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ANTENNA

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Fig. 1.

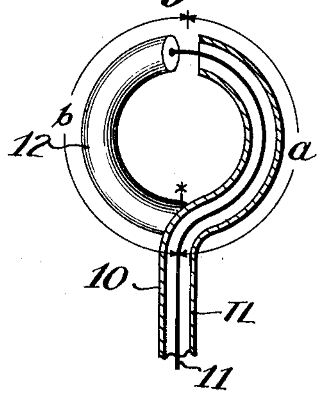

Fig. 2.

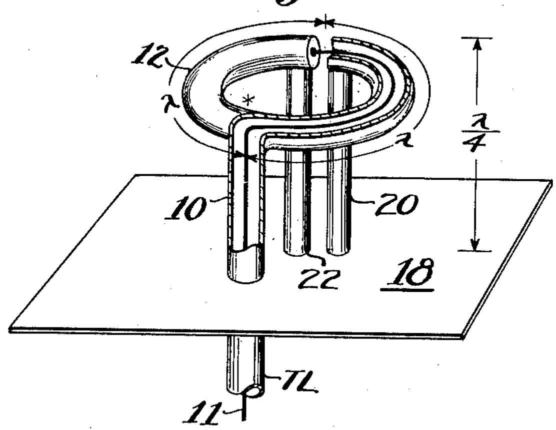

Fig. 4.

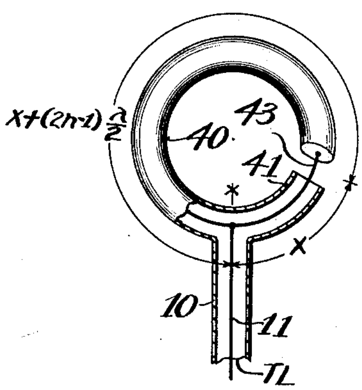

Fig. 3.

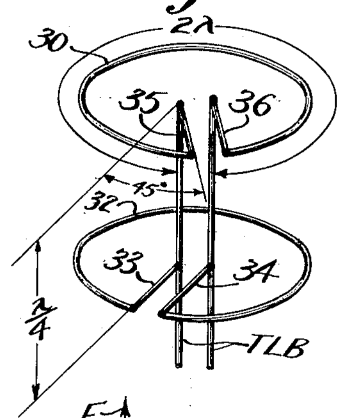

Fig. 5.

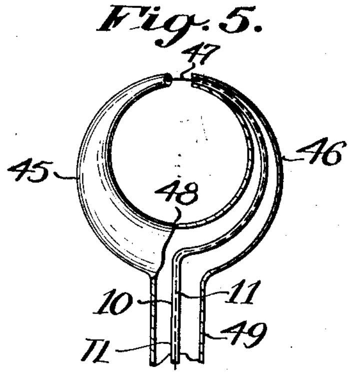

Fig. 6.

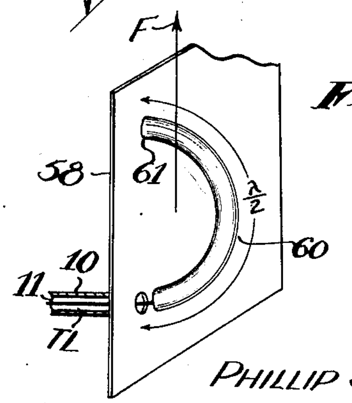

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Fig. 7.

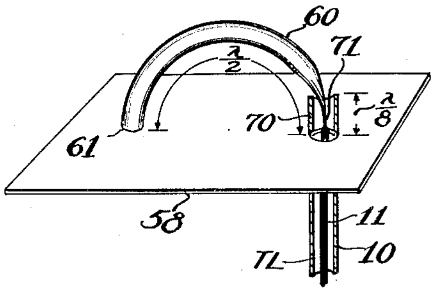

Fig. 8.

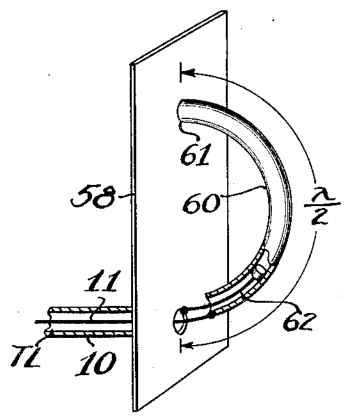

Fig. 9.

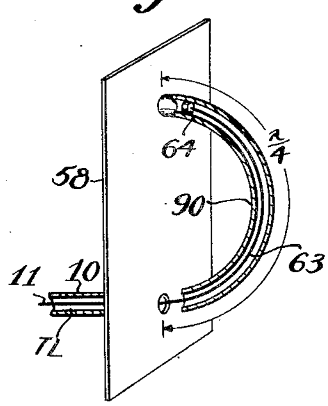

Fig. 10.

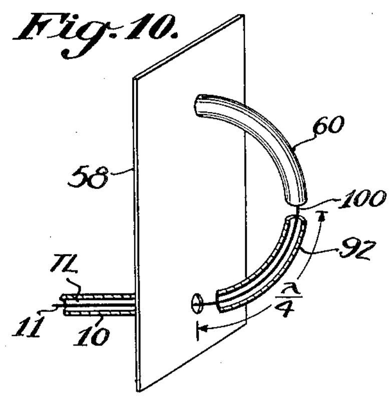

Fig. 12.

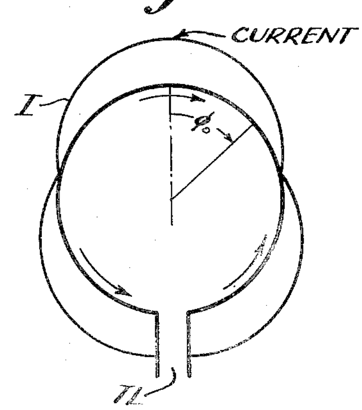

Fig. 11.

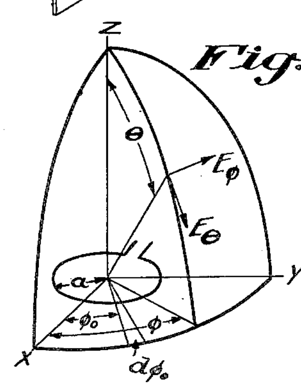

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Fig. 13.

