Rodolfo Kurchan's Square Puzzle #577

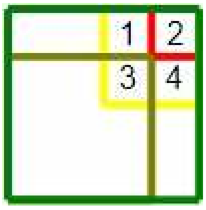
Complete list of maximal solutions for up to 15 squares,
computed by Hermann Jurksch and Hugo Pfoertner.
Illustrations by Rainer Rosenthal. (Autumn 2020)

Introduction

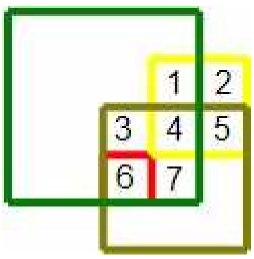
In 1997 R.K. wrote (https://www.puzzlefun.online/puzzle-fun-18):

> Using N different squares (from 1x1 to NxN)
> form the largest quantity of 1x1 squares.
>
> 577) The largest squares contains the smaller ones.
> 578) The smaller squares can be placed anywhere.

My best solutions for 577) are:
1x1 = 1, 2x2 = 1, 3x3 = 2, 4x4 = 4, 5x5 = 8, 6x6 = 10, 7x7 = 15.



My best solutions for 578) are:
1x1 = 1, 2x2 = 1, 3x3 = 4, 4x4 = 7, 5x5 = 12, 6x6 = 17, 7x7 = 23.



In 2020 R. K. added to the Online Encyclopedia of Integer sequences (https://oeis.org/OEIS):

A336659: maximal number of 1x1 cells for N squares (puzzle #577)
A336660: maximal number of 1x1 cells for N squares (puzzle #578)

(We write "cell" instead of "square" to avoid confusion.)

New results

In the following we restrict ourselves to puzzle #577.
Sequence A336659 has entries for N = 1 to 15:

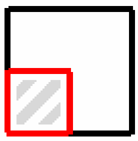
1, 1, 2, 4, 8, 10, 15, 22, 28, 34, 41, 52, 60, 70, 83

Sequence A336782 gives the number of best configurations:

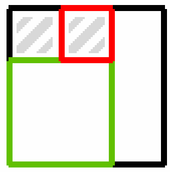
1, 1, 1, 1, 1, 16, 5, 1, 1, 4, 12, 8, 24, 2, 1

Here is the list of illustrations for the best configurations.
(See the catalogue of coordinates https://oeis.org/A336782/a336782.txt)

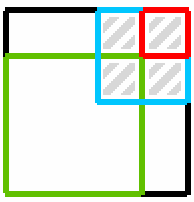
1 squares 1 cells
V1: unique



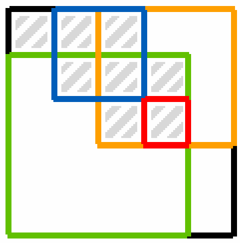
2 squares 1 cells
V1: unique



3 squares 2 cells
V1: unique

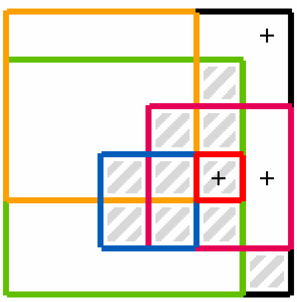


4 squares 4 cells
V1: unique



5 squares 8 cells
V1: unique

Some configurations have the same list of larger squares, but differ only in the position of the smallest square. We call them "1-siblings" and show only one of them in full. The possible locations are marked with a little cross (+):

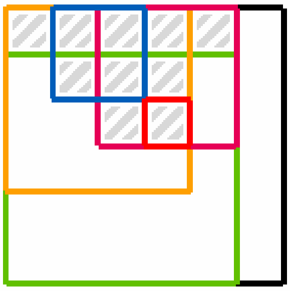


6 squares 10 cells
V2: 3 1-siblings

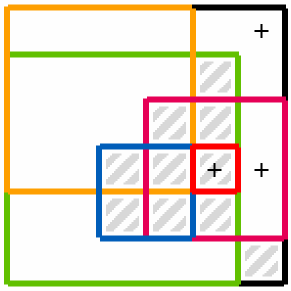
Example: for 6 squares (full list see below) there are three 1-siblings for variation V2 as shown on the left.
The catalogue has (x1,x2) = (4,2), (5,2) and (5,5):

n = 6, 16 solutions, 1X1 cells: 10

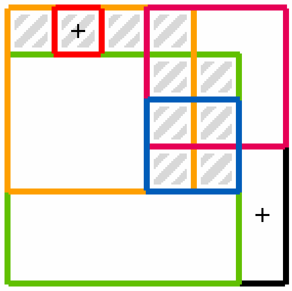
x6	y6	x5	y5	x4	y4	x3	y3	x2	y2	x1	y1	
0	0	0	0	0	2	2	3	1	4	3	3	← V1
0	0	0	0	0	2	3	1	2	1	4	2	} V2
..	..	..	..	..	..	..	..	..	..	5	2	
..	..	..	..	..	..	..	..	..	..	5	5	



