

Trunk Generating Functions for n -ominoes of Dimension $n-3$

Robert A. Russell
New York, NY

June 14, 2022

Golomb [1, pages 81-84] shows that we can represent multidimensional polyominoes as graphs, each vertex of the graph being the center of a cell, and each edge being the line segment connecting the centers of adjacent cells. The dimension of the graph (and its source polyomino) is that of the convex hull of the cell centers. Golomb notes that the number of unoriented n -ominoes of dimension $n-1$ is equivalent to the number of free trees with n nodes ([A000055](#)). For unoriented polyominoes, chiral pairs are counted as one.

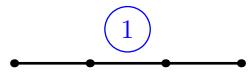
To enumerate n -ominoes of dimension $n-3$, we want to reduce the dimension of each polyomino graph by a removal process in order to obtain a small number of irreducible canonical graphs that we shall call *trunks*. First we align the graph edges with the Cartesian coordinate axes. In the removal process, an edge that is not parallel to any other edge is removed by changing the coordinate along an axis parallel to it to be the same for all vertices. In the first removal process, we remove any vertex and the single edge to which it is attached if that edge is not parallel to another edge of the graph. Our second removal involves a vertex connected to exactly two edges, neither of which is parallel to any other edge. We can eliminate this vertex by removing just one of the two edges. We shall not do this if it causes a new edge to arise elsewhere by bringing two other vertices into proximity. Note that each removal reduces the order of the polyomino by one and also reduces its dimension by one, so that the difference between the order of the polyomino and its dimension is unchanged.

It is not difficult to determine all the trunks that can arise for n -ominoes of dimension $n-3$. For each trunk we can determine a generating function that will indicate the exact number of n -ominoes that reduce to that trunk. There are separate generating functions for unoriented, chiral, and asymmetric n -ominoes. In addition, we can distinguish trunks of orthoplex polyominoes from those that have at least one coordinate with more than two values, which we call *extended* polyominoes. We sum together the generating functions for each trunk to obtain the generating function for all n -ominoes of dimension $n-3$. There are 56 trunks for orthoplex polyominoes and 173 for extended polyominoes. For multidimensional polyominoes we use all 229 trunks.

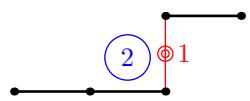
We have done this process for n -ominoes of dimension $n-2$. The results for multidimensional polyominoes are [A036364](#) (unoriented), [A036365](#) (chiral), and [A036366](#) (asymmetric). For orthoplex polyominoes we have [A036367](#) (unoriented), [A036368](#) (chiral), and [A036369](#) (asymmetric).

References

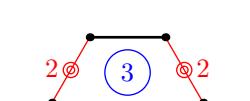
- [1] S. W. Golomb. *Polyominoes*. Princeton University Press, second edition, 1994.



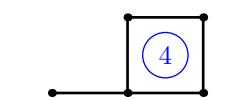
$$(B(x)^4 + B(x^2)^2) / 2 = x^4 + 2x^5 + 8x^6 + 22x^7 + 69x^8 + 194x^9 + 564x^{10} + 1584x^{11} + 4484x^{12} + 12550x^{13} + 35204x^{14} + 98412x^{15} + 275422x^{16} + 770416x^{17} + 2157334x^{18} + 6044684x^{19} + 16956389x^{20} + \dots$$



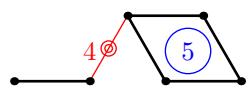
$$B(x)^5 / (1 - B(x)) = x^5 + 6x^6 + 27x^7 + 105x^8 + 378x^9 + 1291x^{10} + 4261x^{11} + 13714x^{12} + 43342x^{13} + 135091x^{14} + 416616x^{15} + 1274158x^{16} + 3871219x^{17} + 11699563x^{18} + 35206822x^{19} + 105573966x^{20} + \dots$$



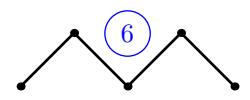
$$(B(x)^6 / (1 - B(x))^2 + B(x^2)^3 / (1 - B(x^2))) / 2 = x^6 + 4x^7 + 24x^8 + 101x^9 + 425x^{10} + 1614x^{11} + 5960x^{12} + 21072x^{13} + 72836x^{14} + 245942x^{15} + 817338x^{16} + 2677208x^{17} + 8671683x^{18} + 27814967x^{19} + 88503701x^{20} + \dots$$



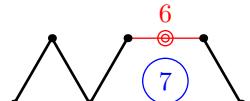
$$B(x)^5 = x^5 + 5x^6 + 20x^7 + 70x^8 + 230x^9 + 721x^{10} + 2200x^{11} + 6575x^{12} + 19385x^{13} + 56575x^{14} + 163952x^{15} + 472645x^{16} + 1357550x^{17} + 3888820x^{18} + 11119325x^{19} + 31753269x^{20} + \dots$$



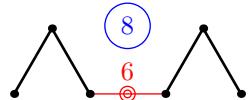
$$B(x)^6 / (1 - B(x)) = x^6 + 7x^7 + 35x^8 + 148x^9 + 570x^{10} + 2061x^{11} + 7139x^{12} + 23957x^{13} + 78516x^{14} + 252664x^{15} + 801513x^{16} + 2513669x^{17} + 7810743x^{18} + 24087497x^{19} + 73820697x^{20} + \dots$$



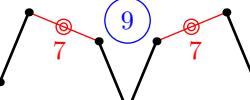
$$B(x) (B(x)^4 + B(x^2)^2) / 2 = x^5 + 3x^6 + 12x^7 + 38x^8 + 124x^9 + 377x^{10} + 1144x^{11} + 3381x^{12} + 9933x^{13} + 28851x^{14} + 83424x^{15} + 239929x^{16} + 688166x^{17} + 1968692x^{18} + 5623855x^{19} + 16046607x^{20} + \dots$$



$$B(x)^6 / (1 - B(x)) = x^6 + 7x^7 + 35x^8 + 148x^9 + 570x^{10} + 2061x^{11} + 7139x^{12} + 23957x^{13} + 78516x^{14} + 252664x^{15} + 801513x^{16} + 2513669x^{17} + 7810743x^{18} + 24087497x^{19} + 73820697x^{20} + \dots$$



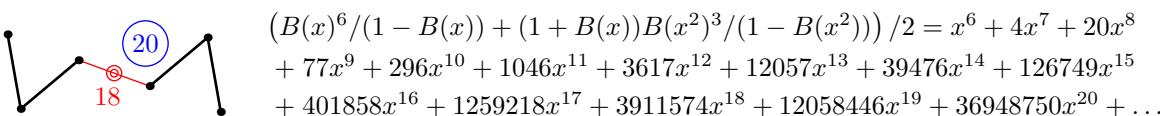
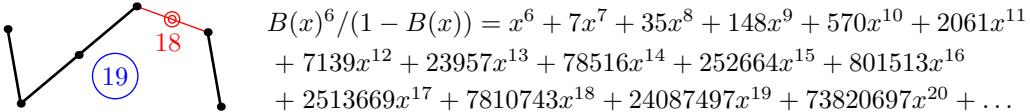
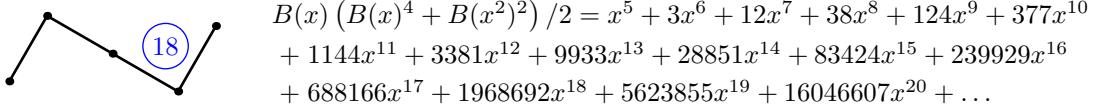
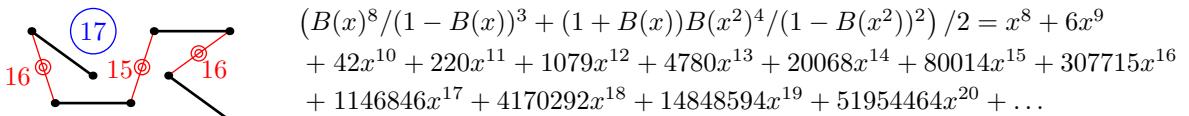
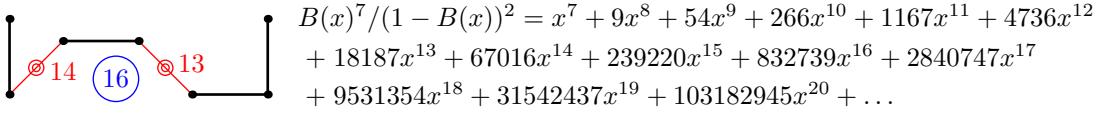
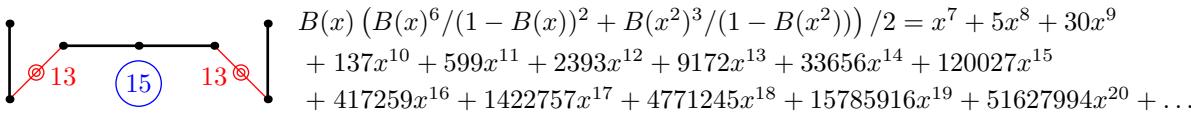
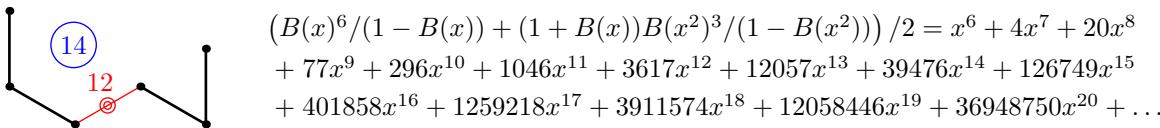
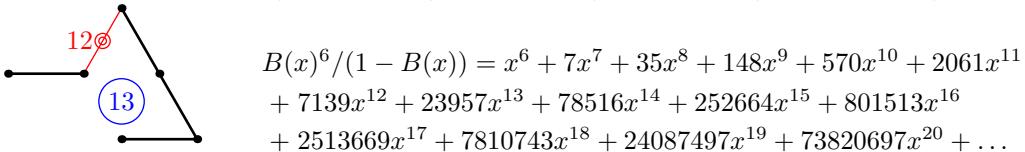
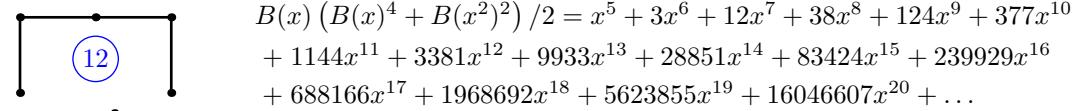
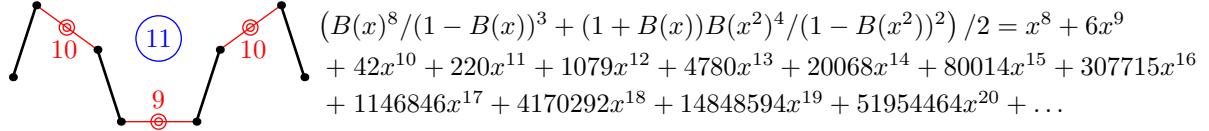
$$(B(x)^6 / (1 - B(x)) + (1 + B(x))B(x^2)^3 / (1 - B(x^2))) / 2 = x^6 + 4x^7 + 20x^8 + 77x^9 + 296x^{10} + 1046x^{11} + 3617x^{12} + 12057x^{13} + 39476x^{14} + 126749x^{15} + 401858x^{16} + 1259218x^{17} + 3911574x^{18} + 12058446x^{19} + 36948750x^{20} + \dots$$