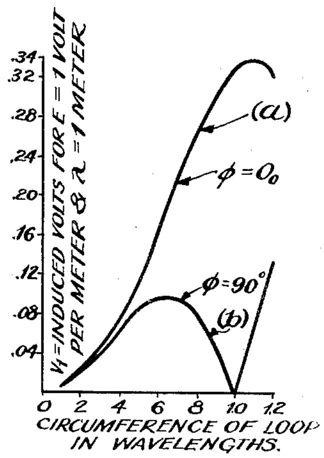


Fig. 14.

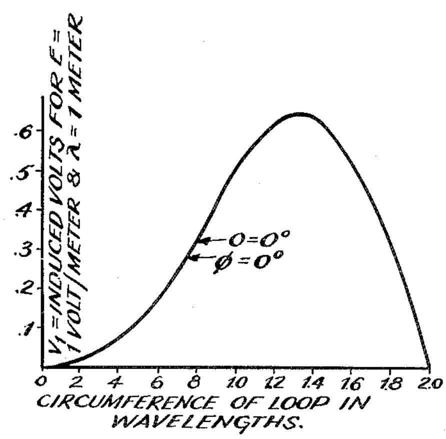


Fig. 17.

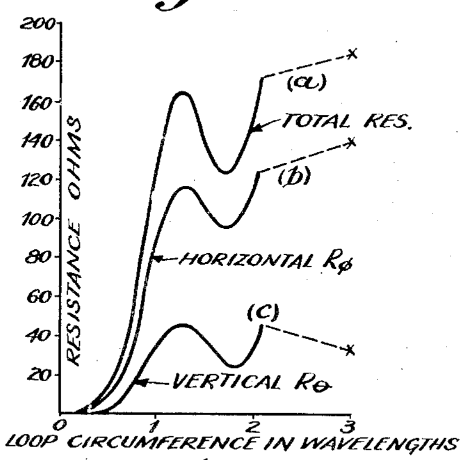
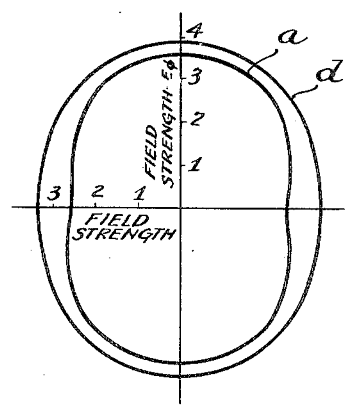


Fig. 15.



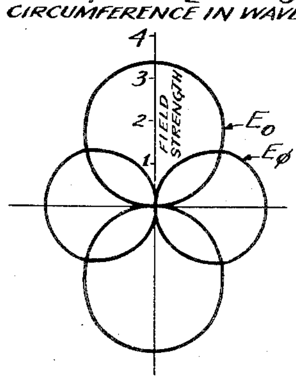


Fig. 16.

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UNITED STATES PATENT OFFICE

2,615,134

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Philip S. Carter, Port Jefferson, N. Y., assignor to Radio Corporation of America, a corporation of Delaware

Application January 9, 1946, Serial No. 639,998

3 Claims. (Cl. 250—33.67)

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The present invention relates to ultra short wave antennas and, more particularly, to loop antennas having dimensions large in terms of the operating wavelength.

An object of the present invention is the improvement of the band width of antennas.

Another object of the invention is improving the efficiency of short wave antennas.

Still another object of the present invention is the provision of an antenna which may be mounted on an airplane without presenting objectionably large wind resistance.

A further object of the present invention is the provision of an airplane antenna which is less hazardous to ground crew personnel than previously known types.

Still a further object of the present invention is the provision of an antenna which, if desired, produces a non-directional radiation pattern for broadcasting use.

Still a further object of the present invention is the provision of an antenna which is mechanically stable and electrically grounded.

Still another object of the present invention is the provision of a loop antenna which may be energized from a transmitter by a coaxial line without the use of line balance converters or similar matching structures.

The foregoing objects and others which may appear from the following detailed description are attained by providing a loop antenna, having a circumference of the order of one or a few wave-lengths at the operating frequency. The loop is preferably a single turn arrangement using a large diameter conductor. The antenna may be coupled to a coaxial transmission line and the antenna itself may utilize elements of the said coaxial transmission line as elements of the loop construction. In a modified form a half loop working against a metallic ground plane may be employed.

The present invention will be more fully understood by reference to the following detailed description which is accompanied by a drawing in which:

Fig. 1 illustrates in simplified diagrammatic form an embodiment of the present invention, while

Fig. 2 illustrates a modification of the form of construction shown in Fig. 1, particularly useful for broadcasting use.

Fig. 3 illustrates a further modification of the invention wherein a plurality of horizontal loops are vertically stacked to produce a more uniform radiation pattern in the horizontal plane.

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Fig. 4 illustrates a modified form of feeder construction for the antenna of Fig. 1.

Fig. 5 illustrates a further modification of the present invention whereby the band pass characteristic of the antenna is improved.

Fig. 6 illustrates in simplified diagrammatic form a further modification of the present invention utilizing a half loop operating against a metallic ground plane.

Fig. 7 illustrates a further modification of Fig. 6 whereby the band width of the antenna is increased.

Figs. 8, 9 and 10 illustrate further modifications of the method of feeding the antennas of Fig. 6.

Fig. 11 is a perspective view of a horizontal loop antenna and the reference axes radiating from the antenna useful in understanding the following figures.

Fig. 12 is a diagrammatic representation of the current distribution of a loop antenna utilizing the principles of the present invention.

Fig. 13 is a curve illustrating the relationship of the voltage induced in a receiving loop antenna versus the circumference of the loop in terms of operating wavelength.

Fig. 14 is a curve illustrating the relationship between the induced voltage in a loop antenna and the circumference of the loop for energy arriving along a direction perpendicular to the plane of the loop, while

Fig. 15 illustrates the horizontal and vertical directivity patterns of a horizontal loop antenna, utilizing the principles of the present invention.

