Generalization of GA-FFT for synthesizing unequally spaced linear array shaped pattern including coupling effects

Pengfei You[1], Yanhui Liu[1,*], Kaida Xu[1], Chunhui Zhu[1], Qing Huo Liu[2]

[1]Department of Electronic Science, Xiamen University, Fujian 361005, China
[2]Department of Electrical and Computer Engineering, Duke University, Durham, NC 27708, USA
[*]Corresponding Email: yanhuiliu@xmu.edu.cn

Abstract: Antenna arrays with shaped patterns have drawn significant attention for their wide applications in radar, sonar and communication systems. The combination of genetic algorithm (GA) and fast Fourier transform (FFT) has been used to synthesize shaped pattern of antenna array's factor without considering coupling effects. In this work, the GA-FFT method is generalized by integrating with a new virtual active element pattern expansion (VAEPE) method which approximates each active element pattern as the radiation pattern by exciting several equally spaced virtual elements surrounding the real element position. The generalized GA-FFT can be applicable for the shaped pattern synthesis of unequally spaced linear arrays including mutual coupling and platform effect. Several synthesis examples with different pattern shapes and different antenna structures are given to demonstrate the effectiveness, accuracy and robustness of the proposed method.

1. Introduction

Antenna arrays with shaped patterns have many applications in radar, sonar and communication systems. The synthesis of shaped power patterns is a highly nonlinear problem, and most of shaped pattern synthesis methods can be considered as iterative optimization techniques, such as the alternating projection method [1, 2], genetic algorithm (GA) [3–5], particle swarm optimization (P-SO) [6, 7], differential evolution algorithm (DEA) [8, 9] and modified tabu search algorithm (MT-SA) [10]. Despite the success of these techniques, they are usually time consuming since a large number of repeated calculations of the pattern of the array are required. To overcome this problem, some hybrid methods have been presented in which the fast Fourier transform (FFT) is adopted to speed up the calculation of the pattern of the array at each iteration [11–13]. In particular, the combination of FFT and GA presented in [11] can significantly improve the efficiency of the GA-based synthesis method while maintains the flexibility of the GA optimization for dealing with the synthesis constraints on the pattern shape, the dynamic range ratio (DRR) of excitation amplitude and so on. However, due to the limitation of the FFT, the GA-FFT method can be only applied to the synthesis of equally spaced array without considering element mutual coupling and platform effect. Recently, the nonuniform FFT has been applied to accelerate the pattern calculation for an unequally spaced array [14–16]. However, the nonuniform FFT deals only with the array factor, and it cannot be applicable when the pattern of the array including coupling effects is considered.

As is well known, the mutual coupling effect may be significant in practical antenna arrays. There are plenty of papers talking about mutual coupling effects from different points of view [17]. For example, many mutual coupling correction techniques have been presented in different applications such as direction-of-arrival (DOA) estimation [18–22], adaptive beamforming [23–25], antenna array manifold repairment [21,22,26] and static array pattern synthesis [7,12,27–31]. Due to the significance of the mutual coupling, incorporating the mutual coupling into the pattern synthesis should be very useful. In this situation, the element pattern in the array should be replaced with the active element pattern (AEP) which is defined as the pattern of the array when only one element is excited and all the others are connected to matching loads [32, 33]. The AEP usually differs from each other. Consequently, the FFT and nonuniform FFT synthesis techniques cannot be applicable. Recently, we presented an active element pattern expansion (AEPE) method [34]. This method approximates the active element pattern as a subarray's pattern by exciting the neighboring elements which have significant coupling effects. In this way, the pattern of an equally spaced linear array including coupling effects can be efficiently calculated by using the FFT. With the help of the AEPE, the GA-FFT is extended in [34] to synthesize a focused beam pattern of the mutually coupled equally spaced array. However, since the AEPE method utilizes the neighboring elements at real positions to expand the AEP, it is compatible with the FFT only in the case of equally spaced arrays.

Here we introduce a virtual active element pattern expansion (VAEPE) method in which each AEP is considered as the pattern by exciting several equally spaced virtual elements surrounding the real element position. By combining the VAEPE method, the GA-FFT can be further generalized to synthesize unequally spaced arrays including coupling effects. This generalized GA-FFT method can control the pattern shape and sidelobe level accurately by considering the mutual coupling as well as platform effect. Several shaped pattern synthesis examples are given to demonstrate the effectiveness and accuracy of the proposed method. The first example is to synthesize a flat-top power pattern for a 30-element dipole array, and then synthesizing a flat-top pattern with part of mainlobe enhanced for special application is considered for the same array. Another example is to synthesize a cosecant-squared pattern for a 13-element microstrip patch array. In addition, to evaluate the performance of the proposed method when considering the platform effect, the cosecant-squared pattern synthesis example is double checked when the same array is mounted in a complicated platform with several metal scatters.

The rest of this paper is organized as follows. Firstly we describe the idea of the VAEPE method and the generalized GA-FFT method in Section 2. And then the synthesis examples with different pattern shape requirements are presented in Section 3. Some conclusions are given in Section 4.

2. Formulation

2.1. The Virtual Active Element Pattern Expansion (VAEPE) Method

Let us consider an unequally spaced linear array of N elements, aligned along X-axis. The far-filed array's pattern can be written as

$$f(u) = \sum_{n=1}^{N} w_n g_n(u) e^{j\beta x_n u} \tag{1}$$

where $j = \sqrt{-1}$, $u = \cos\theta$, and θ is the angle between the direction of observation and the linear array. $\beta = 2\pi/\lambda$ is the wavenumber in free space, λ is the wavelength, x_n is the position of the

nth element, and $g_n(u)$ is the phase-adjusted active element pattern (AEP) of the nth antenna at an operating frequency (the coordinate system's origin locates at each element). The AEP is the array pattern by exciting only one element and connecting all the others to matching loads, and all the AEPs can be obtained by measurement or full wave simulation. Obviously, the pattern calculated by (1) has included both the mutual coupling between antenna elements and platform effect. However, due to the unequal spacing and nonuniform platform effect, the AEPs are usually different from each other. Consequently, the FFT technology can not be used directly, cannot the NUFFT method either. For the unequally spaced array, the LS-AEPE method we presented in [34] is still not suitable when followed by FFT directly.