$$B(x) (B(x)^6 / (1 - B(x))^2 + B(x^2)^3 / (1 - B(x^2))) / 2 = x^7 + 5x^8 + 30x^9 + 137x^{10} + 599x^{11} + 2393x^{12} + 9172x^{13} + 33656x^{14} + 120027x^{15} + 417259x^{16} + 1422757x^{17} + 4771245x^{18} + 15785916x^{19} + 51627994x^{20} + \dots$$



$$B(x)^7 / (1 - B(x))^2 = x^7 + 9x^8 + 54x^9 + 266x^{10} + 1167x^{11} + 4736x^{12} + 18187x^{13} + 67016x^{14} + 239220x^{15} + 832739x^{16} + 2840747x^{17} + 9531354x^{18} + 31542437x^{19} + 103182945x^{20} + \dots$$



- $B(x) \left(B(x)^6 / (1 - B(x))^2 + B(x^2)^3 / (1 - B(x^2)) \right) / 2 = x^7 + 5x^8 + 30x^9 + 137x^{10} + 599x^{11} + 2393x^{12} + 9172x^{13} + 33656x^{14} + 120027x^{15} + 417259x^{16} + 1422757x^{17} + 4771245x^{18} + 15785916x^{19} + 51627994x^{20} + \dots$
- $B(x)^7 / (1 - B(x))^2 = x^7 + 9x^8 + 54x^9 + 266x^{10} + 1167x^{11} + 4736x^{12} + 18187x^{13} + 67016x^{14} + 239220x^{15} + 832739x^{16} + 2840747x^{17} + 9531354x^{18} + 31542437x^{19} + 103182945x^{20} + \dots$
- $(B(x)^8 / (1 - B(x))^3 + (1 + B(x))B(x^2)^4 / (1 - B(x^2))^2) / 2 = x^8 + 6x^9 + 42x^{10} + 220x^{11} + 1079x^{12} + 4780x^{13} + 20068x^{14} + 80014x^{15} + 307715x^{16} + 1146846x^{17} + 4170292x^{18} + 14848594x^{19} + 51954464x^{20} + \dots$
- $B(x) \left(B(x)^4 + B(x^2)^2 \right) / 2 = x^5 + 3x^6 + 12x^7 + 38x^8 + 124x^9 + 377x^{10} + 1144x^{11} + 3381x^{12} + 9933x^{13} + 28851x^{14} + 83424x^{15} + 239929x^{16} + 688166x^{17} + 1968692x^{18} + 5623855x^{19} + 16046607x^{20} + \dots$
- $B(x)^6 / (1 - B(x)) = x^6 + 7x^7 + 35x^8 + 148x^9 + 570x^{10} + 2061x^{11} + 7139x^{12} + 23957x^{13} + 78516x^{14} + 252664x^{15} + 801513x^{16} + 2513669x^{17} + 7810743x^{18} + 24087497x^{19} + 73820697x^{20} + \dots$
- $(B(x)^6 / (1 - B(x)) + (1 + B(x))B(x^2)^3 / (1 - B(x^2))) / 2 = x^6 + 4x^7 + 20x^8 + 77x^9 + 296x^{10} + 1046x^{11} + 3617x^{12} + 12057x^{13} + 39476x^{14} + 126749x^{15} + 401858x^{16} + 1259218x^{17} + 3911574x^{18} + 12058446x^{19} + 36948750x^{20} + \dots$
- $B(x) \left(B(x)^6 / (1 - B(x))^2 + B(x^2)^3 / (1 - B(x^2)) \right) / 2 = x^7 + 5x^8 + 30x^9 + 137x^{10} + 599x^{11} + 2393x^{12} + 9172x^{13} + 33656x^{14} + 120027x^{15} + 417259x^{16} + 1422757x^{17} + 4771245x^{18} + 15785916x^{19} + 51627994x^{20} + \dots$
- $B(x)^7 / (1 - B(x))^2 = x^7 + 9x^8 + 54x^9 + 266x^{10} + 1167x^{11} + 4736x^{12} + 18187x^{13} + 67016x^{14} + 239220x^{15} + 832739x^{16} + 2840747x^{17} + 9531354x^{18} + 31542437x^{19} + 103182945x^{20} + \dots$
- $(B(x)^8 / (1 - B(x))^3 + (1 + B(x))B(x^2)^4 / (1 - B(x^2))^2) / 2 = x^8 + 6x^9 + 42x^{10} + 220x^{11} + 1079x^{12} + 4780x^{13} + 20068x^{14} + 80014x^{15} + 307715x^{16} + 1146846x^{17} + 4170292x^{18} + 14848594x^{19} + 51954464x^{20} + \dots$
- $B(x)^3 \left(B(x)^2 + B(x^2) \right) / 2 = x^5 + 4x^6 + 15x^7 + 49x^8 + 155x^9 + 469x^{10} + 1397x^{11} + 4088x^{12} + 11861x^{13} + 34146x^{14} + 97867x^{15} + 279487x^{16} + 796454x^{17} + 2266084x^{18} + 6441864x^{19} + 18302970x^{20} + \dots$

$$B(x)^6 / (1 - B(x)) = x^6 + 7x^7 + 35x^8 + 148x^9 + 570x^{10} + 2061x^{11} + 7139x^{12} + 23957x^{13} + 78516x^{14} + 252664x^{15} + 801513x^{16} + 2513669x^{17} + 7810743x^{18} + 24087497x^{19} + 73820697x^{20} + \dots$$

$$B(x)^4 (B(x)^2 + B(x^2)) / (2(1 - B(x))) = x^6 + 6x^7 + 28x^8 + 112x^9 + 415x^{10} + 1454x^{11} + 4914x^{12} + 16157x^{13} + 52070x^{14} + 165198x^{15} + 517778x^{16} + 1607106x^{17} + 4949252x^{18} + 15144071x^{19} + 46094110x^{20} + \dots$$

$$B(x)^4 (B(x)^2 + B(x^2)) / (2(1 - B(x))) = x^6 + 6x^7 + 28x^8 + 112x^9 + 415x^{10} + 1454x^{11} + 4914x^{12} + 16157x^{13} + 52070x^{14} + 165198x^{15} + 517778x^{16} + 1607106x^{17} + 4949252x^{18} + 15144071x^{19} + 46094110x^{20} + \dots$$

$$B(x)^7 / (1 - B(x))^2 = x^7 + 9x^8 + 54x^9 + 266x^{10} + 1167x^{11} + 4736x^{12} + 18187x^{13} + 67016x^{14} + 239220x^{15} + 832739x^{16} + 2840747x^{17} + 9531354x^{18} + 31542437x^{19} + 103182945x^{20} + \dots$$

$$B(x)^7 / (1 - B(x))^2 = x^7 + 9x^8 + 54x^9 + 266x^{10} + 1167x^{11} + 4736x^{12} + 18187x^{13} + 67016x^{14} + 239220x^{15} + 832739x^{16} + 2840747x^{17} + 9531354x^{18} + 31542437x^{19} + 103182945x^{20} + \dots$$

$$B(x)^5 (B(x)^2 + B(x^2)) / (2(1 - B(x))^2) = x^7 + 8x^8 + 45x^9 + 211x^{10} + 892x^{11} + 3513x^{12} + 13165x^{13} + 47529x^{14} + 166752x^{15} + 571932x^{16} + 1926183x^{17} + 6390777x^{18} + 20941734x^{19} + 67909538x^{20} + \dots$$

$$B(x)^3 (B(x)^4 / (1 - B(x))^2 + B(x^2)^2 / (1 - B(x^2))) / 2 = x^7 + 6x^8 + 33x^9 + 150x^{10} + 636x^{11} + 2512x^{12} + 9502x^{13} + 34615x^{14} + 122655x^{15} + 424606x^{16} + 1442829x^{17} + 4826604x^{18} + 15937389x^{19} + 52044695x^{20} + \dots$$