Fig. 16 is a further field strength pattern of the present invention, and

Fig. 17 is a family of curves illustrating the relationship between the radiation resistance of a loop antenna versus the loop circumference in wavelengths.

Referring now to Fig. 1, there is shown a method of feeding a loop antenna which is large in terms of the operating wavelength, from a single coaxial line. Transmission line TL includes an outer sheath 10 and an inner conductor 11. At point X the transmission line TL is bent around substantially half the circumference of a circle. For the remainder of the circumference of the circle a conductor 12 is provided having the same diameter as sheath 10 and electrically connected to sheath 10 at point X. At a point on the circumference of the circle opposite point X where conductor 12 and outer sheath 10 are in end to end relationship, the inner conductor 11 of the transmission line TL extends beyond sheath 10 and is connected to the end of conductor 12. Thus, distances a and b are equal and the opposing ends of conductor 12 and sheath 10 are equi-distant from a point of zero reference potential as may be represented by point X. Therefore, radio frequency energy, unbalanced with respect to ground, flowing in line TL may be used to energize the antenna.

The radiation pattern of the loop antenna of Fig. 1 varies somewhat with the dimensions of the loop in terms of the operating wavelength. The effect of variations in the dimensions will be discussed later with reference to following figures and mathematical developments. However, it may be remarked at this point that tests of the loop of Fig. 1 show comparatively small variations of impedance over a wide frequency band.

In case it is desired to radiate energy substantially uniformly in a horizontal plane, the construction shown in Fig. 2 may be used. Transmission line TL in Fig. 2 feeds the loop antenna generally similar in construction to that shown in Fig. 1 and similar elements have similar reference characters applied thereto. However, at point X the antenna of Fig. 2 is bent at right angles so that transmission line TL is vertical while the plane of the loop itself is horizontal. At the adjacent ends of outer sheath 10 and conductor 12, conductive vertical supporting posts 20 and 22 are provided. All three supports, that is line TL and posts 20, 22 are connected at their lower ends to the metallic ground plane 18. The length of the supporting means for the antenna are so chosen as to be equal to an odd multiple of a quarter wavelength as indicated in Fig. 2. The entire loop is effectively insulated from the ground plane 18 for operating frequency currents while at the same time, for direct current, lightning strokes etc., the entire structure may be considered grounded. The antenna of Fig. 2 has a circumference of two wavelengths at the operating frequency. The effect on the pattern of the increase in circumference will be discussed in more detail later.

Fig. 3 illustrates another construction of the loop antenna of the present invention whereby it may be utilized for broadcast transmission or reception. Here two large diameter conductors are bent into nearly closed circles, each having a circumference of two wavelengths, thus forming loops 30 and 32. Between a pair of adjacent ends of loop 32 is connected the vertical central two wire balanced transmission line TLB by means of conductors 33 and 34 parallel to a radius of loop 32 and preferably in the plane of the loop. The balanced transmission line TLB continues vertically a distance equal to an odd multiple of a quarter wavelength to the center of loop 30. It is there connected to the opposing ends of loop 30 by means of conductors 35, 36 parallel to a radius of loop 30. Transmission line TLB is twisted between loops 30 and 32 so that the two radii mentioned above lie in vertical planes intersecting along the vertical axis of the antenna at an angle of forty-five degrees. Due to the forty-five degree displacement and the phase quadrature feeding relationship obtained by the quarter wave spacing between loops 30 and 32, perfect circle distribution of the radiated energy is attained in the horizontal plane. This may be demonstrated as follows:

From one loop alone, the field E is given by the relationship $E_1=A\cos 2\phi$, where ϕ is a measure of the angular displacement around the center of the loop in the horizontal plane. The second loop alone gives a field E_2 determined by the expression;

$$E_2=\pm jA\cos 2(\phi\pm 45°)=\mp jA\sin 2\phi \quad (1)$$

Adding the two fields together, the total field is:

$$E_t=A\,(\cos 2\phi\mp j\sin 2\phi)=A\epsilon^{\mp j2\phi} \quad (2)$$

Therefore, the field $|E_t|=A$ (3)

($|E_t|$ indicates the magnitude of E_t)

Though only one method of energizing the antenna of Fig. 3 is shown, any scheme giving a balanced feed may be used. Though the loops in Fig. 3 are shown as having a circumference of 2λ, a pair of loops having a circumference of 1λ may be used, in which case, they are fed in phase quadrature, by the same method as for the 2λ loop, but the feed points are now spaced 90° around the circumference. A similar mathematical consideration shows that the field is again perfectly circular. Any number of antennas may be vertically stacked in order to obtain as much vertical directivity as desired with uniform horizontal field strength.

Fig. 4 illustrates a modified manner of feeding the antenna of Fig. 1. Here the transmission line TL having a sheath 10 and an inner conductor 11 is split into two parallel connected branches 40 and 41 at point X. The two branches are bent around the circumference of a circle, branch 40 being longer than branch 41 by an odd multiple of a half wavelength. At the point where branches 40 and 41 oppose each other, the inner conductors are connected together, thus in effect providing a single continuous inner ring 43 connected at point X to inner conductor 11 of transmission line TL. Here again the loop antenna is energized in a balanced relationship from an unbalanced transmission line TL.

Fig. 5 illustrates a way in which a further improvement in the band width of the antenna of Fig. 1 may be attained. Here the loop antenna itself is formed of two hollow conductive cones 45 and 46. Each is so bent that its axis is half the circumference of a circle and the cones are so arranged that their apexes oppose each other at 47 while at 48 base ends of the cones are electrically connected together. At point 48 a suitable supporting pipe or hollow pole 49 is provided. Supporting pole 49 is preferably hollow so that transmission line TL may pass up through 49 into one of the cones, say cone 46, to its apex. There the outer conductor 10 of transmission line TL is connected to the apex of bent cone 46. The inner conductor 11 of transmission line TL passes across the gap at 47 and is connected to the apex of bent cone 45. Preferably the circumference of the loop of Fig. 5 is chosen to be a multiple of the operating wavelength at the midband of the frequency band for which it is designed. The radiation pattern and electrical advantages of Fig. 1 are retained in the modification of Fig. 5 with the further advantage that an increased broad band effect is also attained.