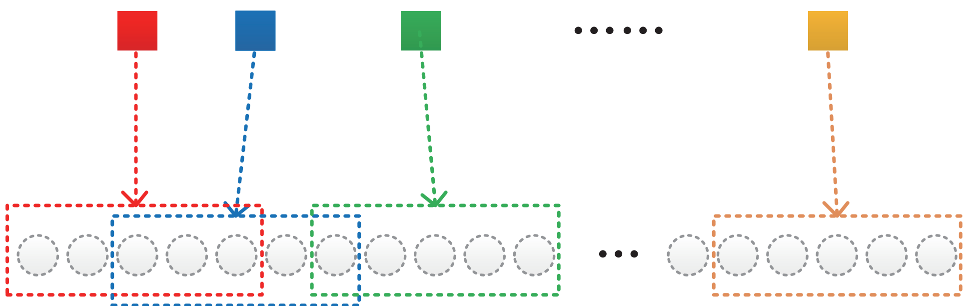

Fig. 1. *Schematic of nonuniform array active element pattern expansion.*

To overcome the problem mentioned above, we introduce a virtual AEP expansion (VAEPE) method in which each AEP of a real antenna element is assumed to be the radiation pattern by exciting several equally spaced virtual elements surrounding to the real element position, as shown in Fig. 1. It should be emphasized that the virtual equally spaced array is just an equivalent model instead of a real existence. Mathematically, the nth AEP $g_n(u)$ can be approximated as

$$\begin{aligned} g_n(u) &\approx g_s(u) \sum_{q=-Q/2}^{Q/2} c_{nq} e^{j\beta(v_n - x_n + qd)u} \\ &= g_s(u) e^{j\beta(v_n - x_n)u} \sum_{q=-Q/2}^{Q/2} c_{nq} e^{j\beta qdu} \end{aligned} \tag{2}$$

where $(Q+1)$ is an odd positive integer which represents the expansion size for each AEP, $d = \frac{x_N - x_1}{rN-1}$ is the element spacing of the virtual array, the over sampling factor r is a real number which is always greater than 1, $v_n = [\frac{x_n - x_1}{d}]d + x_1$ is the position of the nearest virtual element of the nth real element, $[x]$ denotes the nearest integer to x, and c_{nq} is the excitation of the qth nearest virtual element of the nth real element. $g_s(u)$ is called the virtual element pattern, and it is the same for different virtual elements. We choose $g_s(u)$ as the average of all the phase-adjusted AEPs. That is,

$$g_s(u) = \frac{1}{N} \sum_{n=1}^{N} g_n(u) \tag{3}$$

In addition, c_{nq} can be obtained by solving the following minimization problem:

$$\min_{\mathbf{c}_n} \|\mathbf{g}_n - \mathbf{Z}_n \mathbf{c}_n\|_2^2 \tag{4}$$

where

$$\mathbf{c}_n = [c_{n,-Q/2}, c_{n,-Q/2+1}, \cdots, c_{n,Q/2}]^T \tag{5}$$

$$\mathbf{g}_n = [g_n(u_1), g_n(u_2), \cdots, g_n(u_M)]^T \tag{6}$$

and

$$\mathbf{Z}_n = \begin{bmatrix} p_1 e^{j\beta d u_1(-Q/2)} & \dots & p_1 e^{j\beta d u_1(Q/2)} \\ \vdots & & \vdots \\ p_M e^{j\beta d u_M(-Q/2)} & \dots & p_M e^{j\beta d u_M(Q/2)} \end{bmatrix} \tag{7}$$

$$p_m = g_s(u_m) e^{j\beta(v_n - x_n)u_m} \tag{8}$$

The least-square solution to the above problem is given by $\mathbf{c}_n = (\mathbf{Z}_n^H \mathbf{Z}_n)^{-1} \mathbf{Z}_n^H \mathbf{g}_n$. Furthermore, the mathematical form in (5) is not a circulant form, because all the expansion coefficients for different active element patterns are calculated separately and there does not exist a Toepliz matrix structure due to complicated mutual coupling and platform effects in the unequally spaced antenna array.

2.2. *Generalized GA-FFT for Synthesizing Shaped Pattern of Unequally Spaced Coupled Linear Array*

By integrating (1) with (2), we can reformulate the array pattern as

$$f(u) = g_s(u) e^{j\beta u(x_1 - Qd/2)} \sum_{l=0}^{[rN]+Q-1} a_l e^{j\beta l d u} \tag{9}$$

where

$$a_l = \sum_{\substack{l=[\frac{x_n - x_1}{d}]+q+Q/2 \\ -Q/2 \le q \le Q/2}} c_{nq} w_n \tag{10}$$

By uniformly sampling (9) with $u_m = m\Delta_{\text{FFT}} = 2\pi m / M\beta d$ for $m = -M/2 : 1 : (M/2 - 1)$, we obtain that

$$f(u_m) = g_s(u_m) e^{j\beta u_m(x_1 - Qd/2)} \sum_{l=0}^{[rN]+Q-1} a_l e^{j2\pi l m/M} \tag{11}$$

Clearly, the above summation can be evaluated by applying the standard FFT. Hence, the array pattern including mutual coupling effect can be calculated by the FFT with the help of the VAEPE method. For obtaining a pattern with M sampling points, this method requires only $(Q+1)rN + K/2 \log_2 K + M$ complex multiplications provided that the radix-2 algorithm is used ($Q \ll N$, $K = 2^{\lceil log_2 M \rceil}$, and $\lceil x \rceil$ denotes the smallest integer greater than or equal to x). However, we mention that direct summation of (1) with the same M sampling points costs MN complex multiplications. As an illustration, in Fig.2 we plot the number of multiplications required by direct summation versus that by the VAEPE-FFT when the parameters are chosen as $M = 8N$, $r = 2$ and $Q = 6$. As can be seen that the VAEPE-FFT costs much less multiplications especially when the array with a large number of elements is considered. This is extremely useful for the GA-based synthesis technique since a large number of array patterns for all the individuals at each generation need to be calculated.