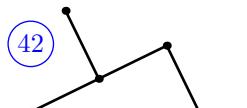
$$B(x)^8 / (1 - B(x))^3 = x^8 + 11x^9 + 77x^{10} + 432x^{11} + 2121x^{12} + 9512x^{13} + 39959x^{14} + 159766x^{15} + 614594x^{16} + 2292276x^{17} + 8336478x^{18} + 29689260x^{19} + 103887195x^{20} + \dots$$

$$B(x)^4 (B(x)^4 / (1 - B(x))^2 + B(x^2)^2 / (1 - B(x^2))) / (2(1 - B(x))) = x^8 + 8x^9 + 50x^{10} + 259x^{11} + 1214x^{12} + 5268x^{13} + 21643x^{14} + 85131x^{15} + 323602x^{16} + 1196060x^{17} + 4319712x^{18} + 15300599x^{19} + 53308738x^{20} + \dots$$

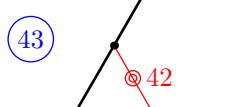
$$B(x)^4 (B(x)^4 / (1 - B(x))^2 + B(x^2)^2 / (1 - B(x^2))) / (2(1 - B(x))) = x^8 + 8x^9 + 50x^{10} + 259x^{11} + 1214x^{12} + 5268x^{13} + 21643x^{14} + 85131x^{15} + 323602x^{16} + 1196060x^{17} + 4319712x^{18} + 15300599x^{19} + 53308738x^{20} + \dots$$

(41) 

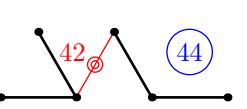
$$B(x)^5 \left(B(x)^4 / (1 - B(x))^2 + B(x^2)^2 / (1 - B(x^2)) \right) / (2(1 - B(x))^2) = x^9 + 10x^{10} + 71x^{11} + 412x^{12} + 2121x^{13} + 10016x^{14} + 44388x^{15} + 187177x^{16} + 758607x^{17} + 2976174x^{18} + 11364369x^{19} + 42412681x^{20} + \dots$$

(42) 

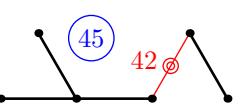
$$B(x)^5 = x^5 + 5x^6 + 20x^7 + 70x^8 + 230x^9 + 721x^{10} + 2200x^{11} + 6575x^{12} + 19385x^{13} + 56575x^{14} + 163952x^{15} + 472645x^{16} + 1357550x^{17} + 3888820x^{18} + 11119325x^{19} + 31753269x^{20} + \dots$$

(43) 

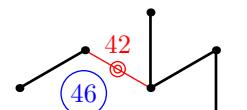
$$B(x)^6 / (1 - B(x)) = x^6 + 7x^7 + 35x^8 + 148x^9 + 570x^{10} + 2061x^{11} + 7139x^{12} + 23957x^{13} + 78516x^{14} + 252664x^{15} + 801513x^{16} + 2513669x^{17} + 7810743x^{18} + 24087497x^{19} + 73820697x^{20} + \dots$$

(44) 

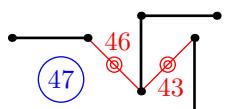
$$B(x)^6 / (1 - B(x)) = x^6 + 7x^7 + 35x^8 + 148x^9 + 570x^{10} + 2061x^{11} + 7139x^{12} + 23957x^{13} + 78516x^{14} + 252664x^{15} + 801513x^{16} + 2513669x^{17} + 7810743x^{18} + 24087497x^{19} + 73820697x^{20} + \dots$$

(45) 

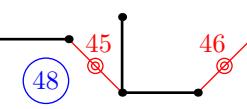
$$B(x)^6 / (1 - B(x)) = x^6 + 7x^7 + 35x^8 + 148x^9 + 570x^{10} + 2061x^{11} + 7139x^{12} + 23957x^{13} + 78516x^{14} + 252664x^{15} + 801513x^{16} + 2513669x^{17} + 7810743x^{18} + 24087497x^{19} + 73820697x^{20} + \dots$$

(46) 

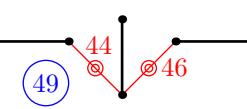
$$B(x)^6 / (1 - B(x)) = x^6 + 7x^7 + 35x^8 + 148x^9 + 570x^{10} + 2061x^{11} + 7139x^{12} + 23957x^{13} + 78516x^{14} + 252664x^{15} + 801513x^{16} + 2513669x^{17} + 7810743x^{18} + 24087497x^{19} + 73820697x^{20} + \dots$$

(47) 

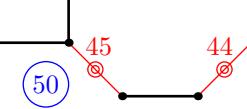
$$B(x)^7 / (1 - B(x))^2 = x^7 + 9x^8 + 54x^9 + 266x^{10} + 1167x^{11} + 4736x^{12} + 18187x^{13} + 67016x^{14} + 239220x^{15} + 832739x^{16} + 2840747x^{17} + 9531354x^{18} + 31542437x^{19} + 103182945x^{20} + \dots$$

(48) 

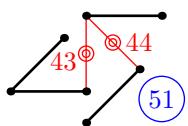
$$B(x)^7 / (1 - B(x))^2 = x^7 + 9x^8 + 54x^9 + 266x^{10} + 1167x^{11} + 4736x^{12} + 18187x^{13} + 67016x^{14} + 239220x^{15} + 832739x^{16} + 2840747x^{17} + 9531354x^{18} + 31542437x^{19} + 103182945x^{20} + \dots$$

(49) 

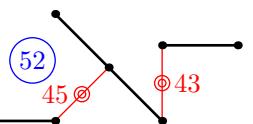
$$B(x)^7 / (1 - B(x))^2 = x^7 + 9x^8 + 54x^9 + 266x^{10} + 1167x^{11} + 4736x^{12} + 18187x^{13} + 67016x^{14} + 239220x^{15} + 832739x^{16} + 2840747x^{17} + 9531354x^{18} + 31542437x^{19} + 103182945x^{20} + \dots$$

(50) 

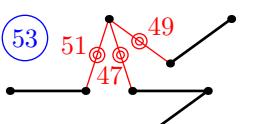
$$B(x)^7 / (1 - B(x))^2 = x^7 + 9x^8 + 54x^9 + 266x^{10} + 1167x^{11} + 4736x^{12} + 18187x^{13} + 67016x^{14} + 239220x^{15} + 832739x^{16} + 2840747x^{17} + 9531354x^{18} + 31542437x^{19} + 103182945x^{20} + \dots$$



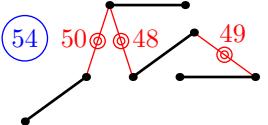
$$B(x)^7/(1-B(x))^2 = x^7 + 9x^8 + 54x^9 + 266x^{10} + 1167x^{11} + 4736x^{12} + 18187x^{13} + 67016x^{14} + 239220x^{15} + 832739x^{16} + 2840747x^{17} + 9531354x^{18} + 31542437x^{19} + 103182945x^{20} + \dots$$



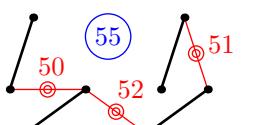
$$B(x)^7/(1-B(x))^2 = x^7 + 9x^8 + 54x^9 + 266x^{10} + 1167x^{11} + 4736x^{12} + 18187x^{13} + 67016x^{14} + 239220x^{15} + 832739x^{16} + 2840747x^{17} + 9531354x^{18} + 31542437x^{19} + 103182945x^{20} + \dots$$



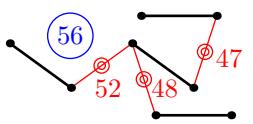
$$B(x)^8/(1-B(x))^3 = x^8 + 11x^9 + 77x^{10} + 432x^{11} + 2121x^{12} + 9512x^{13} + 39959x^{14} + 159766x^{15} + 614594x^{16} + 2292276x^{17} + 8336478x^{18} + 29689260x^{19} + 103887195x^{20} + \dots$$



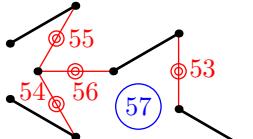
$$B(x)^8/(1-B(x))^3 = x^8 + 11x^9 + 77x^{10} + 432x^{11} + 2121x^{12} + 9512x^{13} + 39959x^{14} + 159766x^{15} + 614594x^{16} + 2292276x^{17} + 8336478x^{18} + 29689260x^{19} + 103887195x^{20} + \dots$$



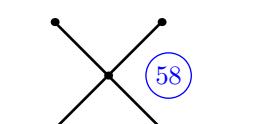
$$B(x)^8/(1-B(x))^3 = x^8 + 11x^9 + 77x^{10} + 432x^{11} + 2121x^{12} + 9512x^{13} + 39959x^{14} + 159766x^{15} + 614594x^{16} + 2292276x^{17} + 8336478x^{18} + 29689260x^{19} + 103887195x^{20} + \dots$$



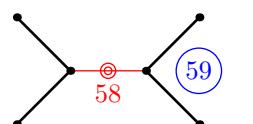
$$B(x)^8/(1-B(x))^3 = x^8 + 11x^9 + 77x^{10} + 432x^{11} + 2121x^{12} + 9512x^{13} + 39959x^{14} + 159766x^{15} + 614594x^{16} + 2292276x^{17} + 8336478x^{18} + 29689260x^{19} + 103887195x^{20} + \dots$$