In Fig. 6 I have shown a modified form of the present invention which is particularly useful on airplanes. The antenna comprises a conductor bent in the form of a half loop 60 having a semi-circumference of one-half of the operating wavelength and connected at point 61 to a conductive ground sheet 58. At the opposite end of the semi-circumference the inner conductor 11 of transmission line TL is connected to the semi loop 60. The outer sheath 10 of transmission line TL is connected to the ground sheet **58**. Ground sheet **58** may be the outer surface of the fuselage of an airplane or it may be an upper or lower wing surface as desired.

As indicated by arrow F the antenna of Fig. 6 has a maximum of radiation in the direction along a line from the feed point toward point **61**. However, the radiation minimum in the right angle direction is not greatly less than the maximum. Such an arrangement provides an airplane antenna having a low wind resistance. Also, due to its small extension from the surface of sheet **58**, and shape, ground crew personnel are much less liable to eye and other injuries than when the common whip antenna is used. Furthermore, the smooth outline of the antenna and its small projection render it less likely to become entangled with cables which may be employed around the landing field.

The modification shown in Fig. 7 adds a sleeve feed to the antenna of Fig. 6 to widen its band width. Here the transmission line TL extends above ground sheet **58** a distance of the order of one-eighth of the operating wavelength in the form of a sleeve **70** and an inner conductor **71**. The inner conductor **71** is directly connected in series between semi-loop **60** and inner conductor **11** of transmission line TL. The inner conductor **71** preferably has a diameter less than the diameter of inner conductor **11**. Thus, the eighth wave section comprised by **70**, **71** is arranged to have a characteristic impedance of a value lying between the impedance of the transmission line and the impedance of semi-loop antenna itself. Otherwise, the construction of the antenna of Fig. 7 is the same as that of Fig. 6.

Similarly, the general construction of the further modification shown in Fig. 8 resembles that shown in Fig. 6, the only distinction being in the manner of feeding the antenna. Here the inner conductor **11** of transmission line TL is connected to the free end of semi-circular conductor **60**. The end of conductor **60** is hollow a distance equal to approximately a quarter of the operating wavelength from the free end. Supplemental conductor **62** is concentrically arranged within the hollow portion and is connected to ground plane **58** near point of emergence of inner conductor **10** of transmission line TL and connected to semi-circular conductor **60** at the bottom of the hollow portion.

Thus a quarter wave section of transmission line is placed effectively in shunt with the antenna at the feed point. This acts as a compensation circuit to widen the frequency band. At the midband frequency where the length is exactly a quarter wavelength the impedance is infinite but when the frequency is higher than that of the midband frequency it presents a capacitive reactance in shunt with the feed point, thus compensating for the inductive reactance of the antenna. At a frequency somewhat lower than the midband frequency the shunt line presents an inductive reactance in shunt with the antenna compensating for the capacitive reactance of the antenna.

Fig. 9 illustrates a further modification of the invention. Herein the semi-circular conductor **90** has an overall length of one quarter of the operating wavelength. Thus, the semi-loop taken in conjunction with the electrical image formed by ground plane **58** acts as a loop having a circumference of a half wavelength rather than a one wavelength circumference loop as previously described. Conductor **90** is hollow through nearly its entire length. Coaxially arranged within hollow conductor **90** is an extension **63** of the inner conductor **11** of transmission line TL. The interior quarter wave section acts as a compensation circuit to widen the frequency band. By suitably varying the length of extension conductor **63** within conductor **90** from the point of entrance to shorting plate **64**, the reactive component of the antenna may be tuned out over at least a portion of the operating band. The effect of the change in circumference upon the characteristics of the antenna will be discussed later by reference to the impedance characteristics and directivity patterns.

The half loop antenna shown in Fig. 10 is again a half wave semi-loop. The inner conductor **11** of transmission line TL extends half way around the semi-circumference of the loop where it meets and is connected to conductor **60**. It is surrounded by a curved supplemental sleeve **92** insulated from both conductor **60** and ground sheet **58**. Thus, the effective feed point of the loop is moved around to point **100** instead of being closely adjacent to the ground plane as in previously discussed modifications.

In the following mathematical development of the properties of large loop antenna, it should be understood that all loops are considered to lie in a horizontal plane with the axis of the loop pointing upward; that is, along the Z axis in Fig. 11. The feed terminals are assumed to lie on the negative X axis. The radius of the loop is given by a. The spherical coordinate system shown in Fig. 11 is used to specify directions with respect to the loop antenna. The position of the feed point as shown in Fig. 11 is important since it determines the current distribution in the loop. Fig. 11 further shows the polarizations of the electric vector, namely E_θ and E_ϕ, E_θ being the electric vector of a vertically polarized wave while E_ϕ indicates the electric vector of a horizontally polarized wave. In this application we define a vertically polarized wave as one whose magnetic vector is horizontal, i. e. the electric vector lies in a vertical plane containing the direction of propagation or ray. The object of this definition is to avoid limiting the direction of propagation to the horizontal.

The current distribution in the loop is assumed to be sinusoidal as shown in Fig. 12. This is a close approximation to the actual distribution except at the current minimum points where the actual current is finite rather than zero. The current distribution in the loop of Fig. 12 where the circumference is one and one-quarter wavelengths is shown to be symmetrical with respect to a point opposite the feed point and is proportional to cos $(Ka\phi_0)$ where

$$K=\frac{2\pi}{\lambda}$$

a is the loop radius and ϕ_0 is the angle to the reference point which is taken as diametrically opposite to the feed point. Regardless of the circumference of the loop the current distribution will always be symmetrical with respect to the aforementioned point opposite the feed position.

Curve a in Fig. 13 shows the voltage induced in a receiving loop by a horizontally polarized wave coming from the direction $\phi=0$, $\theta=90°$ (as indicated in Fig. 11). The voltage is given in terms of field strength per wavelength. Thus, if the field strength is 10 microvolts per meter and the wavelength is 2 meters, the voltage in a half wave loop would be $10 \times 2 \times .1205 = 2.41$ microvolts. Curve *b* of Fig. 13 is a similar curve for a direction $\phi=90°$, $\theta=90°$ (Fig. 11). In this direction the pick-up becomes zero when the circumference of the loop is one wavelength or any odd multiple thereof while it becomes a maximum when the circumference is an even multiple of a wavelength.