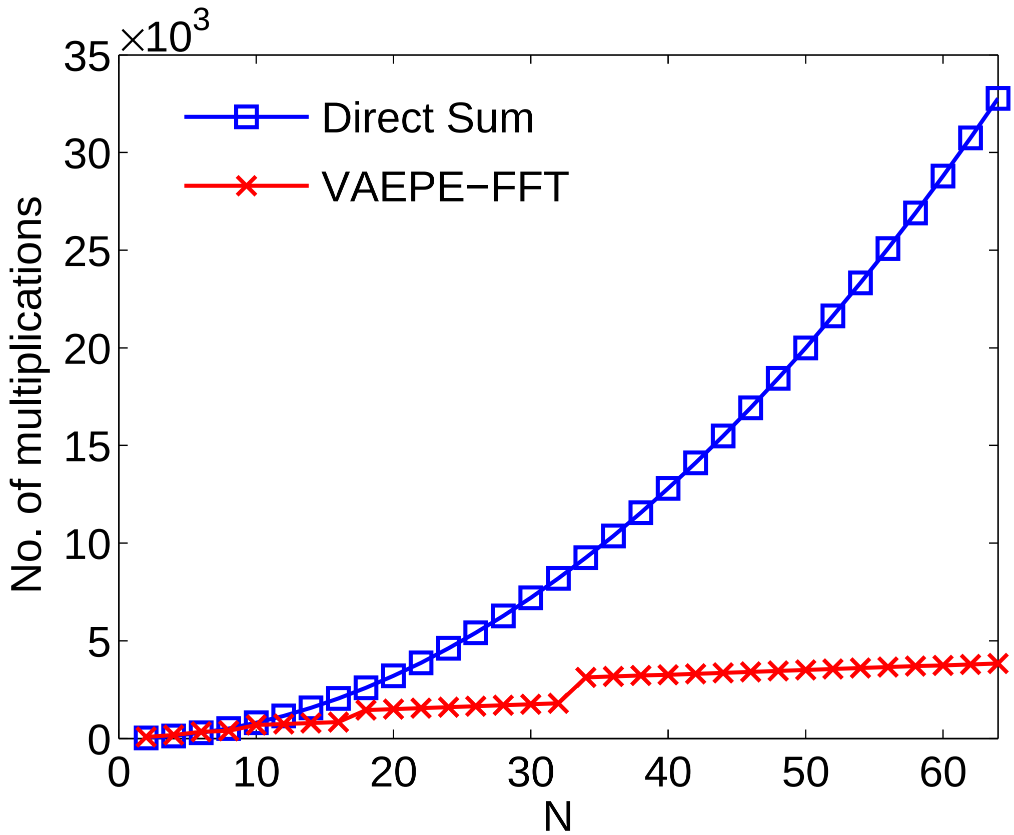

Fig. 2. *The number of complex multiplications required (*$M = 8N$*,* $r = 2$*,* $Q = 6$*).*

Then, the problem of synthesizing a linear array shaped power pattern by optimizing the excitations can be formulated as follows,

$$\begin{aligned} \underset{w_n}{\text{minimize}} \quad & \|P(u_m) - P_d(u_m)\|_2^2, u_m \in \textit{shaped region} \\ \text{subject to} \quad & L(u_m) \le P(u_m) \le U(u_m), u_m \in [-1, 1] \\ & \max\{w_n\}/\min\{w_n\} \le DRR \end{aligned} \tag{12}$$

where $P(u) = |f(u)|^2$ is the power pattern function to be synthesized, $P_d(u)$ is the desired power pattern function, $L(u)$ is the lower bound, and $U(u)$ is the upper bound. With appropriate choice of $P_d(u)$, $L(u)$ and $U(u)$, some complicated radiation requirements such as mainlobe shaping and sidelobe level (SLL) control can be easily described. And the DRR constraint can be also added to control the maximum-to-minimum excitation ratio.

As is well known, the power pattern synthesis is a highly nonlinear problem. Here we choose the real-code GA algorithm presented in [34] as a global optimization tool. The optimization variable is a $2N$-dimensional vector for a N-element array. In this vector, the first N elements are real-valued numbers ranging from 0 to 1 which represent the excitation amplitudes, and the last N elements are real-valued numbers ranging from $-\pi$ to π which represent the excitation phases. However, for the pattern synthesis problem, we only need to care about the relative values of the amplitudes and phases, which means that there are only $(2N - 2)$ degrees of freedom for the array with fixed element positions. The objective function is minimizing the error between desired pattern and synthesised pattern over the specified shaped region. Two constraints are used: one is to control both the power pattern shape and sidelobe (SLL) distribution by presetting the upper and lower bounds, and the other is to control the maximum-to-minimum excitation ratio if required. In the synthesis, the calculation of array's pattern for each individual in every generation is accomplished by the FFT via VAEPE. This method significantly generalizes the original GA-FFT to be suited for the synthesis of unequally spaced array including coupling effects. Therefore, we call it the generalized GA-FFT method.

