# Ripple Numbers

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## 1 Introduction

A ripple number can be thought of as a rectangular wave, which covers a seed or its preceding wave fully in all directions.

For example, take the number 3, and represent it with 3 hashes (#). Then the ripple waves for 3 look as follows.

```
%%%%%%%%%
%&&&&&&&%
%&@@@@@&%
%&@###@&%
%&@@@@@&%
%&&&&&&&%
%%%%%%%%%
```

It can be observed that there are 12@, 20&, 28% which are covering each other.

So, if number 3 is considered as the **seed**, then the first ripple is containing 12 elements, then next ripple 20, and the next one 28 elements and so on. The first ripple that is formed on top of the seed value can be called as **base ripple**, as it forms the basis for subsequent ripples.

# 2 Formulation of Ripples

The following sections derive formulas for the base ripple, and its succeeding ripples.

## 2.1 Base Ripple

For any seed **s** $>$ **0**, the base ripple **b** consists of **2s+6** elements.

The proof goes as follows:

Let's take **s** number of seed elements, and arrange them horizontally in a single row. Then it forms a matrix with **1 row** and **s columns**.

If those elements are to be fully covered horizontally and vertically, the count of rows and columns have to be increased by 2, forming a matrix with **(1+2) rows** and **(s+2) columns.**

So the total number of elements in the matrix are, **(1+2)*(s+2) = 3(s+2) = 3s+6**.

But out of these **3s+6** elements, **s** elements are to be deducted, as they belong to the seed.

Then the total number of elements in the base ripple will become, **3s+6 - s = 2s+6**.

## 2.2 Subsequent Ripples

Once the base ripple is formed on top of the seed elements, to add any subsequent ripples, it would take only 8 more elements than the previous ripple.

The proof goes as follows:

From the previous section, it is known that for any seed with **s** elements, the base ripple is **formed over** (1+2) rows and (s+2) columns. If another ripple has to be formed on top of the base ripple, it will be formed over **(1+2+2) rows and (s+2+2) columns**, making it a total of **5(s+4)** elements.

But, out of these 5(s+4) elements, 2s+6 elements belong to the base ripple and s elements belong to the seed, which are to be deducted.

So, **5(s+4) - ([2s+6] + s) = 5(s+4) - (3s+6) = 5s - 3s + 20 - 6 = 2s+14**.

So, the ripple above the base ripple contains a total of **2s+14** elements.

If we subtract it from the base ripple, it becomes

**(2s+14) - (2s+6) = 14 - 6 = 8**, which is a constant value.

So, after the base ripple, each subsequent ripple will be formed with 8 additional elements than the previous ripple, and can be expressed as a formula as shown below.

Ripple **r(i) = b+8*i**, where **b** is the number of elements in base ripple, and **i** is the set of positive integers starting from 1.

## 2.3 Summary of Formulae

To summarise the formulae that are derived so far,

1) For any seed s > 0, the value of Base Ripple is calculated as, **b = 2s+6**

2) The values of Subsequent Ripples are calculated as, **r(i) = b+8i,** where **b** is the value of base ripple, and **i** is the set of positive integers starting from 1.

### 2.4 Ripple Number Series

Following are the ripple number series (first 11 terms) for the seed values 1 to 9, based on the formulae derived so far.

{1, 8, 16, 24, 32, 40, 48, 56, 64, 72, 80}
{2, 10, 18, 26, 34, 42, 50, 58, 66, 74, 82}
{3, 12, 20, 28, 36, 44, 52, 60, 68, 76, 84}
{4, 14, 22, 30, 38, 46, 54, 62, 70, 78, 86}
{5, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88}
{6, 18, 26, 34, 42, 50, 58, 66, 74, 82, 90}
{7, 20, 28, 36, 44, 52, 60, 68, 76, 84, 92}
{8, 22, 30, 38, 46, 54, 62, 70, 78, 86, 94}
{9, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96}

It can be observed that the ripple waves will form squares instead of rectangles, if and only if, the seed value is 1. When seed is 1, base ripple is formed over 3 rows and 3 columns, next ripple over 5 rows and 5 columns etc.

And when the seed value is 2, it generates the uniform sequence n+8, for any preceding value of n.

## 3 Sum Series

Starting with the seed value, if we add each term of the sequence recursively, it forms the sum series as follows.

{09, 25, 49, 81, 121, 169, 225, 289, 361, 441}
{12, 30, 56, 90, 132, 182, 240, 306, 380, 462}
{15, 35, 63, 99, 143, 195, 255, 323, 399, 483}
{18, 40, 70, 108, 154, 208, 270, 340, 418, 504}
{21, 45, 77, 117, 165, 221, 285, 357, 437, 525}
{24, 50, 84, 126, 176, 234, 300, 374, 456, 546}

It can be observed that the sum series is forming squares of odd numbers, for the sequence with seed value 1.

And also it can be observed that the whole series is forming the odd number tables vertically (3 table, 5 table, 7 table etc.)

So, the sum series has got the special property of generating all odd number tables, without any need to perform multiplication.

## 3.1 Formulation of the Sum Series

The reason for sum series forming odd number tables is, the series generate composite numbers of the form **(1+x)*(s+x)** where x is the set of even numbers starting from 2. As x is always even, (1+x) will always be odd, and so generates odd number tables only.

The proof for it is partially provided in Sections 2.1 and 2.2 already. Here goes the full proof.

It is shown in Section 2.1 that, for a seed with s elements, the base ripple and seed together form a matrix of **(1+2) rows** and **(s+2) columns**. If the next ripple is added, it forms a matrix with **(1+2+2) rows and (s+2+2) columns**, which is nothing but **(1+4) rows and (s+4) columns**. If we add the next ripple on top of it, it forms the matrix with **(1+6) rows and (s+6) columns** etc.

So by theory of induction, the sum series form composite numbers of the form **(1+x)*(s+x)** where x is the set of even numbers starting from 2.

# 4 Prime Numbers and Ripple Numbers

As the sum series is forming odd number tables, it can be used for sieving prime numbers. The following Java program generates all prime numbers starting from 3, for any n > 3.

## 4.1 Sieve Algorithm

```
/*
 * This Java program generates all odd primes below n, for any n >3
 */
/**
 * @author Rayan Ivaturi
 */
public class RippleSieve {
    public static void main(String[] args) {
        final int n = 100;
        boolean[] primes = new boolean[n];
        int limit = (n - 6) / 3;
        for (int seed = 1; seed <= limit; seed += 2) {
            int ripple = 2 * seed + 6;
            int sum = seed + ripple;
            while (sum < n) {
                primes[sum] = true;
                ripple += 8;
                sum += ripple;
            }
        }

        for (int i = 3; i < n; i += 2) {
            if (!primes[i]) {
                System.out.println(i);
            }
        }
```

```
    }
}
```

The computational efficiency of Ripple Sieve exactly matches with Sieve of Eratosthenes, and checks exactly the same number of composites. But Ripple Sieve will perform one more extra addition for getting the ripple term in each iteration, making it little slower than Eratosthenes Sieve for generating prime numbers.

## 4.2 Notes

While the Ripple Sieve is not as efficient as Eratosthenes, but it's time complexity almost matches with another very elegant, but very less known prime sieving algorithm developed by the Indian Mathematician, S. P. Sundaram. Please follow the links below to know more about it.

https://en.wikipedia.org/wiki/Sieve_of_Sundaram
https://luckytoilet.wordpress.com/2010/04/18/the-sieve-of-sundaram