The curve in Fig. 14 shows the voltage induced in a receiving loop when the wave direction is perpendicular to the plane of the loop, that is, along the axis where $\theta=0°$ and $\phi=0°$. Instead of using the curve of Figs. 13 and 14 for determining the voltage induced in the receiving loop, the voltage may be determined from the following equations:

First, the situation will be considered where a very small loop compared to a wavelength is used.

Let V_0 be the voltage induced in such a loop, then

$$V_0 = jK\pi a^2 E_\phi = j\frac{2\pi}{\lambda} \times \text{area} \times E_\phi \text{ volts} \quad (4)$$

where

$$K = \frac{2\pi}{\lambda}$$

a=radius, and E_ϕ=field strength in volts per unit of length. The voltage induced in any sized loop by a horizontally polarized wave coming from a direction (θ,ϕ) is given by the following equation in the form of an infinite series:

$$V = -jV_0 \times \frac{4}{\pi} \sin (Ka\pi) \left\{ j\frac{J_1(Ka \sin \theta)}{(Ka)^2} + \frac{2[J_0(Ka \sin \theta) - J_2(Ka \sin \theta)]}{1-(Ka)^2} \right\} \cos \phi - \frac{2J_1(Ka \sin \theta) - J_3(Ka \sin \theta)}{2^2-(Ka)^2} \cos 2\phi - \frac{2J_2(Ka \sin \theta) - J_4(Ka \sin \theta)}{3^2-(Ka)^2} \cos 3\phi + j\frac{2[J_3(Ka \sin \theta) - J_5(Ka \sin \theta)]}{4^2-(Ka)^2} \sin 4\phi + \ldots \quad (5)$$

where $J_n(u)$=the Bessel function of the first kind of order n (and argument u). In the case where the loop is an even multiple m of a wavelength in circumference, the foregoing expression becomes:

$$|V| = |V_0| [J_{m-1}(m \sin \theta) - J_{m+1}(m \sin \theta)] \cos m\phi \quad (6)$$

wherein the circumference is $m\lambda$.

When the wave is polarized in the horizontal plane, that is, when $\theta=90°$, the voltage becomes:

$$|V| = |V_0| [J_{m-1}(m) - J_{m+1}(m)] \cos m\phi \quad (7)$$

If the loop is one wavelength in circumference (7) becomes:

$$|V| = |V_0| \times 0.7652 \cos \phi \quad (8)$$

while for two wavelengths in circumference, (7) becomes the following:

$$|V| = |V_0| \times 0.4478 \cos 2\phi \quad (9)$$

In case the wave is vertically polarized, the equation for the induced voltage is as follows:

$$V = -j_2 V_0 \times \frac{4}{\pi} \sin (Ka\pi) \cos \theta \left\{ -j\frac{J_1^1(Ka \sin \theta)}{1-(Ka)^2} \sin \phi - \frac{J_2^1(Ka \sin \theta)}{2^2-(Ka)^2} \sin 2\phi + j\frac{J_3^1(Ka \sin \theta)}{3^2-(Ka)^2} \sin 3\phi + \ldots \right\} \quad (10)$$

Where the loop has a circumference which is an integral multiple (m) of a wavelength and the received wave is vertically polarized, the voltage becomes the following:

$$|V| = |V_0| [J_{m-1}(m \sin \theta) - J_{m+1}(m \sin \theta)] \cos \theta \cos m\phi \quad (11)$$

Fig. 15, curve *c*, shows the horizontal field strength pattern, that is, the pattern in the plane of the antenna for a loop having a half wave circumference. It will be noted that, when the loop is used for transmitting, the radiation is greater in the direction of a diameter through the feed point. Fig. 15, curve *d*, shows the vertical pattern in the plane ϕ=zero, that is, with the vertical plane passing through the feed point. There is no vertically polarized field (E_θ) in this plane. The vertical patterns for both horizontally polarized (E_ϕ) and vertically polarized (E_θ) radiation in the vertical plane $\phi=90°$ are shown in Fig. 16. In this plane, that is, the plane at right angles to that through the feed point, the horizontally polarized radiation is a maximum in the horizontal direction and zero vertically while the vertical polarized radiation E_θ is zero horizontally and a maximum vertically. In the regions between horizontal and vertical, the radiation is elliptically polarized since E_θ and E_ϕ are in phase quadrature.

Where the circumference of the loop is increased to one wavelength, the horizontal pattern is a simple figure of eight or cosine curve with maxima along the diameter passing through the feed point. The vertical pattern in the plane $\phi=0$ (vertical plane passing through the feed point) is the same as that shown in Fig. 15*c*. The radiation is a maximum in the vertical direction while the E_θ (vertically polarized radiation) is zero everywhere in this plane. The vertical radiation pattern in the plane $\phi=90°$, that is, the vertical perpendicular to the plane passing through the feed point, is a simple figure of eight or cosine curve. All radiation is vertically polarized (E_θ radiation), the horizontally polarized radiation (E_ϕ) being zero everywhere. It will be noted that this situation is quite different from the half wave circumference loop which radiates considerable horizontally polarized energy in this plane.

Where a loop has a circumference equal to a multiple of a wavelength, the horizontal patterns are simple cos $m\phi$ curves where (m) is the integral multiple of the wavelength in the circumference. Thus, the figure of 8 pattern for a loop of one wave circumference becomes in the case of two wave circumference a four-leaf clover pattern.

The variation in radiation resistance with loop circumference is shown in detail in Fig. 17 for loops up to two wavelengths in circumference with the value for three wavelengths also indicated. Curve *a* of Fig. 17 indicates the total variation in radiation resistance with a variation of the loop circumference while curves *b* and *c* indicate respectively the horizontally and vertically polarized radiation resistance components of curve *a*. The values of the radiation resistance between two wavelengths and three wavelengths are not shown in detail but the general shape of the curve is well indicated in the area between one and two wavelengths. It should be noted that for circumferences greater than a half wavelength, the radiation resistance is of the order of that of a half-wave dipole, reaching a maximum of over 160 ohms in the vicinity of one and one quarter wavelengths circumference. The radiation efficiency, it will therefore be noted, is very high in contrast to the extremely low efficiency of small loops.

