

OEIS A342500

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ABSTRACT. This manuscript illustrates the connected unlabeled Hasse diagrams complete up to $V = 6$ elements /nodes refined according to the length of the longest chain [2, A000608,A342500].

We surround each diagram with a solid frame.

Each diagram is followed by 2 integer parameters and an “invariant”:

- R is the number of order relations [1, §1.16] related to the graph. It is the number of arcs in the Hasse diagram, E , plus the number of arcs (possibly zero) that are added if the transitive closure of the incidence matrix is computed, plus the number of nodes, V , to comply with the reflexive cases (1’s on the diagonal of the incidence matrix).
- A is the order of the automorphism group (the number of permutations of rows and columns that keep the incidence matrix fixed). This is an aid to enumerate the labeled diagrams.
- The invariant is Gilbert’s composition of V listing the number of elements on level 1,2,... [2, A000608]. [The level of an element with indegree zero is 1, and the level of any other element is 1 plus the largest level of the elements with incident arcs.]

1 DIAGRAM ON $V = 1$ NODE



1 DIAGRAM ON $V = 2$ NODES

1 diagram with rank 1. $R=3$ $A=1$ 11

3 DIAGRAMS ON $V = 3$ NODES

2 diagrams with rank 1. $R=5$ $A=2$ 12 $R=5$ $A=2$ 21





1 diagram with rank 2. $R=6$ $A=1$ 111

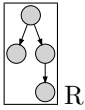
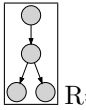
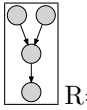
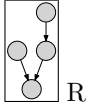
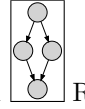
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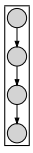
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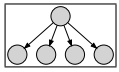
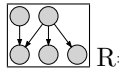

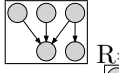
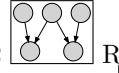
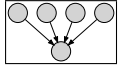
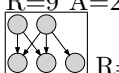
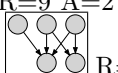
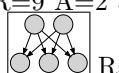
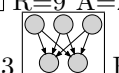
10 DIAGRAMS ON $V = 4$ NODES

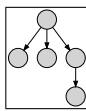
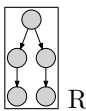
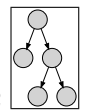

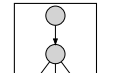
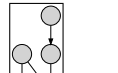
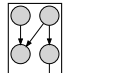
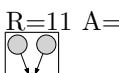
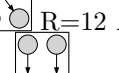
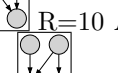
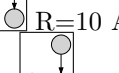
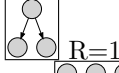
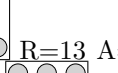
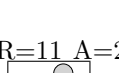
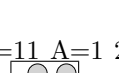
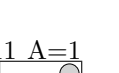




4 diagrams with rank 1.  $R=7$ $A=6$ 13  $R=7$ $A=1$ 22  $R=7$
 $A=6$ 31  $R=8$ $A=4$ 22