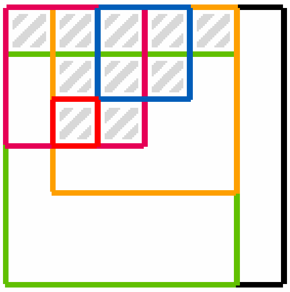
6 squares 10 cells
V1: unique



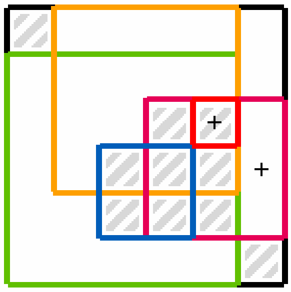
6 squares 10 cells
V2: 3 1-siblings



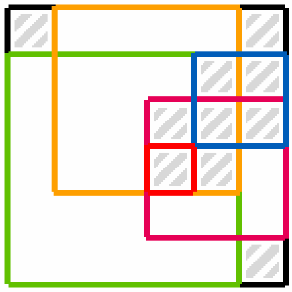
6 squares 10 cells
V3: 2 1-siblings



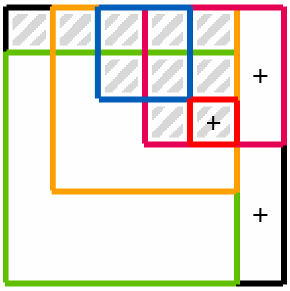
6 squares 10 cells
V4: unique



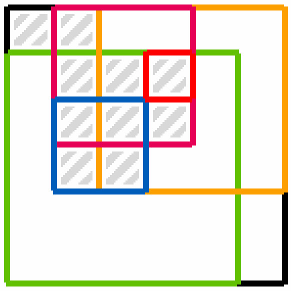
6 squares 10 cells
V5: 2 1-siblings



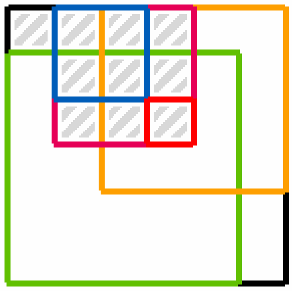
6 squares 10 cells
V6: unique



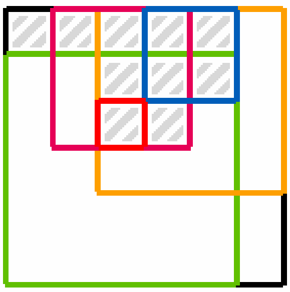
6 squares 10 cells
V7: 3 1-siblings