3. Numerical Results

3.1. Flat-top Power Pattern Synthesis of an Unequally Spaced Dipole Array

As the first example, we consider to synthesize a flat-top power pattern for a 30-element array whose element positions are given in Table 1 of [10]. Assuming all the elements are isotropic, the excitation given by [10] can achieve a flat-top power pattern result satisfying the radiation requirements. However, if consider a practical antenna array, there would exist significant deviation between the synthesized pattern and the real one due to the presence of mutual coupling between elements. In this example, we assume that each element is a dipole antenna, working at the frequency of 1 GHz. Fig. 3 shows the geometry of this array. To compensate for the mutual coupling effects, we use the active element pattern (AEP) for each element dipole which can be obtained by full-wave simulation such as High-Frequency Structure Simulator (HFSS) software [35]. To apply the proposed generalized GA-FFT, at first we use the VAEPE method to expand each AEP as the pattern of some equally spaced virtual elements. The approximation error of the VAEPE method can be calculated as the following

$$\epsilon = \sqrt{\frac{\sum_{n=1}^{N} \|g_n(u) - g_s(u)e^{j\beta(v_n - x_n)u} \sum_{q=-Q/2}^{Q/2} c_{nq} e^{j\beta q d u}\|_2^2}{\sum_{n=1}^{N} \|g_n(u)\|_2^2}} \tag{13}$$

Fig. 4 shows the error ϵ of the VAEPE method versus the $(Q + 1)$ for different virtual element spacing $d = [0.4, 0.3, 0.2]\lambda$ (obtained at $r = [1.25, 1.66, 2.47]$, respectively). As is expected, the error ϵ decreases as Q gets larger for a fixed d. For a fixed Q, the error with $d = 0.3\lambda$ is much lower than that with $d = 0.4\lambda$. However, using a smaller d cannot reduce the error any more. This is because that the virtual array with a smaller d can capture the variation of real antenna array current distribution, but for a fixed Q, a smaller d means a reduced size of the virtual subarray used for approximating the AEP. That is, the smaller aperture counteracts the improvement in accuracy by smaller spacing. So, the parameter d cannot be too small. In this example and the following ones, we use $d = 0.3\lambda$, and $Q = 10$. In this way, the unequally spaced dipole array is dealt to be a 60-element virtual uniform array. Then, the generalized GA-FFT is applied to optimize the excitation amplitudes and phases for this array to achieve the radiation requirements. The synthesized pattern is shown in Fig. 5. As can be seen, this pattern is nearly the same as the method by combining GA and direct summation method. For further comparison, Fig. 5 also shows the synthesized pattern given in [10] and the one obtained by summing all the APEs of dipole antennas with the same weights used in [10]. As can be seen, due to lack of considering the mutual coupling, the real pattern deviates significantly from the synthesized one with isotropic elements. This proves the accuracy of the proposed method which can include the mutual coupling and eliminate such deviation.