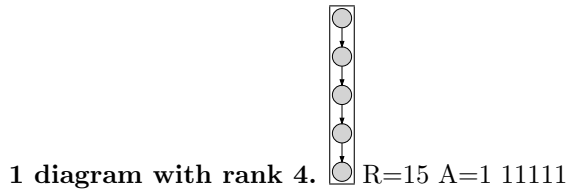
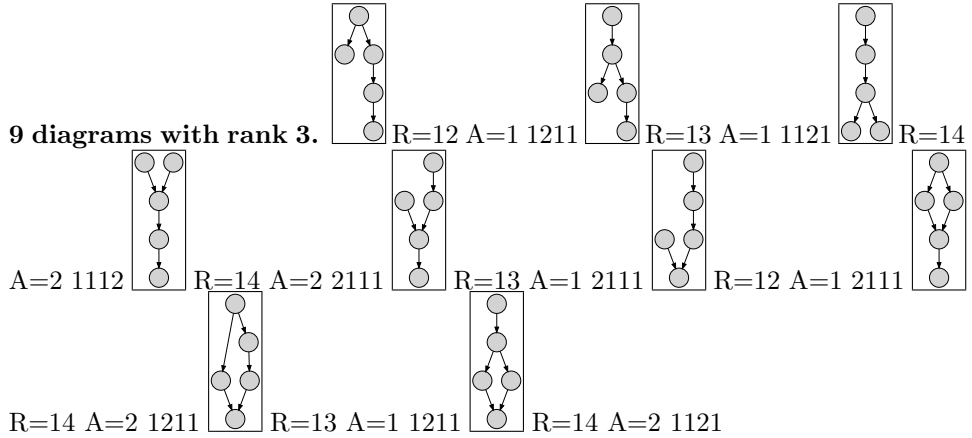
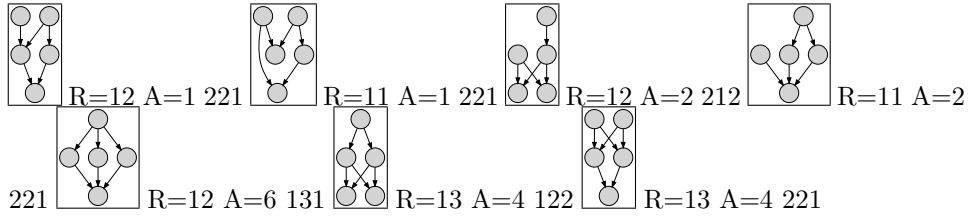
5 diagrams with rank 2.  $R=8$ $A=1$ 121  $R=9$ $A=2$ 112  $R=9$ $A=2$
 $R=8$ $A=1$ 211  $R=9$ $A=2$ 121

