Wolfdieter Lang, May 31 2018

a(n,k) tabf head (staircase) for A305309, for n = 1..10.

The elementwise product of the composition numbers A048996(n, k) and A118851(n, k), the parts of the k-th partitions of n in Abramowitz-Stegun (A-St) order (pp. 831-2 of their handbook).

The row number is n, and m = m(n,k) = A036043(n, k) is the number of parts of the k-th partition of n.

The length of row n is p(n) = A000041(n), the number of partitions of n.

n\k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	2020	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42
1:	1																																									
2:	2	1																																								
3:	3	4	1																																							
4:	4	6	4	6	1																																					
5:	5	8	12	9	12	8	1																																			
6:	6	10	16	9	12	36	8	12	24	10	1																															
7:	7	12	20	24	15	48	27	36	16	72	32	15	40	12	1																											
8:	8	14	24	30	16	18	60	72	48	54	20	96	54	144	16	20	120	80	18	60	14	1																				
9:	9	16	28	36	40	21	72	90	48	60	144	27	24	120	144	192	216	96	25	160	90	360	80	24	180	160	21	84	16	1												
10:	10	18	32	42	48	25	24	84	108	120	72	180	96	108	28	144	180	96	240	576	108	128	216	30	200	240	480	540	480	32	30	240	135	720	240	28	252	280	24	112	18	1
...																																										
n\k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42

The sequence of row lengths is A000041: [1, 2, 3, 5, 7, 11, 15, 22, 30, 42,...] (partition numbers).

The row sums give A001906(n) = Fibonacci(2*n) = [1, 3, 8, 21, 55, 144, 377, 987, 2584, 6765, …].

The triangle obtained by summing the entries with like part numbers m becomes A078812(n, m) = binomial(n+m-1, 2*m-1)

(with offsets n >= 1, m = 1..n).

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E.g. a(5, 4) refers to the fourth partition of n = 5 in this ordering, namely (1^2,3^1) = [1,1,3],

hence a(5,4) = 3*3 = 9 because A048996(5, 4) = 3!/(2!*1!) = 3 and A118851(5, 4).= 3.

a(6, 5) = 3*4 = 12 for the partition (1^2,4^1) = [1,1,4] of n = 6.

a(7, 10) = (4!/(2!*1!*1!))***(1***1*2*3) = 12*6 = 72 from the 10th partition of n = 7 which is

(1^2,2^1,3^1) = [1,1,2,3] with m = 4.

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Example for the set partition of [n]:= {1,2, ..., n} into m = m(n, k) blocks of consecutive numbers corresponding
to a composition obtained from the k-th partition of n with m parts, and the counting of the product of the block lengths:
n=5, k=4, with the partition (1^2,3^1), where m = 2 + 1 = 3, and one of the A04896(5, 4) = 3 compositions is, e.g.,.
1 + 3 + 1. This is mapped (bijectively) to the set partition of [5] with blocks of consecutive numbers {1},{2,3,4},{5}.
Each of the other 2 set partitions has the same block length pattern, and therefore the three set partitions have the
same product of their block lengths, namely 1*1*3 = 3 (from the signature of the underlying partition). Therefore the
entry a(5, 4) = 3*3 = 9 counts the sum of product of the block lengths (or the product of the parts of the partition)
for all 3-block partitions of [5] with consecutive numbers, belonging to the k=4 partition of n = 5, via the
corresponding three part compositions of 5.

e.o.f.