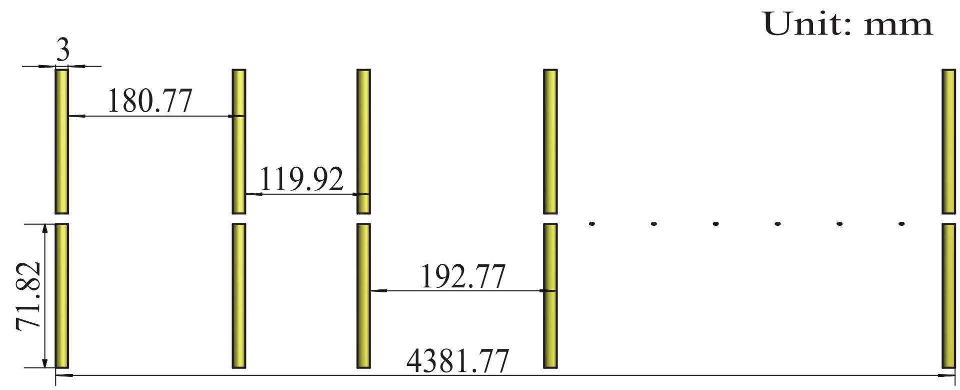

Fig. 3. *Geometry of a 30-element unequally spaced dipole antenna array.*

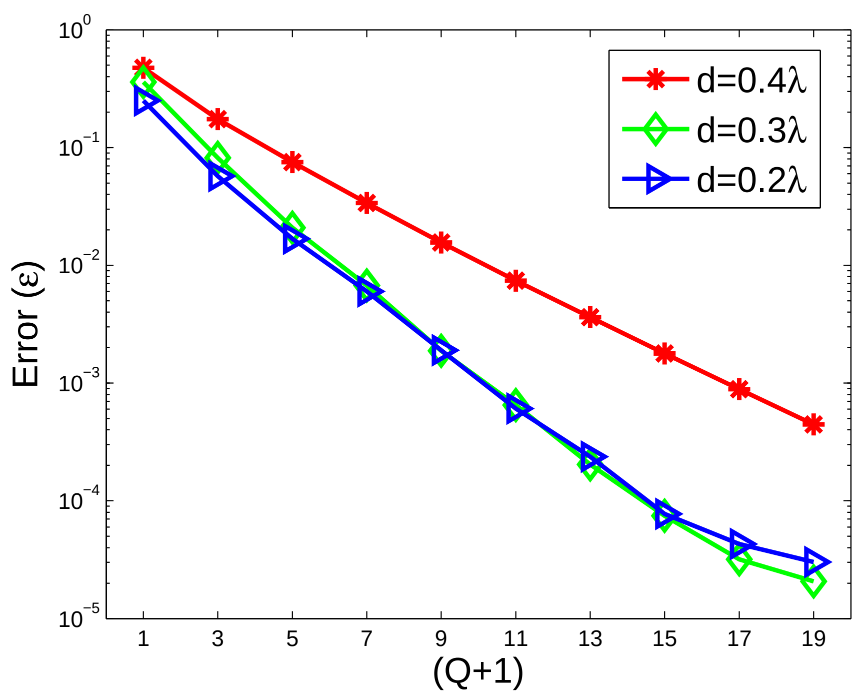

Fig. 4. *Interpolation error ϵ versus $(Q+1)$ for different virtual spacing d.*

Sometimes a more complicated shaped power pattern is desired. For example, the desired radiation is a flat-top power pattern but with a part of mainlobe enhanced over a specified angle region. This may happen in some special applications when longer signal transmission distance or larger radiated power in a communication system should be obtained in a certain angle region. Assume that the flat-top shaped power region is $u \in [-0.390, -0.053] \cup [0.053, 0.390]$, and the pattern level in this region is 8 dB lower than the maximum power level that happens at $u = 0$. The sidelobe level is required to be less than -25 dB for $u \in [-1, -0.439] \cup [0.439, 1]$. In this example, we set the upper and lower pattern bounds as shown in Fig. 6. The proposed generalized GA-FFT is still effective, and the synthesized pattern is also shown in Fig. 6. As can be seen, the obtained pattern is nearly the same as the one obtained by the combination of GA and direct summation method, and both meet the specified pattern bounds very well.

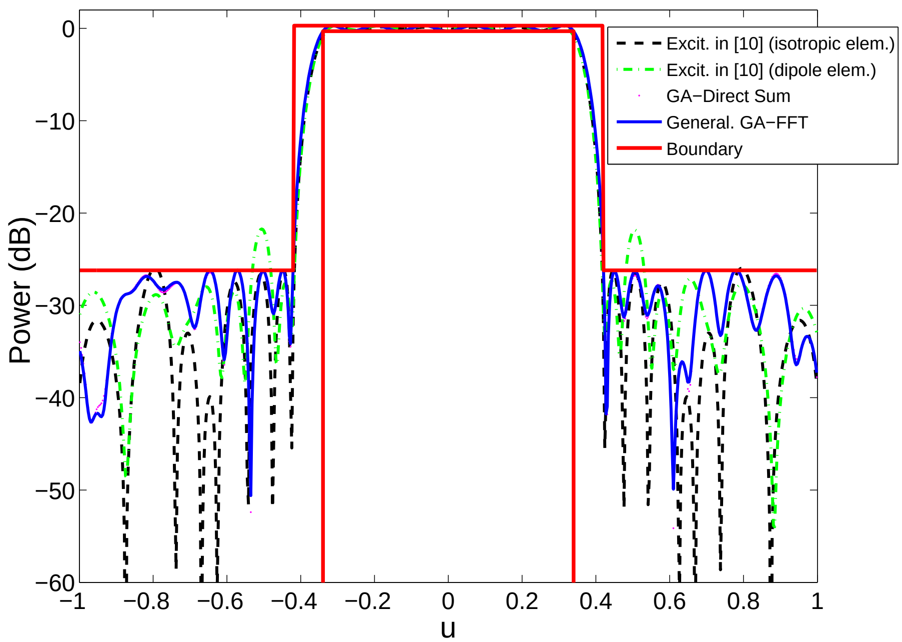

Fig. 5. *The flat-top patterns synthesized by the generalized GA-FFT and the GA-direct summation method, the pattern given in [10] with isotropic elements, and the pattern obtained by using the excitations in Table 1 of [10] with the active element patterns of the 30-element dipole array.*

3.2. Cosecant-squared Power Pattern Synthesis of an Unequally Spaced Patch Array

In the second example, we consider to synthesize a cosecant-squared power pattern for a 13-element unequally spaced array whose element positions are given in Table 2 of [36]. Assume that the element antenna is a microstrip patch antenna operating at the frequency of 10 GHz. The geometry of this antenna array is shown in Fig. 7. Due to the element pattern modulation and mutual coupling of microstrip antenna elements, the real array's pattern with the excitations of [36] obtained with ideally isotropic elements falls below the prescribed lower bound in the shaped region, as shown in Fig. 8. We now apply the generalized GA-FFT to synthesize this pattern. In this example, the dynamic range ratio (DRR) is constrained to be less than 6.9 . The obtained pattern is also shown in Fig. 8. As can be seen, this pattern including mutual coupling strictly meets the prescribed upper and lower bounds, and the mainlobe shape as well as sidelobe distribution are controlled very well. In addition, this result is again very close to the one obtained by combining GA and direct summation. Note that the obtained DRR is 6.9, which is just equal to its bound. This value is much less than the DRR of 11.1 obtained in [36].

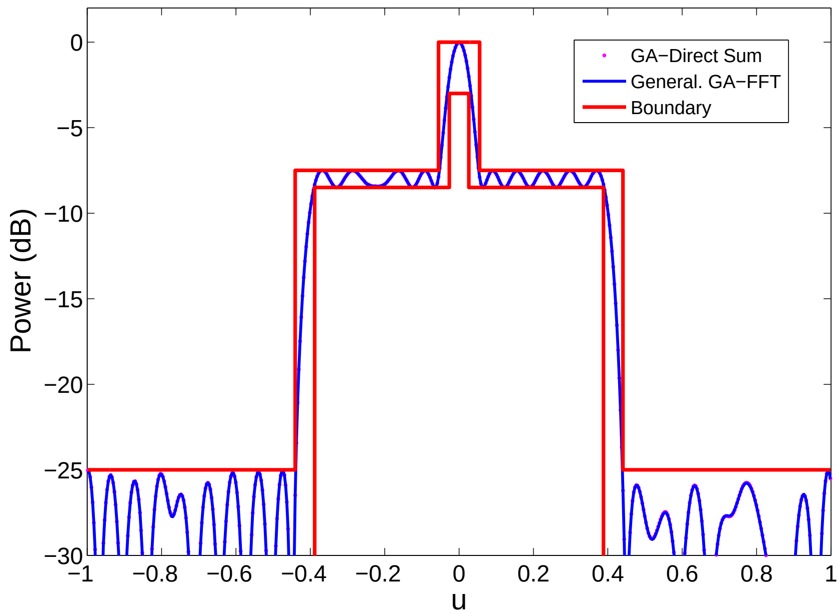

Fig. 6. *The synthesized flat-top power pattern with a part of mainlobe enhanced over a specified region for the 30-element dipole array.*