1 diagram with rank 3.  $R=10$ $A=1$ 1111

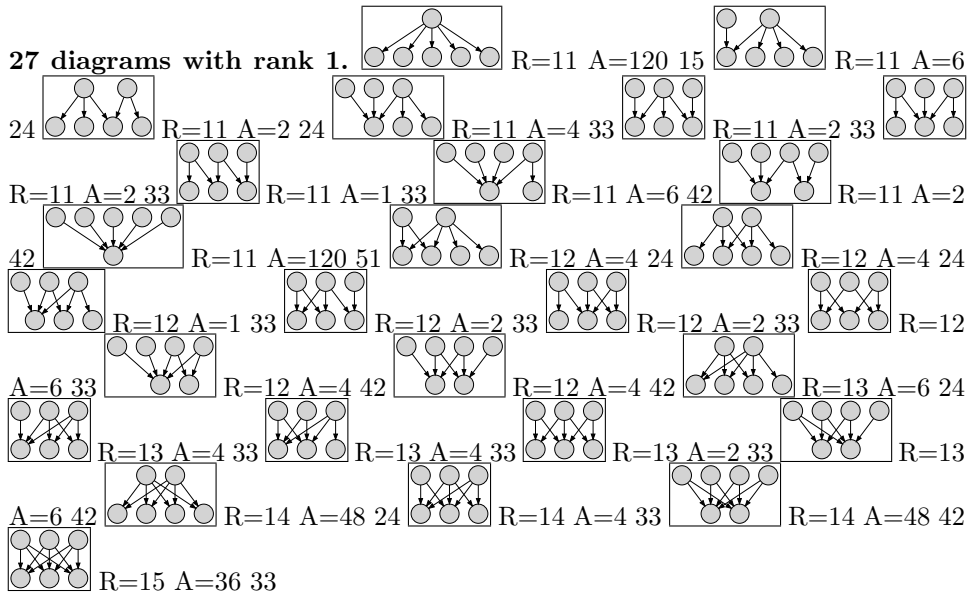
44 DIAGRAMS ON $V = 5$ NODES

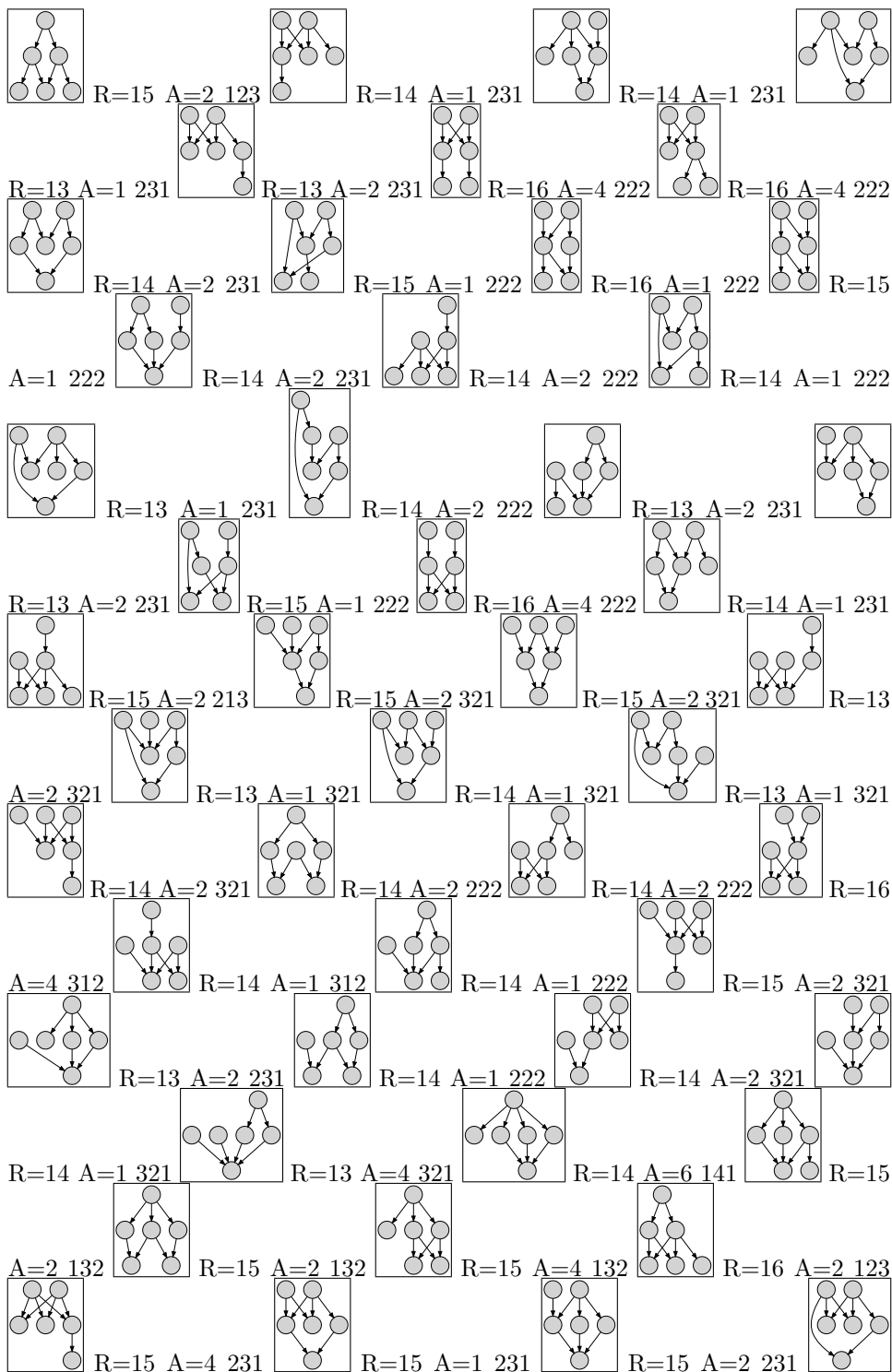
10 diagrams with rank 1.  $R=9$ $A=24$ 14  $R=9$ $A=2$ 23  $R=9$ $A=2$ 23
 $R=9$ $A=2$ 32  $R=9$ $A=2$ 32  $R=9$ $A=24$ 41
 $R=10$ $A=2$ 23  $R=10$ $A=2$ 32  $R=11$ $A=12$ 23  $R=11$
 $A=12$ 32