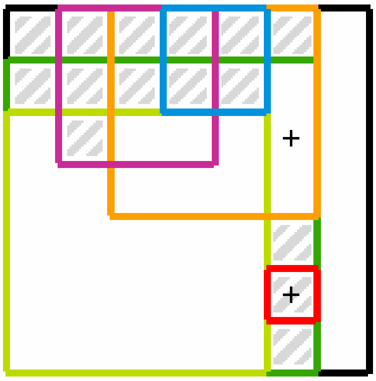
6 squares 10 cells
V8: unique



6 squares 10 cells
V9: unique

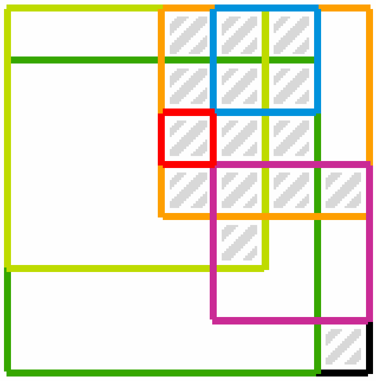


6 squares 10 cells
V10: unique



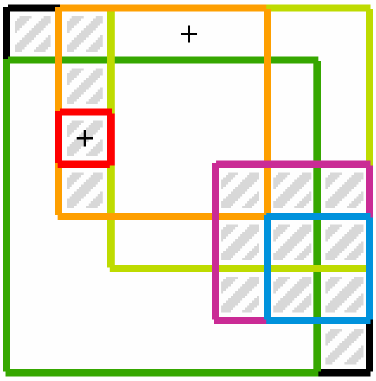
7 squares 15 cells

V1: 2 1-siblings



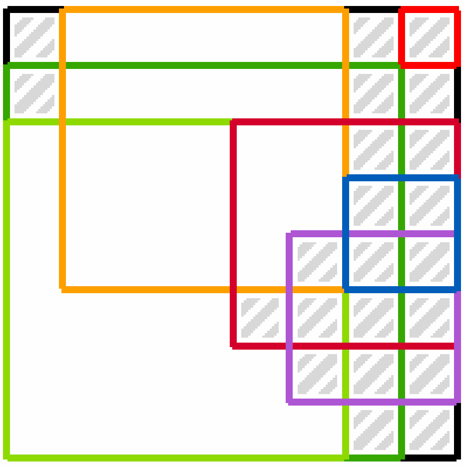
7 squares 15 cells

V2: unique



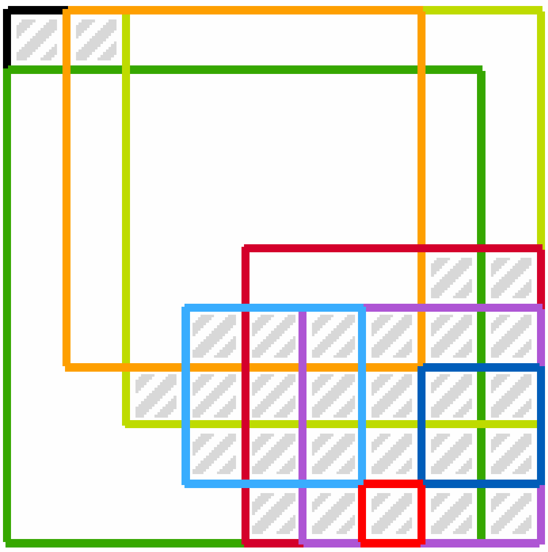
7 squares 15 cells

V3: 2 1-siblings



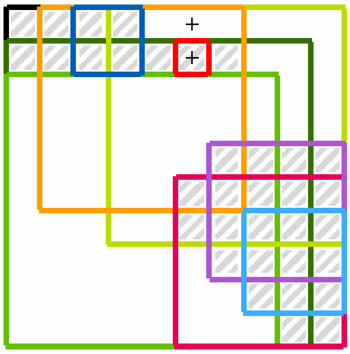
8 squares 22 cells

V1: unique



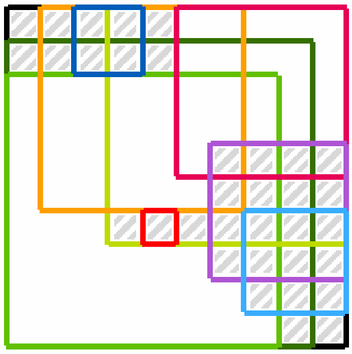
9 squares 28 cells

V1: unique