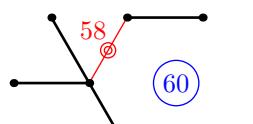
$$B(x)^9/(1-B(x))^4 = x^9 + 13x^{10} + 104x^{11} + 654x^{12} + 3548x^{13} + 17395x^{14} + 79206x^{15} + 340861x^{16} + 1403056x^{17} + 5571416x^{18} + 21478460x^{19} + 80776807x^{20} + \dots$$



$$B(x) (B(x)^4 + 2B(x)^2B(x^2) + 3B(x^2)^2 + 2B(x^4)) / 8 = x^5 + 2x^6 + 7x^7 + 19x^8 + 58x^9 + 162x^{10} + 469x^{11} + 1322x^{12} + 3762x^{13} + 10609x^{14} + 29999x^{15} + 84590x^{16} + 238775x^{17} + 673585x^{18} + 1901600x^{19} + 5369904x^{20} + \dots$$



$$B(x)^2 (B(x)^4 + B(x^2)^2) / (4(1-B(x))) + (1+B(x))B(x^2)^3 / (2(1-B(x^2))) = x^6 + 3x^7 + 14x^8 + 48x^9 + 178x^{10} + 601x^{11} + 2037x^{12} + 6653x^{13} + 21530x^{14} + 68398x^{15} + 215295x^{16} + 670414x^{17} + 2072685x^{18} + 6363694x^{19} + 19435257x^{20} + \dots$$



$$B(x)^4 (B(x)^2 + B(x^2)) / (2(1-B(x))) = x^6 + 6x^7 + 28x^8 + 112x^9 + 415x^{10} + 1454x^{11} + 4914x^{12} + 16157x^{13} + 52070x^{14} + 165198x^{15} + 517778x^{16} + 1607106x^{17} + 4949252x^{18} + 15144071x^{19} + 46094110x^{20} + \dots$$

$$B(x)^2 \left(B(x)^2 + B(x^2) \right)^2 / (8(1-B(x))) + B(x^2)(1+B(x)) \left(B(x^2)^2 + B(x^4) \right) / (4(1-B(x^2))) = x^6 + 3x^7 + 13x^8 + 44x^9 + 157x^{10} + 517x^{11} + 1703x^{12} + 5439x^{13} + 17230x^{14} + 53759x^{15} + 166479x^{16} + 511101x^{17} + 1560229x^{18} + 4736888x^{19} + 14322259x^{20} + \dots$$

$$B(x)^5 \left(B(x)^2 + B(x^2) \right) / (2(1-B(x))^2) = x^7 + 8x^8 + 45x^9 + 211x^{10} + 892x^{11} + 3513x^{12} + 13165x^{13} + 47529x^{14} + 166752x^{15} + 571932x^{16} + 1926183x^{17} + 6390777x^{18} + 20941734x^{19} + 67909538x^{20} + \dots$$

$$B(x) \left(B(x)^2 + B(x^2) \right) \left(B(x)^4 / (1-B(x))^2 + B(x^2)^2 / (1-B(x^2)) \right) / 4 = x^7 + 5x^8 + 27x^9 + 116x^{10} + 480x^{11} + 1841x^{12} + 6826x^{13} + 24392x^{14} + 85107x^{15} + 290529x^{16} + 975511x^{17} + 3228636x^{18} + 10561301x^{19} + 34199641x^{20} + \dots$$

$$B(x) \left(B(x)^6 / (1-B(x))^2 + B(x^2)^3 / (1-B(x^2)) \right) / 2 = x^7 + 5x^8 + 30x^9 + 137x^{10} + 599x^{11} + 2393x^{12} + 9172x^{13} + 33656x^{14} + 120027x^{15} + 417259x^{16} + 1422757x^{17} + 4771245x^{18} + 15785916x^{19} + 51627994x^{20} + \dots$$

$$B(x)^7 / (1-B(x))^2 = x^7 + 9x^8 + 54x^9 + 266x^{10} + 1167x^{11} + 4736x^{12} + 18187x^{13} + 67016x^{14} + 239220x^{15} + 832739x^{16} + 2840747x^{17} + 9531354x^{18} + 31542437x^{19} + 103182945x^{20} + \dots$$

$$B(x)^2 \left(B(x)^6 / (1-B(x))^2 + B(x^2)^3 / (1-B(x^2)) \right) / (2(1-B(x))) = x^8 + 7x^9 + 45x^{10} + 235x^{11} + 1123x^{12} + 4936x^{13} + 20521x^{14} + 81434x^{15} + 311816x^{16} + 1159072x^{17} + 4205526x^{18} + 14951498x^{19} + 52250974x^{20} + \dots$$

$$\left(B(x)^8 / (1-B(x))^3 + B(x^2)^4(1+B(x)) / (1-B(x^2))^2 \right) / 2 = x^8 + 6x^9 + 42x^{10} + 220x^{11} + 1079x^{12} + 4780x^{13} + 20068x^{14} + 80014x^{15} + 307715x^{16} + 1146846x^{17} + 4170292x^{18} + 14848594x^{19} + 51954464x^{20} + \dots$$

$$B(x)^4 \left(B(x)^4 / (1-B(x))^2 + B(x^2)^2 / (1-B(x^2)) \right) / (2(1-B(x))) = x^8 + 8x^9 + 50x^{10} + 259x^{11} + 1214x^{12} + 5268x^{13} + 21643x^{14} + 85131x^{15} + 323602x^{16} + 1196060x^{17} + 4319712x^{18} + 15300599x^{19} + 53308738x^{20} + \dots$$

$$B(x)^2 \left(B(x)^2 + B(x^2) \right) \left(B(x)^4 / (1-B(x))^2 + B(x^2)^2 / (1-B(x^2)) \right) / (4(1-B(x))) = x^8 + 7x^9 + 42x^{10} + 208x^{11} + 947x^{12} + 4002x^{13} + 16098x^{14} + 62151x^{15} + 232516x^{16} + 847376x^{17} + 3022694x^{18} + 10588705x^{19} + 36528856x^{20} + \dots$$

$$\left(B(x)^8 / (1-B(x))^3 + B(x^2)^4(1+B(x)) / (1-B(x^2))^2 \right) / 2 = x^8 + 6x^9 + 42x^{10} + 220x^{11} + 1079x^{12} + 4780x^{13} + 20068x^{14} + 80014x^{15} + 307715x^{16} + 1146846x^{17} + 4170292x^{18} + 14848594x^{19} + 51954464x^{20} + \dots$$

$$(B(x)^8/(1-B(x))^3 + B(x^2)^4(1+B(x))/(1-B(x^2))^2)/2 = x^8 + 6x^9 + 42x^{10} + 220x^{11} + 1079x^{12} + 4780x^{13} + 20068x^{14} + 80014x^{15} + 307715x^{16} + 1146846x^{17} + 4170292x^{18} + 14848594x^{19} + 51954464x^{20} + \dots$$

$$B(x) (B(x)^8/(1-B(x))^4 + 2B(x)^4B(x^2)^2/((1-B(x))^2(1-B(x^2))) + 3B(x^2)^4/(1-B(x^2))^2 + 2B(x^4)^2/(1-B(x^4)))/8 = x^9 + 4x^{10} + 26x^{11} + 129x^{12} + 638x^{13} + 2866x^{14} + 12405x^{15} + 51181x^{16} + 204533x^{17} + 792892x^{18} + 3000851x^{19} + 11116976x^{20} + \dots$$

$$B(x)^9/(1-B(x))^4 = x^9 + 13x^{10} + 104x^{11} + 654x^{12} + 3548x^{13} + 17395x^{14} + 79206x^{15} + 340861x^{16} + 1403056x^{17} + 5571416x^{18} + 21478460x^{19} + 80776807x^{20} + \dots$$

$$B(x)^5 (B(x)^4/(1-B(x))^2 + B(x^2)^2/(1-B(x^2)))/(2(1-B(x))^2) = x^9 + 10x^{10} + 71x^{11} + 412x^{12} + 2121x^{13} + 10016x^{14} + 44388x^{15} + 187177x^{16} + 758607x^{17} + 2976174x^{18} + 11364369x^{19} + 42412681x^{20} + \dots$$

$$B(x)^2 (B(x)^8/(1-B(x))^4 + B(x^2)^4/(1-B(x^2))^2)/(4(1-B(x))) + B(x^2)^5(1+B(x))/(2(1-B(x^2))^3) = x^{10} + 5x^{11} + 42x^{12} + 251x^{13} + 1462x^{14} + 7563x^{15} + 36846x^{16} + 168537x^{17} + 736755x^{18} + 3093901x^{19} + 12578648x^{20} + \dots$$

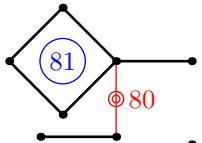
$$B(x)^2 (B(x)^4/(1-B(x))^2 + B(x^2)^2/(1-B(x^2)))^2/(8(1-B(x))) + B(x^2)(1+B(x))(B(x^2)^4/(1-B(x^2))^2 + B(x^4)^2/(1-B(x^4)))/(4(1-B(x^2))) = x^{10} + 5x^{11} + 36x^{12} + 200x^{13} + 1072x^{14} + 5214x^{15} + 24132x^{16} + 106034x^{17} + 448642x^{18} + 1834953x^{19} + 7300326x^{20} + \dots$$

$$(B(x)^2 + B(x^2))(B(x)^4 + B(x^2)^2)/4 = x^6 + 3x^7 + 13x^8 + 42x^9 + 143x^{10} + 446x^{11} + 1395x^{12} + 4223x^{13} + 12710x^{14} + 37702x^{15} + 111219x^{16} + 325692x^{17} + 949992x^{18} + 2759861x^{19} + 7997205x^{20} + \dots$$

