailing address of the ISA/US
Commissioner of Patents and Trademarks
Box PCT
Washington, D.C. 20231

Facsimile No. NOT APPLICABLE

Authorized officer

ALMIS JANKUS

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INTERNATIONAL SEARCH REPORT

International application No.

PCT/US92/11342

Box I Observations where certain claims were found unsearchable (Continuation of item 1 of first sheet)

This international report has not been established in respect of certain claims under Article 17(2)(a) for the following reasons:

1. Claims Nos.:
because they relate to subject matter not required to be searched by this Authority, namely:

2. Claims Nos.:
because they relate to parts of the international application that do not comply with the prescribed requirements to such an extent that no meaningful international search can be carried out, specifically:

3. Claims Nos.:
because they are dependent claims and are not drafted in accordance with the second and third sentences of Rule 6.4(a).

Box II Observations where unity of invention is lacking (Continuation of item 2 of first sheet)

This International Searching Authority found multiple inventions in this international application, as follows:

GROUP I: Claims 1-12 and 17 are drawn to a method of displaying a curve.

GROUP II: Claims 13-16 are drawn to a method and apparatus for producing a color by combining two colors.

1. As all required additional search fees were timely paid by the applicant, this international search report covers all searchable claims. (Telephone Practice)
2. As all searchable claims could be searched without effort justifying an additional fee, this Authority did not invite payment of any additional fee.
3. As only some of the required additional search fees were timely paid by the applicant, this international search report covers only those claims for which fees were paid, specifically claims Nos.:

4. No required additional search fees were timely paid by the applicant. Consequently, this international search report is restricted to the invention first mentioned in the claims; it is covered by claims Nos.:

Remark on Protest

The additional search fees were accompanied by the applicant's protest.

No protest accompanied the payment of additional search fees.