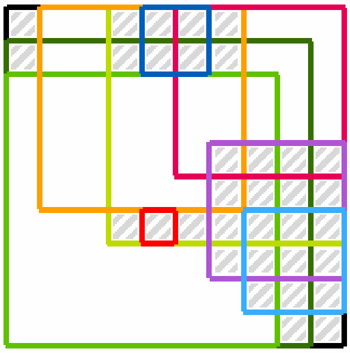
10 squares 34 cells

V1: 2 1-siblings



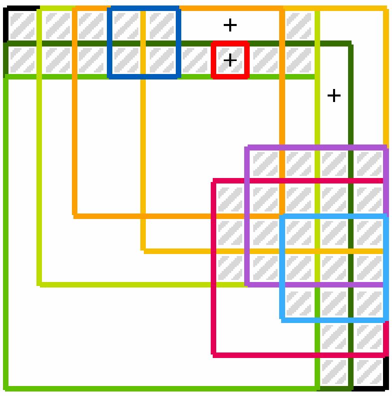
10 squares 34 cells

V2: unique



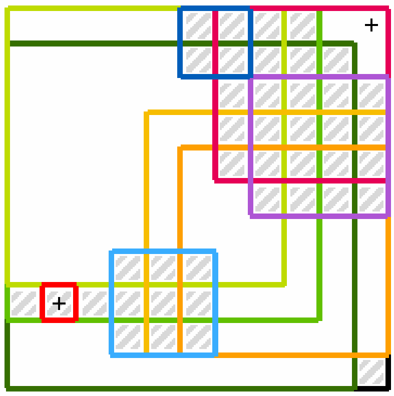
10 squares 34 cells

V3: unique



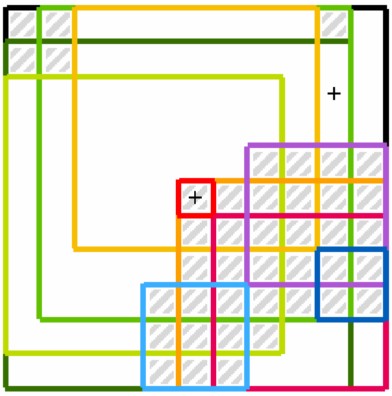
11 squares 41 cells

V1: 3 1-siblings



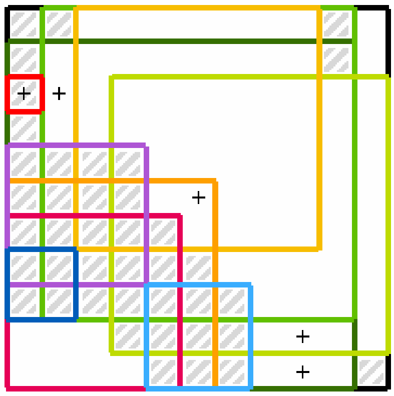
11 squares 41 cells

V2: 2 1-siblings



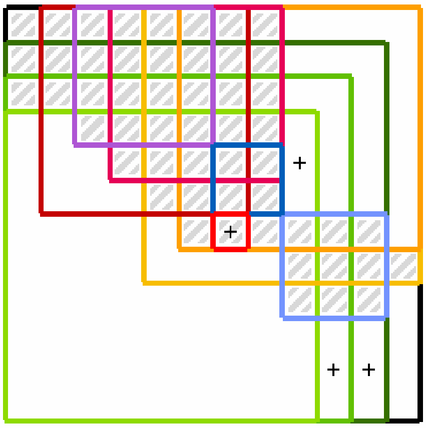
11 squares 41 cells

V3: 2 1-siblings



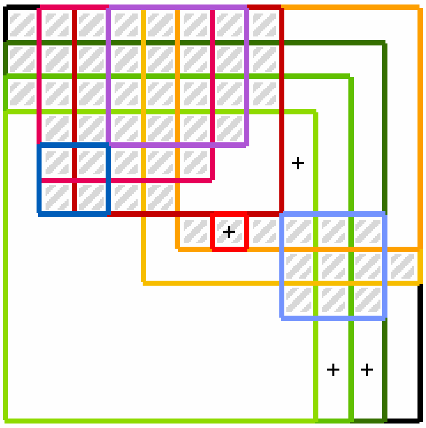
11 squares 41 cells

V4: 5 1-siblings