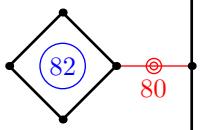
$$B(x)^5 (B(x)^2 + B(x^2))/(2(1-B(x))) = x^7 + 7x^8 + 36x^9 + 156x^{10} + 616x^{11} + 2279x^{12} + 8066x^{13} + 27610x^{14} + 92166x^{15} + 301655x^{16} + 972014x^{17} + 3092752x^{18} + 9739434x^{19} + 30409345x^{20} + \dots$$

$$(B(x)^2 + B(x^2))(B(x)^6/(1-B(x))^2 + B(x^2)^3/(1-B(x^2)))/4 = x^8 + 5x^9 + 31x^{10} + 143x^{11} + 638x^{12} + 2587x^{13} + 10083x^{14} + 37551x^{15} + 135891x^{16} + 478830x^{17} + 1653828x^{18} + 5613094x^{19} + 18782402x^{20} + \dots$$

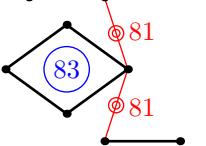
$$B(x)^2 (B(x)^2 + B(x^2))^2/4 = x^6 + 4x^7 + 16x^8 + 54x^9 + 178x^{10} + 556x^{11} + 1709x^{12} + 5138x^{13} + 15289x^{14} + 45016x^{15} + 131718x^{16} + 383264x^{17} + 1111080x^{18} + 3211134x^{19} + 9260556x^{20} + \dots$$



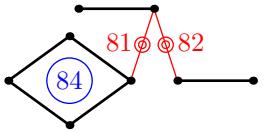
$$B(x)^5 \left(B(x)^2 + B(x^2) \right) / (2(1-B(x))) = x^7 + 7x^8 + 36x^9 + 156x^{10} + 616x^{11} + 2279x^{12} + 8066x^{13} + 27610x^{14} + 92166x^{15} + 301655x^{16} + 972014x^{17} + 3092752x^{18} + 9739434x^{19} + 30409345x^{20} + \dots$$



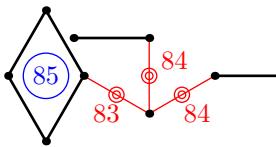
$$B(x)^3 \left(B(x)^2 + B(x^2) \right)^2 / (4(1-B(x))) = x^7 + 6x^8 + 29x^9 + 119x^{10} + 453x^{11} + 1625x^{12} + 5615x^{13} + 18835x^{14} + 61831x^{15} + 199497x^{16} + 635038x^{17} + 1999283x^{18} + 6238185x^{19} + 19320061x^{20} + \dots$$



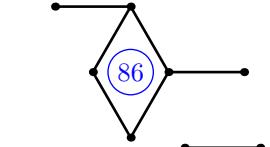
$$B(x)^2 \left(B(x)^2 + B(x^2) \right) \left(B(x)^4 / (1-B(x))^2 + B(x^2)^2 / (1-B(x^2)) \right) / 4 = x^8 + 6x^9 + 34x^{10} + 157x^{11} + 679x^{12} + 2726x^{13} + 10482x^{14} + 38759x^{15} + 139312x^{16} + 488715x^{17} + 1681615x^{18} + 5691814x^{19} + 19002975x^{20} + \dots$$



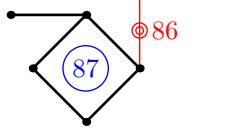
$$B(x)^6 \left(B(x)^2 + B(x^2) \right) / (2(1-B(x))^2) = x^8 + 9x^9 + 55x^{10} + 276x^{11} + 1234x^{12} + 5099x^{13} + 19919x^{14} + 74586x^{15} + 270277x^{16} + 954169x^{17} + 3298025x^{18} + 11202300x^{19} + 37500193x^{20} + \dots$$



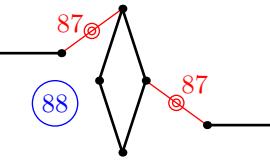
$$B(x)^3 \left(B(x)^2 + B(x^2) \right) \left(B(x)^4 / (1-B(x))^2 + B(x^2)^2 / (1-B(x^2)) \right) / (4(1-B(x))) = x^9 + 8x^{10} + 51x^{11} + 268x^{12} + 1276x^{13} + 5616x^{14} + 23392x^{15} + 93204x^{16} + 358661x^{17} + 1341079x^{18} + 4896891x^{19} + 17525881x^{20} + \dots$$



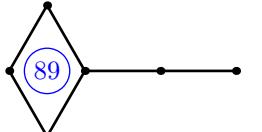
$$(B(x)^6 + B(x^2)^3) / 2 = x^6 + 3x^7 + 15x^8 + 52x^9 + 189x^{10} + 618x^{11} + 2007x^{12} + 6261x^{13} + 19302x^{14} + 58404x^{15} + 175125x^{16} + 519948x^{17} + 1534324x^{18} + 4502091x^{19} + 13158063x^{20} + \dots$$



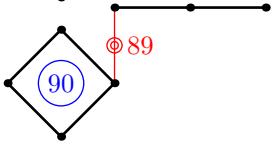
$$B(x)^7 / (1-B(x)) = x^7 + 8x^8 + 44x^9 + 201x^{10} + 825x^{11} + 3150x^{12} + 11435x^{13} + 39981x^{14} + 135856x^{15} + 451449x^{16} + 1473773x^{17} + 4742598x^{18} + 15083315x^{19} + 47505924x^{20} + \dots$$



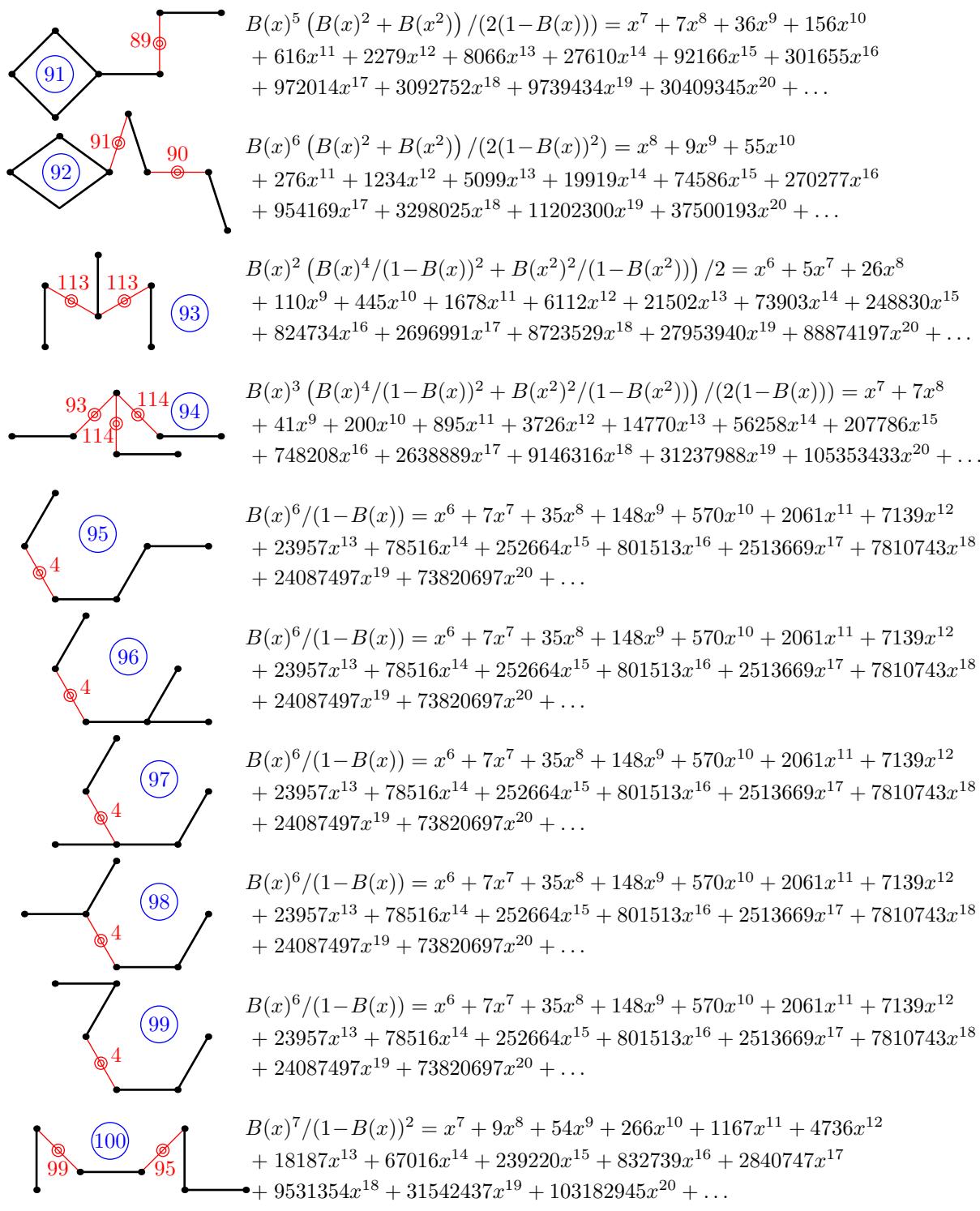
$$(B(x)^8 / (1-B(x))^2 + B(x^2)^4 / (1-B(x^2))) / 2 = x^8 + 5x^9 + 35x^{10} + 171x^{11} + 803x^{12} + 3376x^{13} + 13553x^{14} + 51682x^{15} + 190764x^{16} + 683487x^{17} + 2394761x^{18} + 8229561x^{19} + 27839714x^{20} + \dots$$