The mathematical theory of loop antennas which are large compared to the operating wavelength will now be given with reference to Fig. 11 for the geometry involved.

MATHEMATICAL THEORY

1. *Radiation fields-transmitting loop*

Let a=loop radius, ϕ the angle to the current element with X axis as reference and

$$A_y=\frac{I_0}{c}\frac{\epsilon^{-jKr_0}}{r}\int_{-\pi}^{+\pi}\cos Ka\phi_0\cos\phi_0 \exp [j\{Ka\sin\theta\cos(\phi-\phi_0)\}]d\phi_0 \quad (21)$$

After changing the variable of integration, considerable trigonometric and algebraic manipulation and making use of the well-known expansion:

$$\exp[jKa\cos u]=\sum_{n=-\infty}^{\infty}(j)J_n(Ka\sin\theta)\exp(jnu) \quad (22)$$

we obtain:

$$E_\phi=-j\frac{I_0}{cr}\epsilon^{-jKr_0}X(Ka)^2\sin(Ka\pi)\sum_{n=-\infty}^{\infty}(-j)^nJ_n(Ka\sin\theta)\left\{\frac{\epsilon^{-j(n+1)\phi}}{(n-1)^2-(Ka)^2}+\frac{\epsilon^{-j(n-1)\phi}}{(n-1)^2-(Ka)^2}\right\} \quad (23)$$

$$E_\theta=-\frac{I_0}{cr_0}\epsilon^{-jKr_0}X(Ka)^2\sin(Ka\pi)\sum_{n=-\infty}^{\infty}(-j)^nJ_n(Ka\sin\theta)\left\{\frac{\epsilon^{-j(n+1)\phi}}{(n-1)^2-(Ka)^2}-\frac{\epsilon^{-j(n-1)\phi}}{(n-1)^2-(Ka)^2}\right\} \quad (24)$$

$$K=\frac{2\pi}{\lambda}=\frac{\omega}{c}=\text{wave number}$$

Assume time function=$Re\{\exp(j\omega t)\}$ in accordance with standard practice.

Let $\overline{A}$ be the magnetic vector potential. Gaussian units are used.

Then $\overline{H}$=curl $\overline{A}$ and $\overline{E}=\frac{j}{k}$(grad. div. $\overline{A}+K^2\overline{A}$) (12)

where $J_n(X)$=Bessel Function of nth order with argument (X).

After changing to practical units, combining terms in plus and minus n and making use of the relation

$$J_n^1(X)=\frac{1}{2}[J_{n-1}(X)-J_{n+1}(X)]$$

where $J_n'(X)$ is the derivative, we obtain:

$$E_\phi=-\frac{120I_0}{r_0}\epsilon^{-jKr_0}(Ka)^2\sin(Ka\pi)\left[\frac{1}{2}\frac{J_0(Ka)\sin\theta)}{0-(Ka)^2}+\sum_{n=1}^{\infty}(-j)^n\frac{J_n^1(Ka\sin\theta)}{n^2-(Ka)^2}\cos n\phi\right] \quad (25)$$

$$E_\theta=-\frac{120I_0}{r_0}\epsilon^{-jKr_0}(Ka)^2\sin(Ka\pi)\cos\theta\sum_{n=1}^{\infty}(-j)^n\frac{J_n^1(Ka\sin\theta)}{n^2-(Ka)^2}\sin n\phi \text{ volts/meter} \quad (26)$$

At great distances $E_\phi=-jKA_\phi$ (13)

and

$$E_\theta=-jKA_\theta \quad (14)$$

neglecting terms of order higher than

$$\frac{1}{r_0}$$

where r_0 is the distance from the origin (center of loop).

Now

$$A_\phi=-A_x\sin\phi+A_y\cos\phi \quad (15)$$

and

$$A_\theta=(A_x\cos\phi+A_y\sin\phi)\cos\theta \quad (16)$$

The current distribution is given by

$$i=I_0\cos(Ka\phi_0) \quad (17)$$

The angle ψ between the radius vector to the

SMALL LOOP

When the loop is small so that $Ka \ll 1$ the limits become:

$$E_\phi=30\pi(Ka)^2\times\frac{I_0}{r_0}\epsilon^{-jKr_0}\sin\theta=120\pi^2\times\frac{I_0}{r_0}\frac{\text{Area}}{\lambda^2}\sin\theta\,\epsilon^{-jKr_0}\text{ volts/meter} \quad (27)$$

$$E_\theta=j120\pi(Ka)^3\frac{I_0}{r_0}\epsilon^{-jKr_0}\cos\theta\sin\phi \quad (28)$$

LOOP CIRCUMFERENCE MULTIPLE OF WAVELENGTH

When the circumference is an integral multiple m of a wavelength the above infinite series expressions degenerate to a single term and we have:

$$E_\phi=\pi 60\frac{I_0}{r_0}\epsilon^{-jKr_0}(m)(j)^mJ_m^1(m\sin\theta)\cos m\phi \quad (29)$$

$$E_\theta=60\pi\frac{I_0}{r_0}\epsilon^{-jKr_0}(m)(j)^mJ_m^1(m\sin\theta)\cos\theta\sin m\phi\text{ volts/meter} \quad (30)$$

current element and the radius vector to the distant point P is given by:

$$\cos\psi=\sin\theta\cos(\phi-\phi_0) \quad (18)$$

The phase angle=$Ka\cos\psi=Ka\sin\theta\cos(\phi-\phi_0)$ (19)

We then obtain:

$$A_x=-\frac{I_0}{c}\frac{\epsilon^{-jKr_0}}{r_0}\int_{-\pi}^{+\pi}\cos Ka\phi_0\sin\phi_0\exp [j\{Ka\sin\theta\cos(\phi-\phi_0)\}]d\phi_0 \quad (20)$$

2. *Radiation resistance*

In general power is radiated in both polarizations. The power in each will be separately calculated.