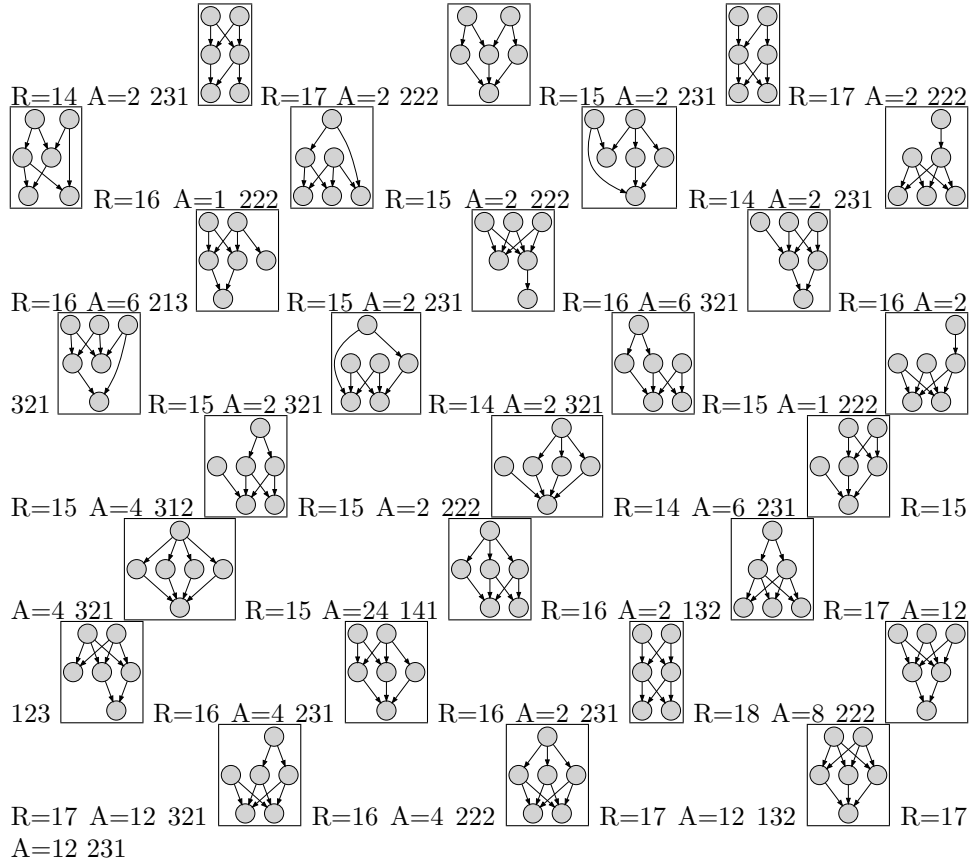
24 diagrams with rank 2.  $R=10$ $A=2$ 131  $R=11$ $A=2$ 122  $R=11$ $A=1$ 221
 $R=11$ $A=2$ 122  $R=12$ $A=6$ 113  $R=10$ $A=1$ 221  $R=10$ $A=1$ 221
 $R=13$ $A=4$ 212  $R=11$ $A=2$ 221  $R=11$ $A=1$ 221  $R=11$ $A=1$
 212  $R=12$ $A=6$ 311  $R=10$ $A=1$ 221  $R=11$ $A=2$ 311  $R=11$ $A=1$
 $R=10$ $A=2$ 311  $R=11$ $A=2$ 131  $R=12$ $A=1$ 122  $R=12$ $A=2$ 221