$$B(x)^4 \left(B(x)^2 + B(x^2) \right) / 2 = x^6 + 5x^7 + 21x^8 + 76x^9 + 259x^{10} + 838x^{11} + 2635x^{12} + 8091x^{13} + 24460x^{14} + 73032x^{15} + 216123x^{16} + 635092x^{17} + 1856500x^{18} + 5404637x^{19} + 15684765x^{20} + \dots$$



$$B(x)^5 \left(B(x)^2 + B(x^2) \right) / (2(1-B(x))) = x^7 + 7x^8 + 36x^9 + 156x^{10} + 616x^{11} + 2279x^{12} + 8066x^{13} + 27610x^{14} + 92166x^{15} + 301655x^{16} + 972014x^{17} + 3092752x^{18} + 9739434x^{19} + 30409345x^{20} + \dots$$



$$B(x)^7/(1-B(x))^2 = x^7 + 9x^8 + 54x^9 + 266x^{10} + 1167x^{11} + 4736x^{12} + 18187x^{13} + 67016x^{14} + 239220x^{15} + 832739x^{16} + 2840747x^{17} + 9531354x^{18} + 31542437x^{19} + 103182945x^{20} + \dots$$

$$B(x)^7/(1-B(x))^2 = x^7 + 9x^8 + 54x^9 + 266x^{10} + 1167x^{11} + 4736x^{12} + 18187x^{13} + 67016x^{14} + 239220x^{15} + 832739x^{16} + 2840747x^{17} + 9531354x^{18} + 31542437x^{19} + 103182945x^{20} + \dots$$

$$B(x)^7/(1-B(x))^2 = x^7 + 9x^8 + 54x^9 + 266x^{10} + 1167x^{11} + 4736x^{12} + 18187x^{13} + 67016x^{14} + 239220x^{15} + 832739x^{16} + 2840747x^{17} + 9531354x^{18} + 31542437x^{19} + 103182945x^{20} + \dots$$

$$B(x)^7/(1-B(x))^2 = x^7 + 9x^8 + 54x^9 + 266x^{10} + 1167x^{11} + 4736x^{12} + 18187x^{13} + 67016x^{14} + 239220x^{15} + 832739x^{16} + 2840747x^{17} + 9531354x^{18} + 31542437x^{19} + 103182945x^{20} + \dots$$

$$B(x)^7/(1-B(x))^2 = x^7 + 9x^8 + 54x^9 + 266x^{10} + 1167x^{11} + 4736x^{12} + 18187x^{13} + 67016x^{14} + 239220x^{15} + 832739x^{16} + 2840747x^{17} + 9531354x^{18} + 31542437x^{19} + 103182945x^{20} + \dots$$

$$B(x)^7/(1-B(x))^2 = x^7 + 9x^8 + 54x^9 + 266x^{10} + 1167x^{11} + 4736x^{12} + 18187x^{13} + 67016x^{14} + 239220x^{15} + 832739x^{16} + 2840747x^{17} + 9531354x^{18} + 31542437x^{19} + 103182945x^{20} + \dots$$

$$B(x)^7/(1-B(x))^2 = x^7 + 9x^8 + 54x^9 + 266x^{10} + 1167x^{11} + 4736x^{12} + 18187x^{13} + 67016x^{14} + 239220x^{15} + 832739x^{16} + 2840747x^{17} + 9531354x^{18} + 31542437x^{19} + 103182945x^{20} + \dots$$

$$(B(x)^7/(1-B(x)) + B(x^2)^3(B(x) + B(x^2))/(1-B(x^2))) / 2 = x^7 + 5x^8 + 25x^9 + 107x^{10} + 428x^{11} + 1610x^{12} + 5796x^{13} + 20174x^{14} + 68345x^{15} + 226733x^{16} + 739270x^{17} + 2377250x^{18} + 7556355x^{19} + 23790687x^{20} + \dots$$

$$B(x)^7/(1-B(x))^2 = x^7 + 9x^8 + 54x^9 + 266x^{10} + 1167x^{11} + 4736x^{12} + 18187x^{13} + 67016x^{14} + 239220x^{15} + 832739x^{16} + 2840747x^{17} + 9531354x^{18} + 31542437x^{19} + 103182945x^{20} + \dots$$

$$B(x)^8/(1-B(x))^3 = x^8 + 11x^9 + 77x^{10} + 432x^{11} + 2121x^{12} + 9512x^{13} + 39959x^{14} + 159766x^{15} + 614594x^{16} + 2292276x^{17} + 8336478x^{18} + 29689260x^{19} + 103887195x^{20} + \dots$$

$$B(x)^6 / (1 - B(x)) = x^6 + 7x^7 + 35x^8 + 148x^9 + 570x^{10} + 2061x^{11} + 7139x^{12} + 23957x^{13} + 78516x^{14} + 252664x^{15} + 801513x^{16} + 2513669x^{17} + 7810743x^{18} + 24087497x^{19} + 73820697x^{20} + \dots$$

$$B(x)^7 / (1 - B(x))^2 = x^7 + 9x^8 + 54x^9 + 266x^{10} + 1167x^{11} + 4736x^{12} + 18187x^{13} + 67016x^{14} + 239220x^{15} + 832739x^{16} + 2840747x^{17} + 9531354x^{18} + 31542437x^{19} + 103182945x^{20} + \dots$$

$$B(x)^6 / (1 - B(x)) = x^6 + 7x^7 + 35x^8 + 148x^9 + 570x^{10} + 2061x^{11} + 7139x^{12} + 23957x^{13} + 78516x^{14} + 252664x^{15} + 801513x^{16} + 2513669x^{17} + 7810743x^{18} + 24087497x^{19} + 73820697x^{20} + \dots$$

$$B(x)^7 / (1 - B(x))^2 = x^7 + 9x^8 + 54x^9 + 266x^{10} + 1167x^{11} + 4736x^{12} + 18187x^{13} + 67016x^{14} + 239220x^{15} + 832739x^{16} + 2840747x^{17} + 9531354x^{18} + 31542437x^{19} + 103182945x^{20} + \dots$$

$$B(x)^3 (B(x)^4 / (1 - B(x))^2 + B(x^2)^2 / (1 - B(x^2))) / 2 = x^7 + 6x^8 + 33x^9 + 150x^{10} + 636x^{11} + 2512x^{12} + 9502x^{13} + 34615x^{14} + 122655x^{15} + 424606x^{16} + 1442829x^{17} + 4826604x^{18} + 15937389x^{19} + 52044695x^{20} + \dots$$

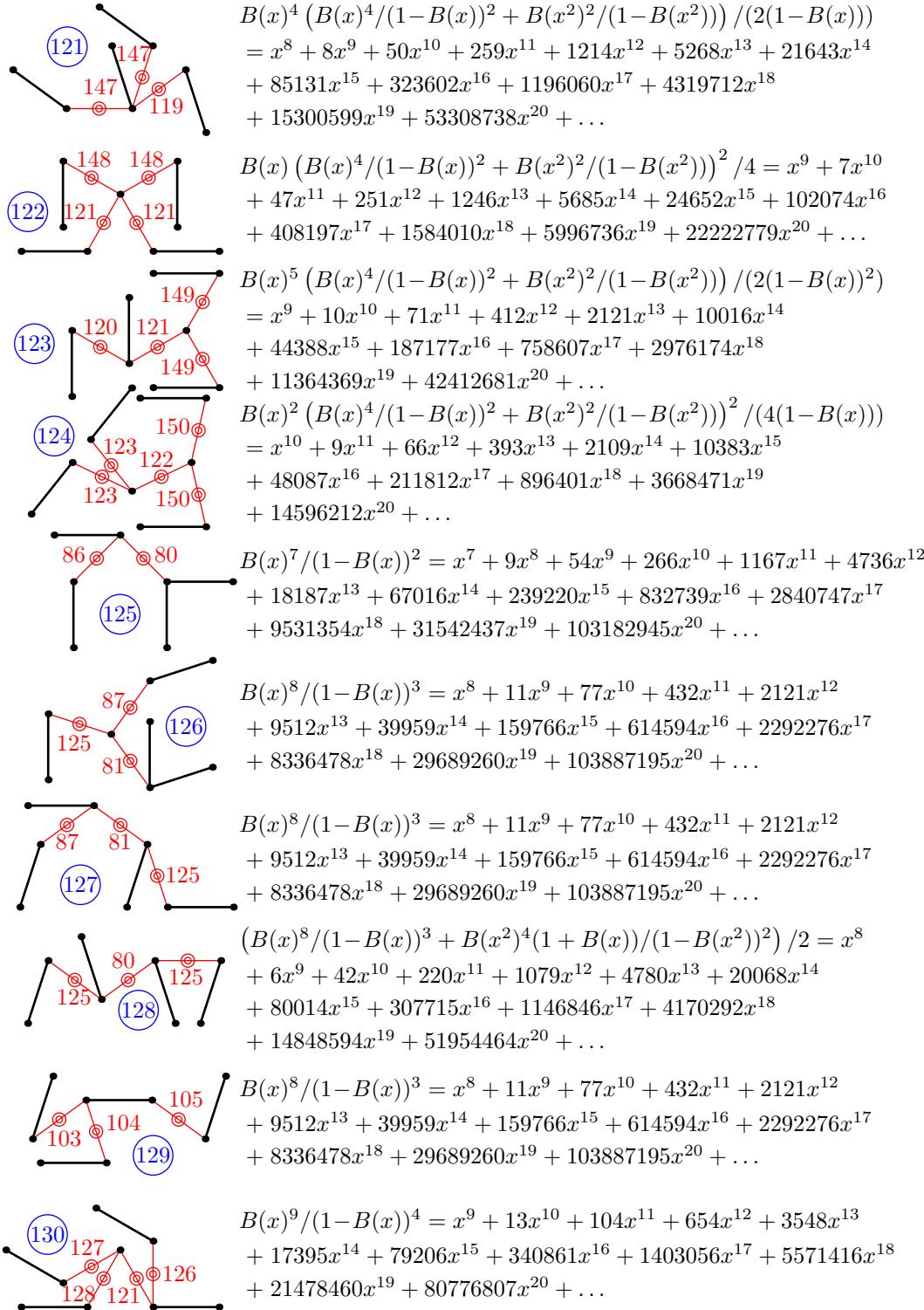
$$B(x)^4 (B(x)^4 / (1 - B(x))^2 + B(x^2)^2 / (1 - B(x^2))) / (2(1 - B(x))) = x^8 + 8x^9 + 50x^{10} + 259x^{11} + 1214x^{12} + 5268x^{13} + 21643x^{14} + 85131x^{15} + 323602x^{16} + 1196060x^{17} + 4319712x^{18} + 15300599x^{19} + 53308738x^{20} + \dots$$

$$B(x)^4 (B(x)^4 / (1 - B(x))^2 + B(x^2)^2 / (1 - B(x^2))) / (2(1 - B(x))) = x^8 + 8x^9 + 50x^{10} + 259x^{11} + 1214x^{12} + 5268x^{13} + 21643x^{14} + 85131x^{15} + 323602x^{16} + 1196060x^{17} + 4319712x^{18} + 15300599x^{19} + 53308738x^{20} + \dots$$