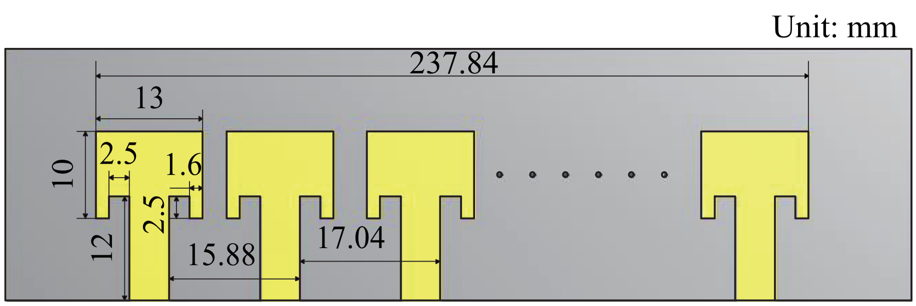

Fig. 7. *Geometry of a 13-element unequally spaced microstrip patch antenna array.*

3.3. Cosecant-squared Power Pattern Synthesis of the Nonuniform Patch Array in Complicated Environment

As the last example, we check the effectiveness of the proposed method for the array in an inhomogeneous platform environment. Assume that the same microstrip antenna array as in the second example is mounted on a metal platform with several metal scatters, as shown in Fig. 9. In this situation, the AEPs will be significantly affected by the platform and scatters, and can be much different depending their locations. We still use the proposed VAEPE method to deal with all these AEPs and apply the generalized GA-FFT to synthesize the same desired pattern as in the second example. With the synthesized excitations, the patterns using approximate AEPs and real AEPs as well as the pattern obtained by HFSS are shown in Fig. 10 for comparison. As can be seen, the synthesized pattern with the approximate AEPs completely meets the prescribed bounds in both

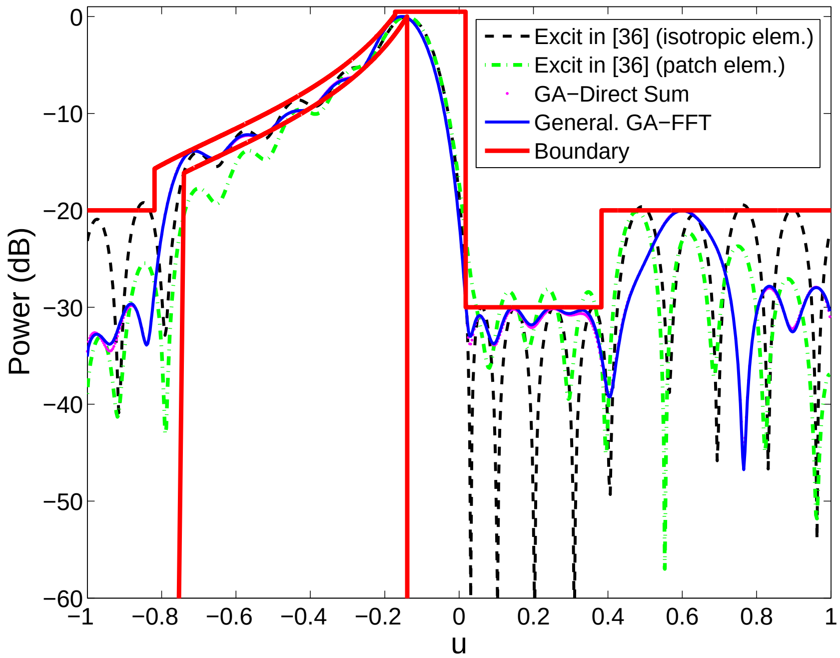

Fig. 8. *The cosecant-squared patterns synthesized by the generalized GA-FFT and the GA-direct summation method, the pattern given in [36] with isotropic elements, and the pattern obtained by using the excitations in Table 2 of [36] with the active element patterns of the 13-element microstrip antenna array.*

mainlobe and sidelobe regions. The pattern with real AEPs has a slight degradation in the low sidelobe region (i.e., -30 dB SLL region) due to the complex platform environment effect, but it is in general acceptable. In addition, this pattern matches very well with the HFSS simulation result. This shows the robustness of the proposed method for practical problems.

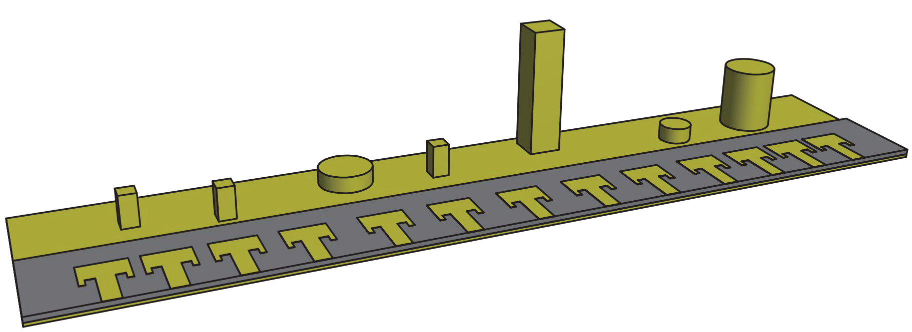

Fig. 9. *The 13-element unequally spaced microstrip patch antenna array mounted on a metal platform and surrounded with some scatters.*