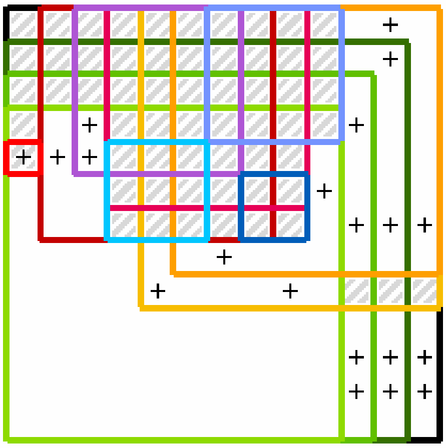
12 squares 52 cells

V1: 4 1-siblings



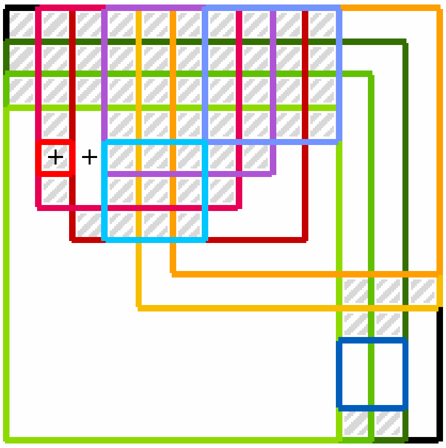
12 squares 52 cells

V2: 4 1-siblings



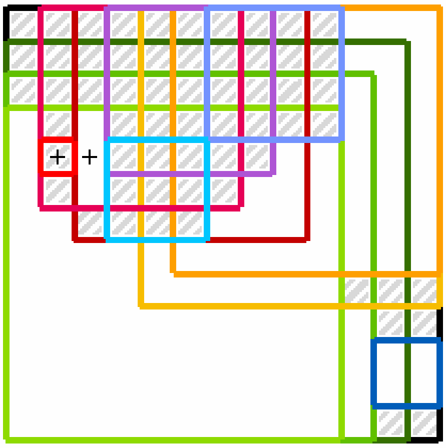
13 squares 60 cells

V1: 20 1-siblings



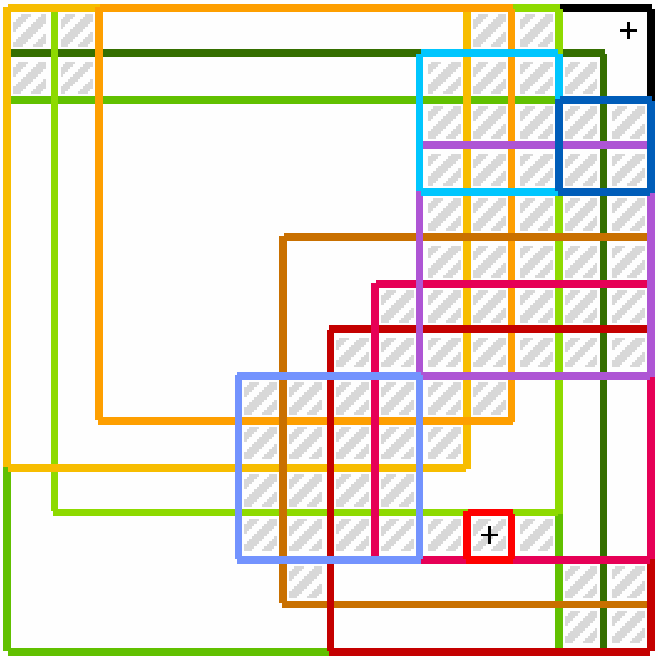
13 squares 60 cells

V2: 2 1-siblings



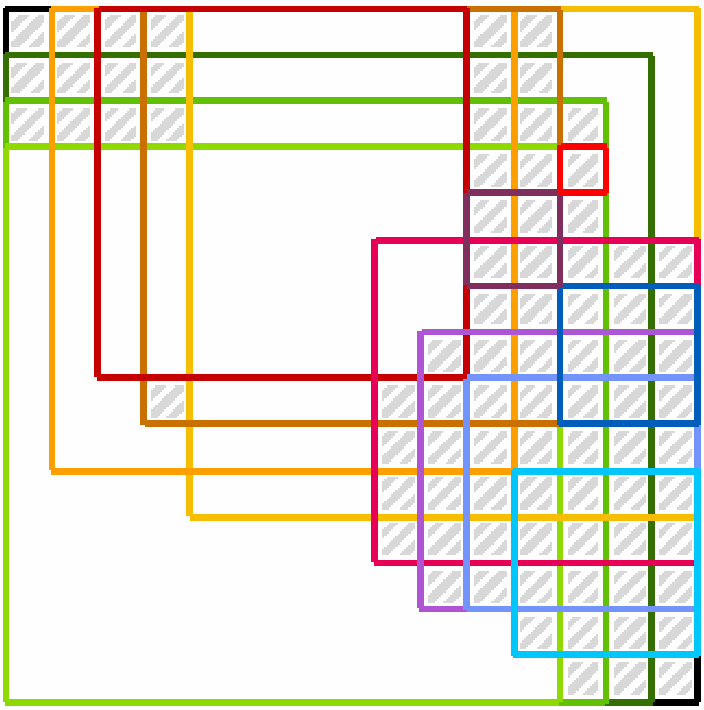
13 squares 60 cells

V3: 2 1-siblings



14 squares 70 cells

V1: 2 1-siblings



15 squares 83 cells

V1: unique

Rainer Rosenthal, Überlingen 2020-10-06, with many thanks to Hugo Pfoertner.