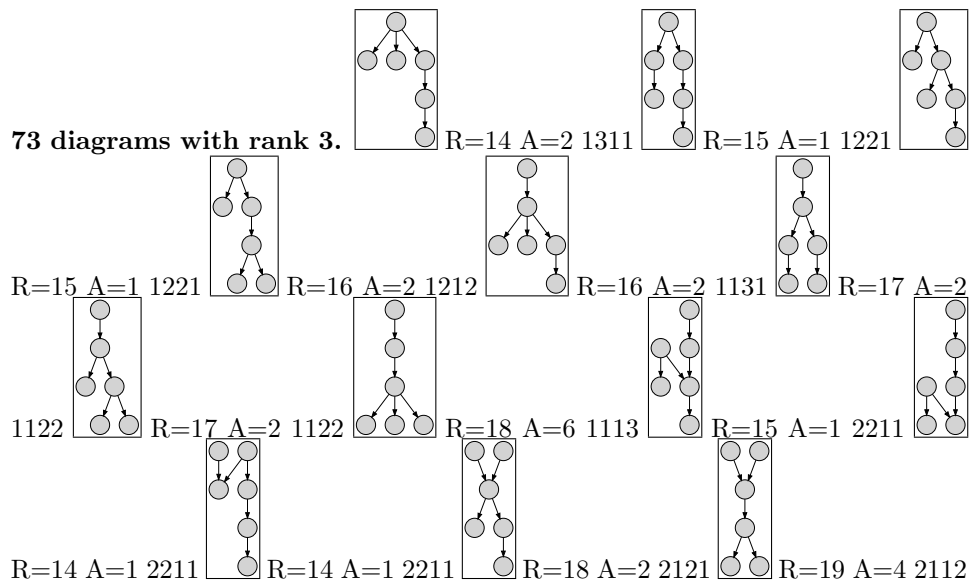
238 DIAGRAMS ON $V = 6$ NODES

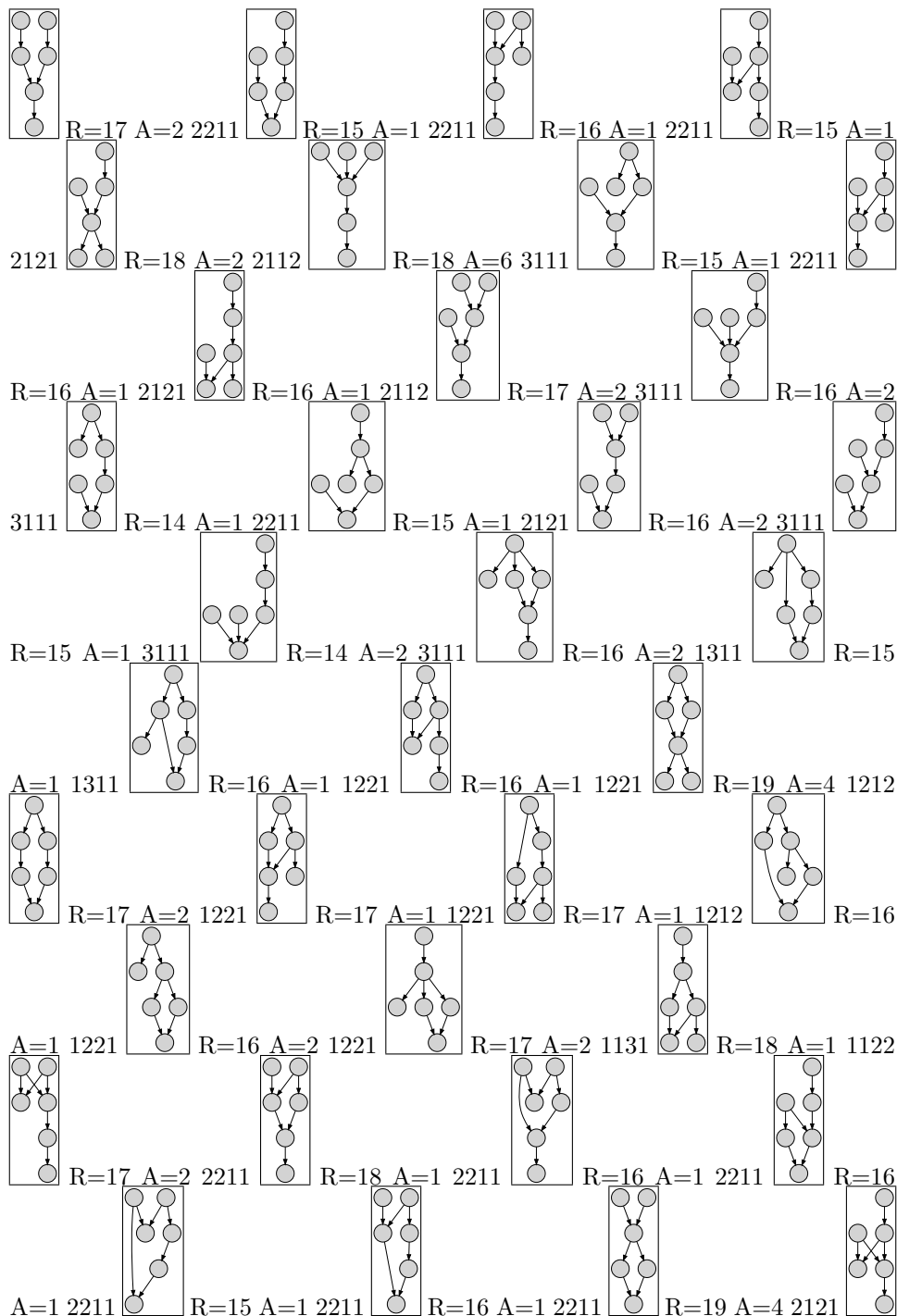