The Poynting Vector, or watts per square meter radiated, is given by

$$\overline{P}=\overline{E}\times\overline{H}=\overline{r_1}\frac{|E|^2}{120\pi} \quad (31)$$

at a great distance. Separating into its two parts P_ϕ and P_θ we have:

$$P_\phi=\frac{|E_\phi|^2}{120\pi}\quad\text{and}\quad P_\theta=\frac{|E_\theta|^2}{120\pi} \quad (32)$$

The radiation resistance is the total power per ampere squared radiated through a sphere. We shall call the two parts of this resistance R_ϕ and R_θ. Then:

$$R_\phi=\frac{r_0^2}{I_0^2}\int_0^\pi\int_0^{2\pi}P_\phi \sin\theta d\phi \quad (33)$$

$$R_\theta=\frac{r_0^2}{I_0^2}\int_0^\pi\int_0^{2\pi}P_\theta \sin d\theta d\phi \quad (34)$$

Hence we obtain:

$$R_\phi=\int_0^\pi\int_0^{2\pi}\frac{120}{\pi}(Ka)^4\sin^2 Ka\pi\left[\frac{1}{2}\frac{J_0^1(Ka\sin\theta)}{0-(Ka)^2}+\sum_{n=1}^{\infty}(-j)^n\frac{J_n^1(Ka\sin\theta)}{n^2-(Ka)^2}\cos n\phi\right]\times[\text{conjugate}]\times\sin\theta d\theta d\phi \quad (35)$$

and a similar type of expression for R_θ. This may be written in the form:

$$R_\phi=\int\int C\times\left[\frac{F_0(\theta)}{2}+\sum_{n=1}^{\infty}F_n(\theta)\cos n\phi\right]\times\left[\frac{F_0\theta}{2}+\sum_{n=1}^{\infty}F_n^*(\theta)\cos n\phi\right]\sin\theta d\theta d\phi \quad (36)$$

where $F_n(\theta)$ is a function of θ only, $F_n^*(\theta)$ its complex conjugate and C a constant.

Due to the orthogonality of the Fourier series we can immediately integrate with respect to ϕ and obtain:

$$R_\phi=\pi C\int_0^\pi\left[\frac{F_0^2(\theta)}{2}+\sum_{n=1}^{\infty}F_n(\theta)\cdot F_n^*(\theta)\right]\sin\theta d\theta \quad (37)$$

$$R_\phi=120(Ka)^4\sin^2(Ka\pi)\left\{\int_0^\pi\tfrac{1}{2}\left[\frac{J_0^1(Ka\sin\theta)}{(Ka)^2}\right]^2\sin\theta d\theta+\sum_{n=1}^{\infty}\int_0^\pi\left[\frac{J_n^1(Ka\sin\theta)}{n^2-(Ka)^2}\right]^2\sin\theta d\theta\right\}\text{ ohms} \quad (38)$$

It simplifies evaluation to change the variable of integration to $u=\cos\theta$. The R_θ part of the resistance is obtained by a similar procedure and we have the following results for the two parts of the radiated power per ampere squared:

$$R_\phi=120(Ka)^4\sin^2(Ka\pi)\left\{\tfrac{1}{2}\int_{-1}^{+1}\left[\frac{J_0^1(Ka\sqrt{1-u^2})}{(Ka)^2}\right]^2du+\sum_{n=1}^{\infty}\int_{-1}^{+1}\left[\frac{J_n^1(Ka\sqrt{1-u^2})}{n^2-(Ka)^2}\right]^2du\right\} \quad (39)$$

$$R_\theta=120(Ka)^4\sin^2(Ka\pi)\left\{\sum_{n=1}^{\infty}\int_{-1}^{+1}\left[\frac{J_n^1(Ka\sqrt{1-u^2})}{n^2-(Ka)^2}\right]^2\cdot u^2du\right.\text{ ohms} \quad (40)$$

SMALL LOOPS

When $Ka\ll1$ the limit of the above formulas become:

$$R_\phi=20\pi^2(Ka)^4=31170\left(\frac{\text{area}}{\lambda^2}\right)\text{ ohms} \quad (41)$$

$$R_\theta=40\pi^2(Ka)^6\text{ ohms} \quad (42)$$

LOOP CIRCUMFERENCE MULTIPLE OF A WAVELENGTH

When the circumference is a multiple m of a wavelength ($Ka=m$)

$$R_\phi=30\pi^2m^2\int_{-1}^{+1}[J_m^1(m\sqrt{1-u^2})]^2du \quad (43)$$

$$R_\theta=30\pi^2m^2\int_{-1}^{+1}[J_m^1(m\sqrt{1-u^2})]^2u^2du \quad (44)$$

3. *Voltage induced in a receiving loop*

The current in the loop (assuming sine wave distribution) is given by

$$i=I_0\cos(Ka\,\phi_0) \quad (45)$$

where ϕ_0 is the position angle, a the loop radius and

$$K=\frac{2\pi}{\lambda}=\frac{\omega}{c}$$

Let E_ϕ and E_θ be the horizontally and vertically polarized field strengths of the wave coming in at the colatitude angle θ and longitude angle ϕ. Using the center of the loop as phase reference the phase angle of the incoming wave is:

$$Ka\cos(\phi-\phi_0)\sin\theta \quad (46)$$

The component of E_ϕ in the direction of the element $ad\phi$ is

$$E_\phi\cos(\phi-\phi_0) \quad (47)$$

The component of E_θ in the direction of the element $ad\,\theta$ is

$$E_\theta\cos\theta\sin(\phi-\phi_0) \quad (48)$$

The voltages dV_ϕ and dV_θ induced in the elements by horizontally and vertically polarized waves are therefore:

$$dV_\phi=E_\phi\cos(\phi-\phi_0)\times ad\phi_0\,\epsilon^{iKa\cos(\phi-\phi_0)\sin\theta} \quad (49)$$

$$dV_\theta=E_\theta\cos\theta\sin(\phi-\phi_0)\times ad\phi_0\,\epsilon^{iKa\cos(\phi-\phi_0)\sin\theta} \quad (50)$$

The total voltages induced are then given by:

$$V_\phi=aE_\phi\int_{-\pi}^{+\pi}\cos(\phi-\phi_0)\,\epsilon^{iKa\cos(\phi-\phi_0)\sin\theta}d\phi_0 \quad (51)$$

$$V_\theta=aE_\theta\cos\theta\int_{-\pi}^{+\pi}\sin(\phi-\phi_0)\,\epsilon^{iKa\cos(\phi-\phi_0)\sin\theta}d\phi_0 \quad (52)$$