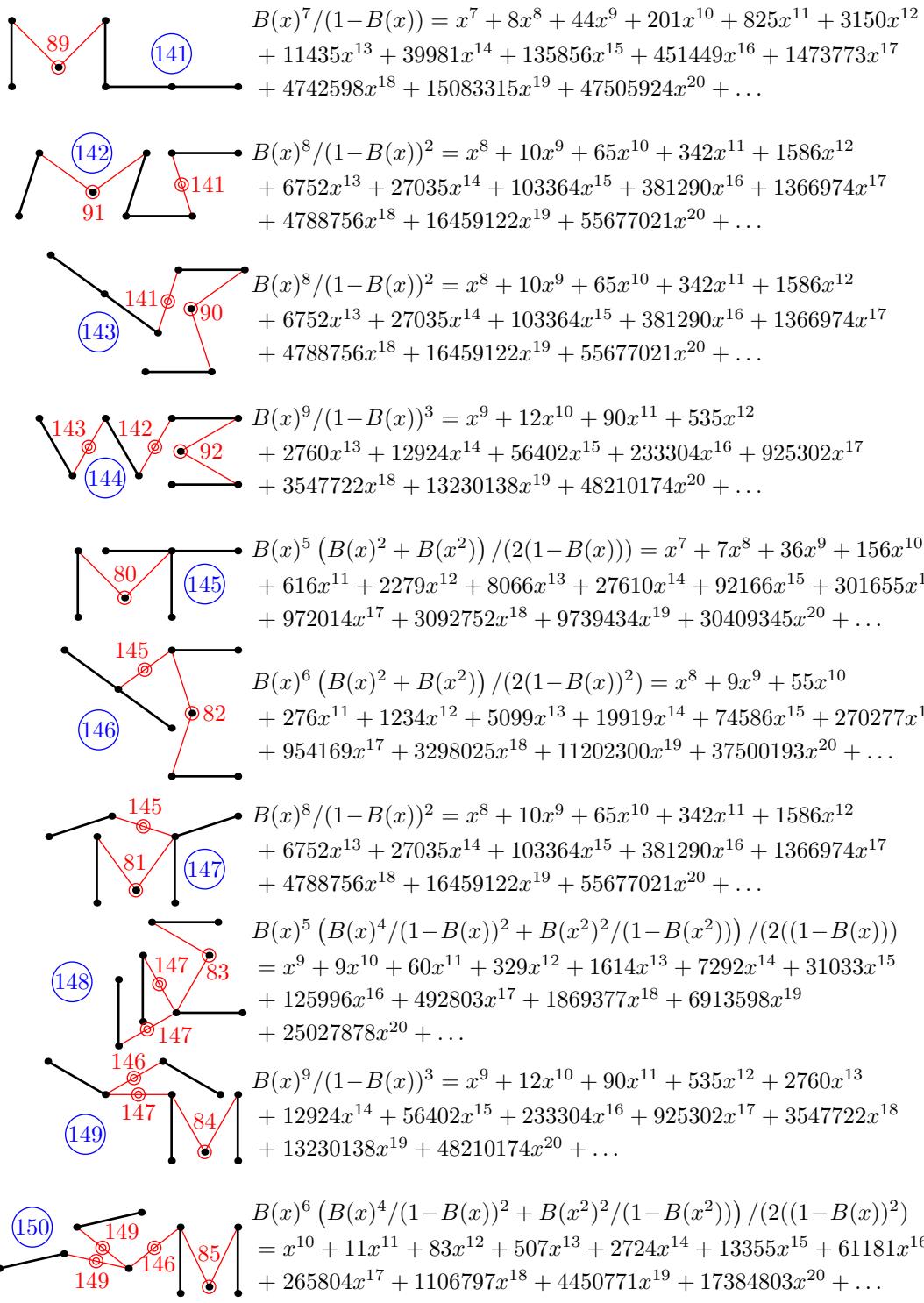
$$B(x)^5 (B(x)^4 / (1 - B(x))^2 + B(x^2)^2 / (1 - B(x^2))) / (2(1 - B(x))^2) = x^9 + 10x^{10} + 71x^{11} + 412x^{12} + 2121x^{13} + 10016x^{14} + 44388x^{15} + 187177x^{16} + 758607x^{17} + 2976174x^{18} + 11364369x^{19} + 42412681x^{20} + \dots$$

$$B(x) (B(x)^2 + B(x^2)) (B(x)^4 / (1 - B(x))^2 + B(x^2)^2 / (1 - B(x^2))) / 4 = x^7 + 5x^8 + 27x^9 + 116x^{10} + 480x^{11} + 1841x^{12} + 6826x^{13} + 24392x^{14} + 85107x^{15} + 290529x^{16} + 975511x^{17} + 3228636x^{18} + 10561301x^{19} + 34199641x^{20} + \dots$$

$$B(x)^2 (B(x)^2 + B(x^2)) (B(x)^4 / (1 - B(x))^2 + B(x^2)^2 / (1 - B(x^2))) / (4(1 - B(x))) = x^8 + 7x^9 + 42x^{10} + 208x^{11} + 947x^{12} + 4002x^{13} + 16098x^{14} + 62151x^{15} + 232516x^{16} + 847376x^{17} + 3022694x^{18} + 10588705x^{19} + 36528856x^{20} + \dots$$



- $B(x)^9/(1-B(x))^4 = x^9 + 13x^{10} + 104x^{11} + 654x^{12} + 3548x^{13} + 17395x^{14} + 79206x^{15} + 340861x^{16} + 1403056x^{17} + 5571416x^{18} + 21478460x^{19} + 80776807x^{20} + \dots$
- $(B(x)^{10}/(1-B(x))^5 + B(x^2)^5(1+B(x))/(1-B(x^2))^3)/2 = x^{10} + 8x^{11} + 72x^{12} + 475x^{13} + 2818x^{14} + 14866x^{15} + 72837x^{16} + 334843x^{17} + 1466743x^{18} + 6169407x^{19} + 25103434x^{20} + \dots$
- $B(x)^7/(1-B(x))^2 = x^7 + 9x^8 + 54x^9 + 266x^{10} + 1167x^{11} + 4736x^{12} + 18187x^{13} + 67016x^{14} + 239220x^{15} + 832739x^{16} + 2840747x^{17} + 9531354x^{18} + 31542437x^{19} + 103182945x^{20} + \dots$
- $B(x)^8/(1-B(x))^3 = x^8 + 11x^9 + 77x^{10} + 432x^{11} + 2121x^{12} + 9512x^{13} + 39959x^{14} + 159766x^{15} + 614594x^{16} + 2292276x^{17} + 8336478x^{18} + 29689260x^{19} + 103887195x^{20} + \dots$
- $B(x)^8/(1-B(x))^3 = x^8 + 11x^9 + 77x^{10} + 432x^{11} + 2121x^{12} + 9512x^{13} + 39959x^{14} + 159766x^{15} + 614594x^{16} + 2292276x^{17} + 8336478x^{18} + 29689260x^{19} + 103887195x^{20} + \dots$
- $B(x)^9/(1-B(x))^4 = x^9 + 13x^{10} + 104x^{11} + 654x^{12} + 3548x^{13} + 17395x^{14} + 79206x^{15} + 340861x^{16} + 1403056x^{17} + 5571416x^{18} + 21478460x^{19} + 80776807x^{20} + \dots$
- $B(x)^7/(1-B(x)) = x^7 + 8x^8 + 44x^9 + 201x^{10} + 825x^{11} + 3150x^{12} + 11435x^{13} + 39981x^{14} + 135856x^{15} + 451449x^{16} + 1473773x^{17} + 4742598x^{18} + 15083315x^{19} + 47505924x^{20} + \dots$
- $B(x)^8/(1-B(x))^2 = x^8 + 10x^9 + 65x^{10} + 342x^{11} + 1586x^{12} + 6752x^{13} + 27035x^{14} + 103364x^{15} + 381290x^{16} + 1366974x^{17} + 4788756x^{18} + 16459122x^{19} + 55677021x^{20} + \dots$
- $B(x)^8/(1-B(x))^2 = x^8 + 10x^9 + 65x^{10} + 342x^{11} + 1586x^{12} + 6752x^{13} + 27035x^{14} + 103364x^{15} + 381290x^{16} + 1366974x^{17} + 4788756x^{18} + 16459122x^{19} + 55677021x^{20} + \dots$
- $B(x)^9/(1-B(x))^3 = x^9 + 12x^{10} + 90x^{11} + 535x^{12} + 2760x^{13} + 12924x^{14} + 56402x^{15} + 233304x^{16} + 925302x^{17} + 3547722x^{18} + 13230138x^{19} + 48210174x^{20} + \dots$