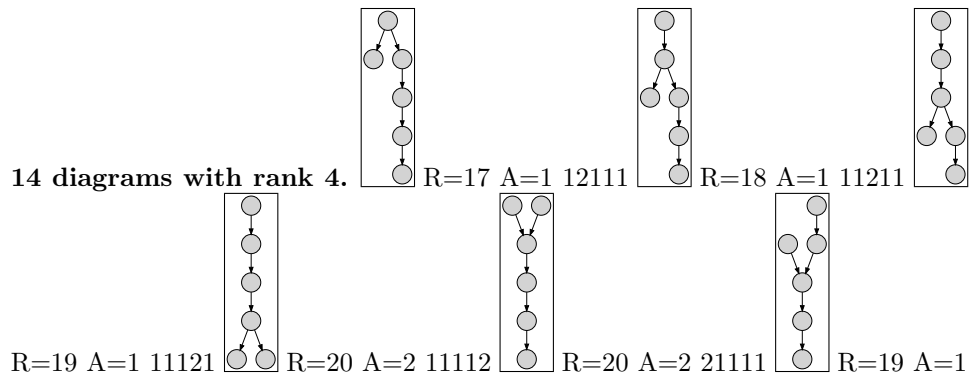
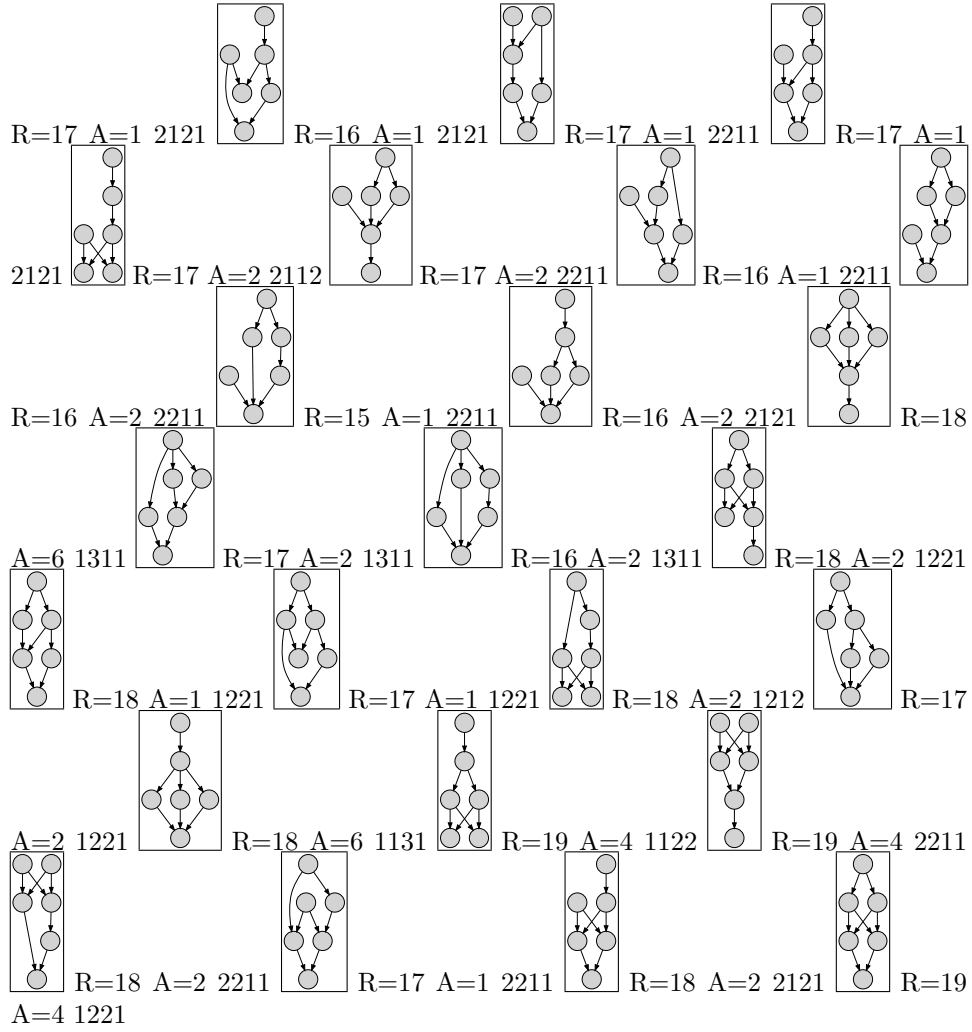