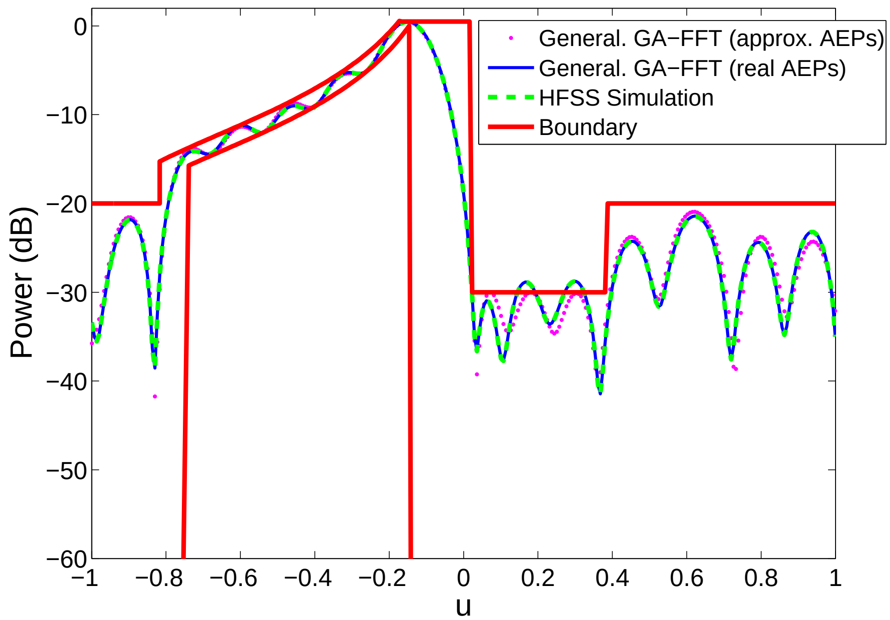

Fig. 10. *The cosecant-squared patterns calculated using the approximate AEPs and real AEPs and the pattern from HFSS simulation, all with the same excitations synthesized by the generalized GA-FFT for the 13-element microstrip antenna array mounted in a metal platform with some scatters.*

4. Conclusion

We have generalized the GA-FFT by integrating with a virtual active element pattern expansion (VAEPE) method. The generalized GA-FFT can be applied to synthesize unequally spaced linear antenna arrays including mutual coupling as well as platform effect, while maintains the similar computational complexity of the original GA-FFT. Numerical examples are given for synthesizing flat-top power pattern with/without a part of enhanced mainlobe for a 30-element dipole array, a cosecant-squared power pattern for a 13-element microstrip patch array, and the same cosecant-squared pattern for the same array but mounted in a complicated platform with several metal scatters. The synthesis results show that the generalized GA-FFT method by including the coupling effects can achieve satisfactory accuracy performance in controlling the pattern shape and sidelobe distribution, even for the antenna array surrounded with metal scatters.

Finally, it should be noted that the proposed VAEPE-FFT technique can be also combined with some other optimization algorithms such as differential evolution (DE) and particle swarm optimization (PSO) algorithms. Hence, this technique would be very useful in many problems related to antenna array pattern calculation.

5. Acknowledgments

This work was supported in part by the National Natural Science Foundation of China (NSFC) under Grant No. 61301009 and 61304110, in part by the Fundamental Research Funds for the Central Universities under Grant No. 20720160081.

6. References

[1] Quijano, J.L.A., Vecchi, G.: ‘Alternating adaptive projections in antenna synthesis’, IEEE Trans. Antennas Propag., 2010, **58**, (3), pp. 727-737

[2] Haddadi, A., Ghorbani, A., Rashed-Mohassel, J.: ‘Cosecant-squared pattern synthesis using a weighted alternating reverse projection method’, IET Microw., Antennas Propag., 2011, **5**, (15), pp. 1789-1795

[3] Haupt, R., Werner, D.: ‘Genetic algorithms in electromagnetics’ (Wiley-IEEE Press, 2007, 1st edn.)

[4] Weile, D.S., Michielssen, E.: ‘Genetic algorithm optimization applied to electromagnetics: a review’, IEEE Trans. Antennas Propag., 1997, **45**, (3), pp. 343-353

[5] Ares-Pena, F.J., Rodriguez-Gonzalez, J.A., Villanueva-Lopez, E., Rengarajan, S.R.: ‘Genetic algorithms in the design and optimization of antenna array patterns’, IEEE Trans. Antennas Propag., 1999, **47**, (3), pp. 506-510

[6] Manica, L., Anselmi, N., Rocca, P., Massa, A.: ‘Robust mask-constrained linear array synthesis through an interval-based particle swarm optimisation’, IET Microw., Antennas Propag., 2013, **7**, (12), pp. 976-984

[7] Pirhadi, A., Rahmani, M.H., Mallahzadeh, A.: ‘Shaped beam array synthesis using particle swarm optimisation method with mutual coupling compensation and wideband feeding network’, IET Microw., Antennas Propag., 2014, **8**, (8), pp. 549-555

[8] Rocca, P., Oliveri, G., Massa, A.: ‘Differential evolution as applied to electromagnetics’, IEEE Antennas Propag. Mag., 2011, **53**, (1), pp. 38-49

[9] Mandal, A., Zafar, H., Das, S., Vasilakos, A.: ‘A modified differential evolution algorithm for shaped beam linear array antenna design’, Prog. Electromagn. Res., 2012, **125**, pp. 439-457

[10] Akdagli, A., Guney, K.: ‘Shaped-beam pattern synthesis of equally and unequally spaced linear antenna arrays using a modified tabu search algorithm’, Microw. Opt. Technol. Lett., 2003, **36**, (1), pp. 16-20

[11] Wang, L.L., Fang, D. G., Sheng, W. X.: ‘Combination of genetic algorithm (GA) and fast Fourier transform (FFT) for synthesis of arrays’, Microw. Opt. Technol. Lett, 2003, **37**, (1), pp. 56-59