$$(B(x)^7/(1-B(x)) + B(x^2)^3(B(x) + B(x^2))/(1-B(x^2))) / 2 = x^7 + 5x^8 + 25x^9 + 107x^{10} + 428x^{11} + 1610x^{12} + 5796x^{13} + 20174x^{14} + 68345x^{15} + 226733x^{16} + 739270x^{17} + 2377250x^{18} + 7556355x^{19} + 23790687x^{20} + \dots$$

$$B(x)^8/(1-B(x))^2 = x^8 + 10x^9 + 65x^{10} + 342x^{11} + 1586x^{12} + 6752x^{13} + 27035x^{14} + 103364x^{15} + 381290x^{16} + 1366974x^{17} + 4788756x^{18} + 16459122x^{19} + 55677021x^{20} + \dots$$

$$(B(x)^9/(1-B(x))^3 + B(x^2)^4(B(x) + B(x^2))/(1-B(x^2))^2) / 2 = x^9 + 7x^{10} + 49x^{11} + 276x^{12} + 1404x^{13} + 6515x^{14} + 28332x^{15} + 116951x^{16} + 463359x^{17} + 1775531x^{18} + 6619033x^{19} + 24114750x^{20} + \dots$$

$$B(x)^7/(1-B(x)) = x^7 + 8x^8 + 44x^9 + 201x^{10} + 825x^{11} + 3150x^{12} + 11435x^{13} + 39981x^{14} + 135856x^{15} + 451449x^{16} + 1473773x^{17} + 4742598x^{18} + 15083315x^{19} + 47505924x^{20} + \dots$$

$$B(x)^8/(1-B(x))^2 = x^8 + 10x^9 + 65x^{10} + 342x^{11} + 1586x^{12} + 6752x^{13} + 27035x^{14} + 103364x^{15} + 381290x^{16} + 1366974x^{17} + 4788756x^{18} + 16459122x^{19} + 55677021x^{20} + \dots$$

$$B(x)^8/(1-B(x))^2 = x^8 + 10x^9 + 65x^{10} + 342x^{11} + 1586x^{12} + 6752x^{13} + 27035x^{14} + 103364x^{15} + 381290x^{16} + 1366974x^{17} + 4788756x^{18} + 16459122x^{19} + 55677021x^{20} + \dots$$

$$B(x)^9/(1-B(x))^3 = x^9 + 12x^{10} + 90x^{11} + 535x^{12} + 2760x^{13} + 12924x^{14} + 56402x^{15} + 233304x^{16} + 925302x^{17} + 3547722x^{18} + 13230138x^{19} + 48210174x^{20} + \dots$$

$$B(x)^5 (B(x)^2 + B(x^2)) / (2(1-B(x))) = x^7 + 7x^8 + 36x^9 + 156x^{10} + 616x^{11} + 2279x^{12} + 8066x^{13} + 27610x^{14} + 92166x^{15} + 301655x^{16} + 972014x^{17} + 3092752x^{18} + 9739434x^{19} + 30409345x^{20} + \dots$$

$$B(x)^8/(1-B(x))^2 = x^8 + 10x^9 + 65x^{10} + 342x^{11} + 1586x^{12} + 6752x^{13} + 27035x^{14} + 103364x^{15} + 381290x^{16} + 1366974x^{17} + 4788756x^{18} + 16459122x^{19} + 55677021x^{20} + \dots$$

$$B(x)^6 (B(x)^2 + B(x^2)) / (2(1-B(x))^2) = x^8 + 9x^9 + 55x^{10} + 276x^{11} + 1234x^{12} + 5099x^{13} + 19919x^{14} + 74586x^{15} + 270277x^{16} + 954169x^{17} + 3298025x^{18} + 11202300x^{19} + 37500193x^{20} + \dots$$

$$B(x)^5 \left(B(x)^4 / (1 - B(x))^2 + B(x^2)^2 / (1 - B(x^2)) \right) / (2((1 - B(x)))$$

$$= x^9 + 9x^{10} + 60x^{11} + 329x^{12} + 1614x^{13} + 7292x^{14} + 31033x^{15}$$

$$+ 125996x^{16} + 492803x^{17} + 1869377x^{18} + 6913598x^{19}$$

$$+ 25027878x^{20} + \dots$$

$$B(x)^9 / (1 - B(x))^3 = x^9 + 12x^{10} + 90x^{11} + 535x^{12} + 2760x^{13}$$

$$+ 12924x^{14} + 56402x^{15} + 233304x^{16} + 925302x^{17} + 3547722x^{18}$$

$$+ 13230138x^{19} + 48210174x^{20} + \dots$$

$$B(x)^6 \left(B(x)^4 / (1 - B(x))^2 + B(x^2)^2 / (1 - B(x^2)) \right) / (2((1 - B(x))^2)$$

$$= x^{10} + 11x^{11} + 83x^{12} + 507x^{13} + 2724x^{14} + 13355x^{15} + 61181x^{16}$$

$$+ 265804x^{17} + 1106797x^{18} + 4450771x^{19} + 17384803x^{20} + \dots$$

$$B(x)^8 / (1 - B(x))^2 = x^8 + 10x^9 + 65x^{10} + 342x^{11} + 1586x^{12}$$

$$+ 6752x^{13} + 27035x^{14} + 103364x^{15} + 381290x^{16} + 1366974x^{17}$$

$$+ 4788756x^{18} + 16459122x^{19} + 55677021x^{20} + \dots$$

$$(B(x)^9 / (1 - B(x))^3 + B(x^2)^4(B(x) + B(x^2)) / (1 - B(x^2))^2) / 2$$

$$= x^9 + 7x^{10} + 49x^{11} + 276x^{12} + 1404x^{13} + 6515x^{14} + 28332x^{15}$$

$$+ 116951x^{16} + 463359x^{17} + 1775531x^{18} + 6619033x^{19}$$

$$+ 24114750x^{20} + \dots$$

$$B(x)^7 / (1 - B(x)) = x^7 + 8x^8 + 44x^9 + 201x^{10} + 825x^{11} + 3150x^{12}$$

$$+ 11435x^{13} + 39981x^{14} + 135856x^{15} + 451449x^{16} + 1473773x^{17}$$

$$+ 4742598x^{18} + 15083315x^{19} + 47505924x^{20} + \dots$$

$$B(x)^8 / (1 - B(x))^2 = x^8 + 10x^9 + 65x^{10} + 342x^{11} + 1586x^{12}$$

$$+ 6752x^{13} + 27035x^{14} + 103364x^{15} + 381290x^{16} + 1366974x^{17}$$

$$+ 4788756x^{18} + 16459122x^{19} + 55677021x^{20} + \dots$$

$$B(x)^8 / (1 - B(x))^2 = x^8 + 10x^9 + 65x^{10} + 342x^{11} + 1586x^{12}$$

$$+ 6752x^{13} + 27035x^{14} + 103364x^{15} + 381290x^{16} + 1366974x^{17}$$

$$+ 4788756x^{18} + 16459122x^{19} + 55677021x^{20} + \dots$$

$$B(x)^9 / (1 - B(x))^3 = x^9 + 12x^{10} + 90x^{11} + 535x^{12} + 2760x^{13}$$

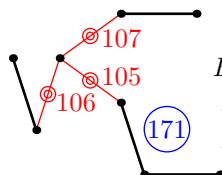
$$+ 12924x^{14} + 56402x^{15} + 233304x^{16} + 925302x^{17} + 3547722x^{18}$$

$$+ 13230138x^{19} + 48210174x^{20} + \dots$$

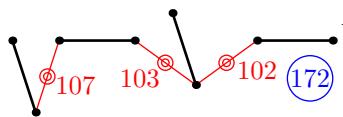
$$B(x)^8 / (1 - B(x))^3 = x^8 + 11x^9 + 77x^{10} + 432x^{11} + 2121x^{12}$$

$$+ 9512x^{13} + 39959x^{14} + 159766x^{15} + 614594x^{16} + 2292276x^{17}$$

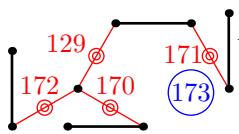
$$+ 8336478x^{18} + 29689260x^{19} + 103887195x^{20} + \dots$$



$$B(x)^8/(1-B(x))^3 = x^8 + 11x^9 + 77x^{10} + 432x^{11} + 2121x^{12} \\ + 9512x^{13} + 39959x^{14} + 159766x^{15} + 614594x^{16} + 2292276x^{17} \\ + 8336478x^{18} + 29689260x^{19} + 103887195x^{20} + \dots$$



$$B(x)^8/(1-B(x))^3 = x^8 + 11x^9 + 77x^{10} + 432x^{11} + 2121x^{12} \\ + 9512x^{13} + 39959x^{14} + 159766x^{15} + 614594x^{16} + 2292276x^{17} \\ + 8336478x^{18} + 29689260x^{19} + 103887195x^{20} + \dots$$



$$B(x)^9/(1-B(x))^4 = x^9 + 13x^{10} + 104x^{11} + 654x^{12} + 3548x^{13} \\ + 17395x^{14} + 79206x^{15} + 340861x^{16} + 1403056x^{17} + 5571416x^{18} \\ + 21478460x^{19} + 80776807x^{20} + \dots$$