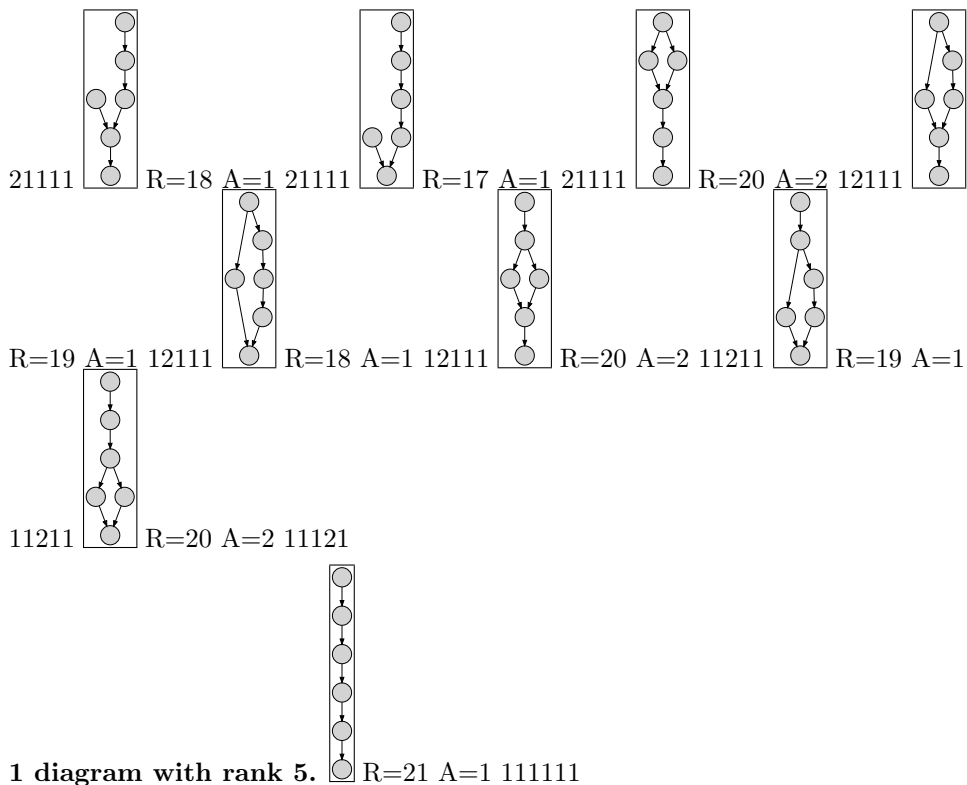


73 diagrams with rank 3.









1 diagram with rank 5.

REFERENCES

1. Louis Comtet, *Advanced combinatorics*, Springer, Berlin, Heidelberg, New York, 1974.
2. O. E. I. S. Foundation Inc., *The On-Line Encyclopedia Of Integer Sequences*, (2021), <https://oeis.org/>. MR 3822822
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