What the terms of A349664 can tell about n.

Number of Solutions	distinct n	Factorizations for n	and/or numbers of the form
0	1	[1]	-
1	2^1	[2]	-
2	-	[aop]	A065091
3	2^2	-	-
4	-	[aop^2]	A001248(>1)
5	2^3	-	-
6	-	[aop^3]	A030078(>1)
7	2^4	or [2 * aop]	A100484(>1)
8	-	[aop^4]	A030514(>1)
9	2^5	-	-
10	-	[aop^5]	A050997(>1)
11	2^6	-	-
12	-	[aop^6] or [aop * adop]	A030516(>1) or A046388
13	2^7	or [2 * aop^2]	A143928
14	-	[aop^7]	A086874
15	2^8	-	-
16	-	[aop^8]	A179645(>1)
17	2^9	or [2 * 2 * aop]	A001749(>1)
18	-	[aop^9]	A179665(>1)
19	2^10	or [2 * aop^3]	A172190(>1)
20	-	[aop^10]	A030629(>1)
21	2^11	-	-
22	-	[aop^11] or [aop^2 * adop]	A079395(>1)
23	2^12	-	-
24	-	[aop^12]	A030631(>1)
25	2^13	or [2 * aop^4]	-
26	-	[aop^13]	A138031(>1)
27	2^14	or [2^3 * aop]	A001749(>1) * 2
28	-	[aop^14]	-
29	2^15	-	-
30	-	[aop^15]	-
31	2^16	or [2^2 * aop^2] or [2 * aop^5]	A069262(>1)
32	-	[aop^16] or [aop^3 * adop]	A030635(>1)

33	2^17	-	-
34	-	[aop^17]	A138032(>1)
35	2^18	-	-
36	-	[aop^18]	A030637(>1)
37	2^19	or [2^4 * aop] or [2 * aop * adop]	A075819
38	-	[aop^19]	-
39	2^20	-	-
40	-	[aop^20] or [aop^2 * adop^2]	-
41	2^21	-	-
42	-	[aop^21] or [aop^4 * adop]	-
43	2^22	or [2 * aop^7]	-
44	-	[aop^22]	-
45	2^23	or [2^2 * aop^3]	-
46	-	[aop^23]	-
47	2^24	or [2^5 * aop]	-
48	-	[aop^24]	-
49	2^25	or [2^3 * aop^2] or [2 * aop^8]	-
:	:	:	-

green:	=	The number of solutions corresponds to one distinct category or one distinct n.
red:	=	The number of solutions corresponds to more than one distinct category or distinct n.
aop	=	any odd prime (primes > 2)
adop	=	any different odd prime (primes > 2 and adop != aop)
addop	=	any double different odd prime (primes > 2 and addop != adop != aop)

Note: There is a similar table in https://oeis.org/wiki/Index_entries_for_number_of_divisors

This table here helps to categorize any Number.
The goal is, that noone must create categories.
The number gives the category through an algorithm by itself.
This helps maybe, to get closer in solving e.g. the Collatz Conjecture.
If we could show, how whole categories behave by using the Collatz rules,
then it is prooven. ;-)

Some first rules, I found:
e.g we have a number of the form [2*aop^4], which is in category 25.
We all know: this number can be divided by two.
But now do the following: (25-1)/3 = 8 and category 8 is [aop^4]. Nice.
There are a few more rules, I found. But more in future.

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