Proceeding in the same manner as for the transmitting case we obtain:

$$V_\phi=4E_\phi Ka^2\sin(Ka\pi)\left[\frac{1}{2}\frac{J_0^1(Ka\sin\theta)}{0-(Ka)^2}+\sum_{n=1}^{\infty}(-j)^n\frac{J_n^1(Ka\sin\theta)}{n^2-(Ka)^2}\cos n\phi\right]\text{ volts} \quad (53)$$

$$V_\theta=4E_\theta Ka^2\sin(Ka\pi)\cos\theta\sum_{n=1}^{\infty}(-j)\frac{J_n^1(Ka\sin\theta)}{n^2-(Ka)^2}\sin n\phi\quad\text{volts} \quad (54)$$

SMALL LOOP

The voltage V_s induced in a small loop is

$$V_s=jK\pi a^2E_\phi\sin\theta \quad (55)$$

as found by taking the limit of the above expression for V_ϕ as a approaches zero.

It will be noted that these expressions are identical with the transmitting field strength formulae except for a constant multiplier.

LOOP CIRCUMFERENCE A MULTIPLE m OF A WAVELENGTH

In a manner similar to the transmitting condition these formulas degenerate into:

$$V_\phi = E_\phi \times m\lambda(j)^m J_m^1(m \sin\theta)\cos m\phi \quad (56)$$

$$V_\theta = E_\theta \times m\lambda(j)^m J_m^1(m \sin\theta)\cos\phi \sin m\phi \quad (57)$$

4. *Uniform current loops*

If we assume the current to be constant all around the loop we obtain:

$$E_\phi = -jK\frac{I_0}{c}\frac{\epsilon^{-jKr_0}}{r_0}\int_{-\pi}^{+\pi}\cos\phi_0 \exp\left[j\{Ka\sin\theta\cos(\phi-\phi_0)\}\right] a\,d\phi_0 \quad (58)$$

$$= -jK\frac{I_0}{c}\frac{\epsilon^{-jKr_0}}{r_0}\times\frac{2\pi a}{14\pi}J_1(Ka\sin\theta) \quad \text{E. S. units} \quad (59)$$

$$= \frac{60\pi I_0}{R_0}(Ka)J_1(Ka\sin\theta) \quad \text{volts/meter} \quad R_0 \text{ in meters} \quad (60)$$

The radiation resistance becomes:

$$R = 60\pi^2(Ka)^2\int_{-1}^{+1}J_1^1(Ka\sqrt{1-u^2})du \quad (61)$$

5. *Half circle half wavelength loop*

By a process similar to that already shown for a complete loop we obtain for a half circle loop

$$E_\phi = -j\frac{60I_0}{R_0}\left\{-J_0^1(\sin\theta)+\pi J_1^1(\sin\theta)\cos\phi + j\sum_{n=2}^{\infty}j^m\left(\frac{1}{m-1}-\frac{1}{m+1}\right)J_m^1(\sin\theta)\cos m\phi\right\} \quad (62)$$

$$E_\theta = j\frac{60I_0}{R_0}\cos\theta\left\{\pi J_1^1(\sin\theta)\sin\phi + j\sum_{m=2}^{\infty}j^m\left(\frac{1}{m-1}-\frac{1}{m+1}\right)J_m^1(\sin\theta\sin m\phi\right\} \quad (63)$$

While I have illustrated a particular embodiment of the present invention, it should be clearly understood that it is not limited thereto since many modifications may be made in the several elements employed and in their arrangement and it is therefore contemplated by the appended claims to cover any such modifications as fall within the spirit and scope of the invention.

What is claimed is:

1. A loop antenna arrangement including a substantially plane conductive surface element, a conductor curved to lie on the circumference of a circle in a plane parallel to said conductive surface element, said circumference being twice the operating wavelength, said conductor being hollow for at least one operating wavelength, there being a gap in said conductor at one end of the hollow portion, a pair of supporting members fixed at one end to said conductive surface element and at the other end to said curved conductor on either side of the gap therein, a further supporting member in the form of an outer conductor of a coaxial transmission line passing through and affixed to said conductive surface element and connected to said curved conductor at a point on the circumference thereof opposite said gap, said supporting members having a length equal to an odd multiple including unity of a quarter of the operating wavelength, said coaxial transmission line having an inner conductor extending through the hollow portion of said curved conductor and across said gap and being connected to the end of said curved conductor opposite to the end of said hollow portion thereof, thereby to provide a substantially uniform directivity pattern in a plane substantially parallel to that of said conductive surface element.

2. A loop antenna arrangement including a substantially plane conductive surface element, a conductor curved to lie on the circumference of a circle in a plane parallel to said conductive surface element, said circumference being at least as great as one wavelength, said conductor being hollow for at least one-half of the length thereof, there being a gap in said conductor at one point along the hollow portion, at least one supporting member fixed to said conductive surface element and to said conductor on one side of the gap therein, a further supporting member in the form of sheath conductor of a coaxial transmission line passing through and affixed to said conductive surface element and connected to said conductor at a point on the circumference thereof removed from the gap, said supporting members having a length equal to an odd multiple including unity of a quarter of the operating wavelength, said coaxial transmission line having an inner conductor extending substantially throughout the hollow portion of said curved conductor and across said gap.

3. A loop antenna arrangement including a conductive surface element, an elongated conductor formed to lie on the perimeter of a closed geometrical figure in a plane substantially parallel to said conductive surface element, said circumference being at least as great as the operating wavelength, said conductor being hollow for at least a portion of the length thereof, there being a gap in said conductor at one point along the hollow portion, a pair of supporting members each fixed at one end to said conductive surface element and the other end to said elongated conductor, at least one of said supporting members being hollow, said supporting members having a length equal to an odd multiple including unity of a quarter of the operating wavelength, a coaxial transmission line having a sheath conductor affixed to said conductive surface element and an inner conductor extending through one of said supporting members, the hollow portion of said arcuate conductor and across said gap.

PHILIP S. CARTER.

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