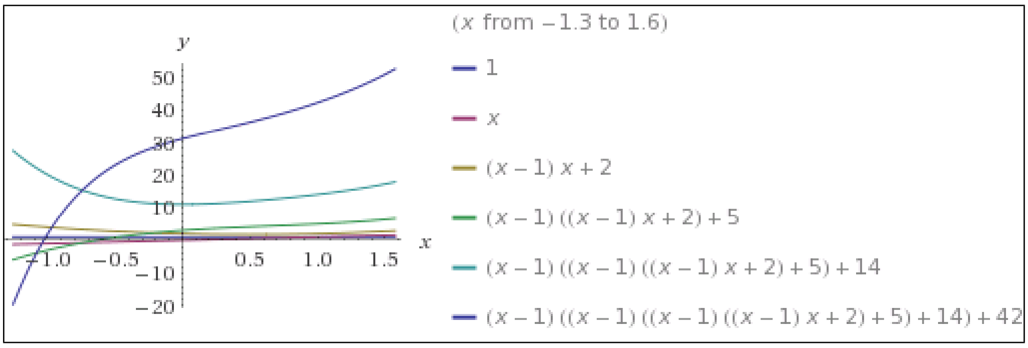
Polynomials C_n(x)

Definition and properties

$$C_n(x) = \sum_{k=0}^{n} \frac{(2k)!(x-1)^{n-k}}{(k+1)!k!}$$



$$G(x,t) = \frac{1-\sqrt{1-4t}}{2t(1+t-xt)} = 1+xt+(x^2-x+2)t^2+(x^3-2x^2+3x+3)t^3+\ldots$$

$$C_n(x) = (x-1)C_{n-1}(x) + C_n(1), C_0(x) = 1$$

$$C_n(1) = \frac{(2n)!}{(n+1)!n!}$$ (A000108)

$$C_n(2) = \sum_{m=0}^{n} C_m(1)$$ (A014137)

Example

The first few polynomials are:

$C_0(x) = 1$

$C_1(x) = x$

$C_2(x) = x^2 - x + 2$

$C_3(x) = x^3 - 2x^2 + 3x + 3$

$C_4(x) = x^4 - 3x^3 + 5x^2 + 11$

$$C_5(x) = x^5 - 4x^4 + 8x^3 - 5x^2 + 11x + 31$$

$$C_6(x) = x^6 - 5x^5 + 12x^4 - 13x^3 + 16x^2 + 20x + 101$$

$$C_7(x) = x^7 - 6x^6 + 17x^5 - 25x^4 + 29x^3 + 4x^2 + 81x + 328$$

$$C_8(x) = x^8 - 7x^7 + 23x^6 - 42x^5 + 54x^4 - 25x^3 + 77x^2 + 247x + 1102$$

$$C_9(x) = x^9 - 8x^8 + 30x^7 - 65x^6 + 96x^5 - 79x^4 + 102x^3 + 170x^2 + 855x + 3760$$

Triangle of coefficients:

$$\begin{array}{ccccccccc}
1 \\
0 & 1 \\
2 & -1 & 1 \\
3 & 3 & -2 & 1 \\
11 & 0 & 5 & -3 & 1 \\
31 & 11 & -5 & 8 & -4 & 1 \\
101 & 20 & 16 & -13 & 12 & -5 & 1 \\
328 & 81 & 4 & 29 & -25 & 17 & -6 & 1 \\
1102 & 247 & 77 & -25 & 54 & -42 & 23 & -7 & 1
\end{array}$$