[12] Pathak, N.N., Basu, B., Mahanti, G.K.: ‘Combination of inverse fast Fourier transform and modified particle swarm optimization for synthesis of thinned mutually coupled linear array of parallel half-wave length dipole antennas’, Prog. Electromagn. Res., 2011, **16**, pp. 105-115

[13] Chatterjee, A., Mahanti, G.K.: ‘Combination of fast Fourier transform and self-adaptive differential evolution algorithm for synthesis of phase-only reconfigurable rectangular array antenna’, Ann Telecommun, 2014, **69**, (9-10), pp. 515-527

[14] Jiang, M.L., Chen R.S., Zhou, L., *et al.*: ‘Synthesis of arrays with genetic algorithm (GA) and nonuniform fast Fourier transform (NFFT)’, APMC - Asia-Pacific Microwave Conf. Proc., December 2005, vol. 4

[15] Liu, J.Z., Zhao, Z.Q., Yang, K., Liu, Q.H.: ‘A hybrid optimization for pattern synthesis of large antenna arrays’, Prog. Electromagn. Res., 2014, **145**, pp. 81-91

[16] Yang, K., Zhao, Z.Q., Liu, Q.H.: ‘Fast pencil beam pattern synthesis of large unequally spaced antenna arrays’, IEEE Trans. Antennas Propag., 2013, **61**, (2), pp. 627-634

[17] Singh, H., Sneha, H. L., Jha, R. M.: ‘Mutual coupling in phased arrays: a review’, Int. J. Antennas Propag., vol. 2013, no. Article ID348123, p. 23

[18] Sellone, F., Serra, A.: ‘A novel online mutual coupling compensation algorithm for uniform and linear arrays’, IEEE Trans. Signal Process, 2007, **55**, (12), pp. 560-573

[19] Parhizgar, N., Alighanbari, A., Masnadi-Shirazi, M. A., Sheikhi, A.: ‘Mutual coupling compensation for a practical VHF/UHF Yagi-Uda antenna array’, IET Microw., Antennas Propag., 2013, **7**, (13), pp. 1072-1083

[20] Huang, Q., Zhou, H., Bao, J., Shi, X.: ‘Mutual coupling calibration for microstrip antenna arrays via element pattern reconstruction method’, IEEE Antennas Wirel. Propag. Lett., 2014, **13**, pp. 51-54

[21] Wu, Y. I., Arada, G. P., Wong, K. T., Tam, W. Y.: ‘Electromagnetic coupling matrix modeling and ESPRIT-based direction finding - a case study using a uniform linear array of identical dipoles’, WISP, 2015, pp. 1-5

[22] Liu, H., Zhao, L., Li, Y., Jing, X.: ‘A sparse-based approach for DOA estimation and array calibration in uniform linear array’, IEEE Sensors J., 2016, **16**, (15), pp. 6018-6027

[23] Lindmark, B.: ‘Comparison of mutual coupling compensation to dummy columns in adaptive antenna systems’, IEEE Trans. Antennas Propag., 2005, **53**, (4), pp. 1332-1336

[24] Ye, Z., Liu, C.: ‘Non-sensitive adaptive beamforming against mutual coupling’, IET Signal Process., 2009, **3**, (1), pp. 1-6

[25] Liao, B., Chan, S. C.: ‘Adaptive beamforming for uniform linear arrays with unknown mutual coupling’, IEEE Antennas Wirel. Propag. Lett., 2012, **11**, pp. 464-467

[26] Kota, J., Madanayake, A., Belostotski, L., *et al.*: ‘A 2-D signal processing model to predict the effect of mutual coupling on array factor’, IEEE Antennas Wirel. Propag. Lett., 2013, **12**, pp. 1264-1267

[27] Kelly, D.F., Stutzman, W.L.: ‘Array antenna pattern modelling methods that include mutual coupling effects’, IEEE Trans. Antennas Propag., 1993, **41**, (12), pp. 1625-1632

[28] Zhang, T., Ser, W.: ‘Robust beampattern synthesis for antenna arrays with mutual coupling effect’, IEEE Trans. Antennas Propag., 2011, **59**, (8), pp. 2889-2895

[29] Karimzadeh, R., Hakkak, M., Haddadi, A., Forooraghi, K.: ‘Conformal array pattern synthesis using the weighted alternating reverse projection method considering mutual coupling and embedded-element pattern effects’, IET Microw., Antennas Propag., 2012, **6**, (6), pp. 621-626

[30] Ercil, E., Yildirim, E., Inal, M. E.: ‘Array antenna pattern synthesis using measured active element patterns and Gram-Schmidt orthogonalization’, Proc. IEEE Int. Symp. Phased Array Syst. Technol., 2010, pp. 357-360

[31] Bencivenni, C., Ivashina, M. V., Maaskant, R., Wettergren, J.: 'Design of maximally sparse antenna arrays in the presence of mutual coupling', IEEE Antennas Wirel. Propag. Lett., 2015, **14**, pp. 159-162

[32] Pozar, D.M.: 'The active element pattern', IEEE Trans. Antennas Propag., 1994, **42**, (8), pp. 1176-1178

[33] Pozar, D.M.: 'A relation between the active input impedance and the active element pattern of a phased array', IEEE Trans. Antennas Propag., 2003, **51**, (9), pp. 2486-2489

[34] You, P., Liu, Y., Huang, X., *et al.*: 'Efficient phase-only linear array synthesis including coupling effect by GA-FFT based on least-square active element pattern expansion method', Electron. Lett., 2015, **51**, (10), pp. 791-792

[35] 'Ansoft HFSS User Guide', http://www.ansys.com, accessed 5 November 2009

[36] Liu, Y., Nie, Z., Liu, Q.H.: 'A new method for the synthesis of non-uniform linear arrays with shaped power patterns', Prog. Electromagn. Res., 2010, **107**, pp. 349-363