Open Problems in the OEIS

Neil J A Sloane

Guest Lecture, Zeilberger Experimental Math Class, May 2 2016

- Puzzles
- Strange recurrences
- Number theory
- Counting problems

PUZZLES

61, 21, 82, 43, 3, ?

(A087409)

Low-Hanging Fruit from the OEIS

Some new problems for the ghosts of Fermat, Gauss, Euler, ...

Strange Recurrences

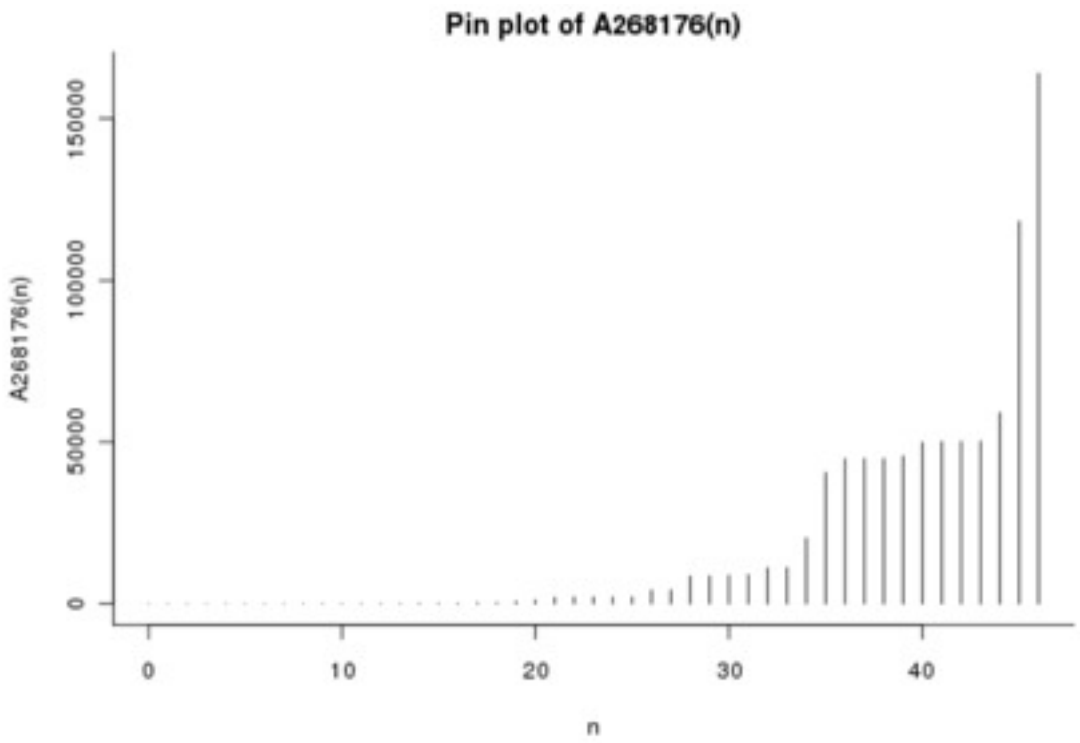
- Modified Fibonacci
- Reed Kelley
- A recurrence that looks ahead
- Van Eck's sequence

Modified Fibonacci

a(n) = a(n-1) + a((a(n-1)-1) mod n) with a(0)=a(1)=1

A268176, Christian Perfect, Jan 2016

Similar to A125204, also not analyzed



Explain!

Reed Kelley's Sequence A214551

14th century Narayana cows sequence A930:

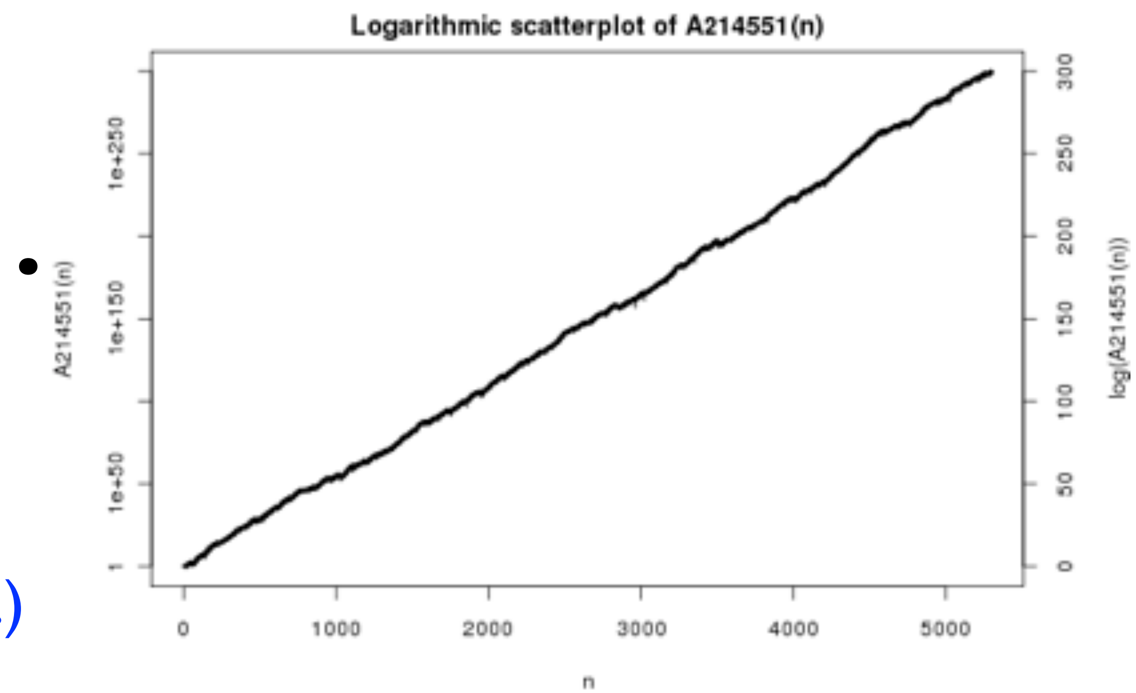
$$a(n) = a(n-1) + a(n-3)$$

1, 1, 1, 2, 3, 4, 6, 9, 13, 19, 28, ...

Reed Kelley, 2012:

$$a(n) = \frac{a(n-1) + a(n-3)}{\gcd\{a(n-1), a(n-3)\}}$$

```
1, 1, 1, 2, 3, 4, 3, 2,
3, 2, 2, 5, 7, 9, 14, 3,...
```

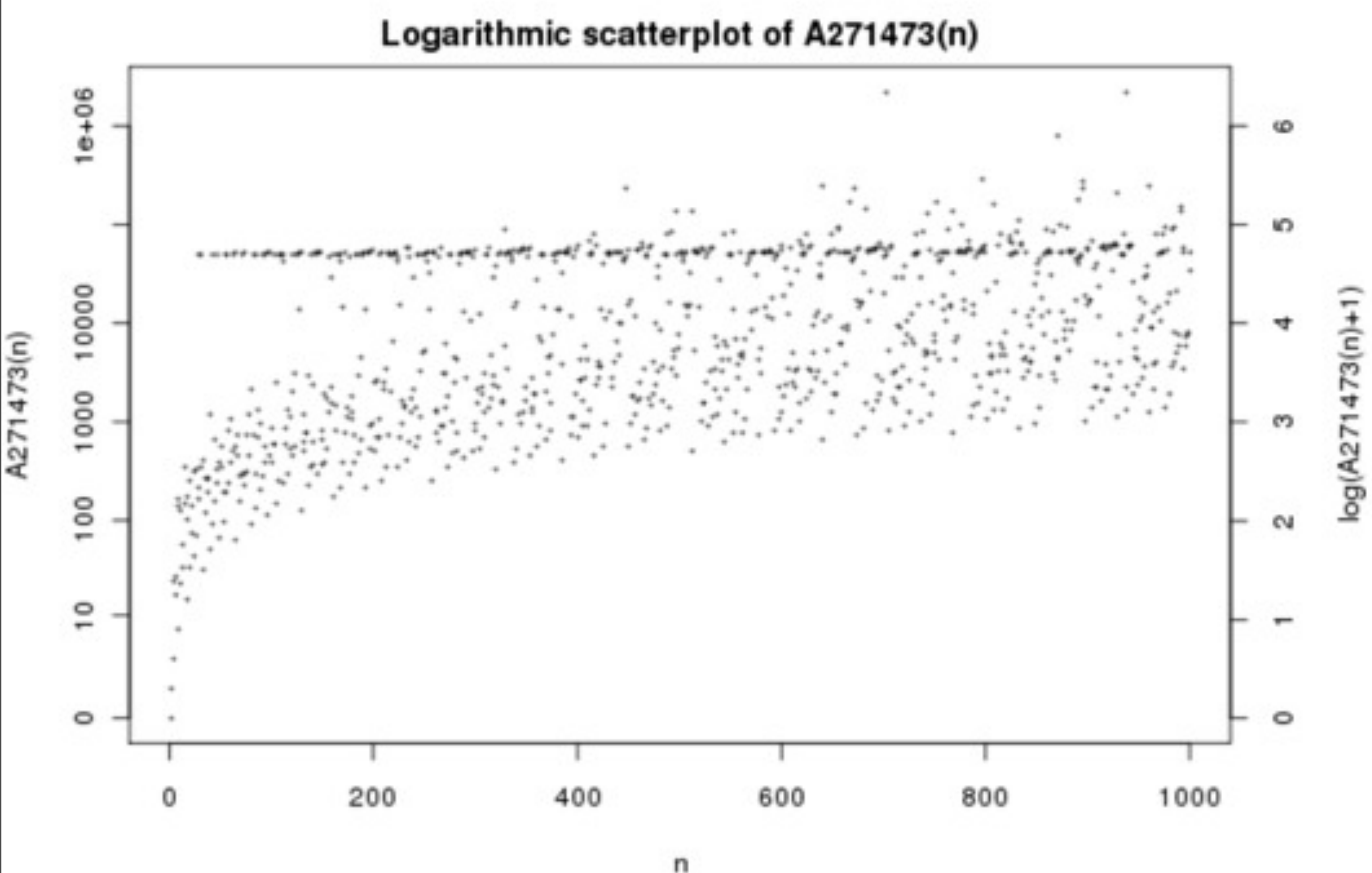


(Have guesses, but nothing is proved.)

A recurrence that looks ahead

a(2k) = k+a(k), a(2k+1) = k+a(6k+4) with a(1)=0.

A271473, suggested by 3x+1 sequence A6370
and new A266569



Apr 8 2016

Explain!

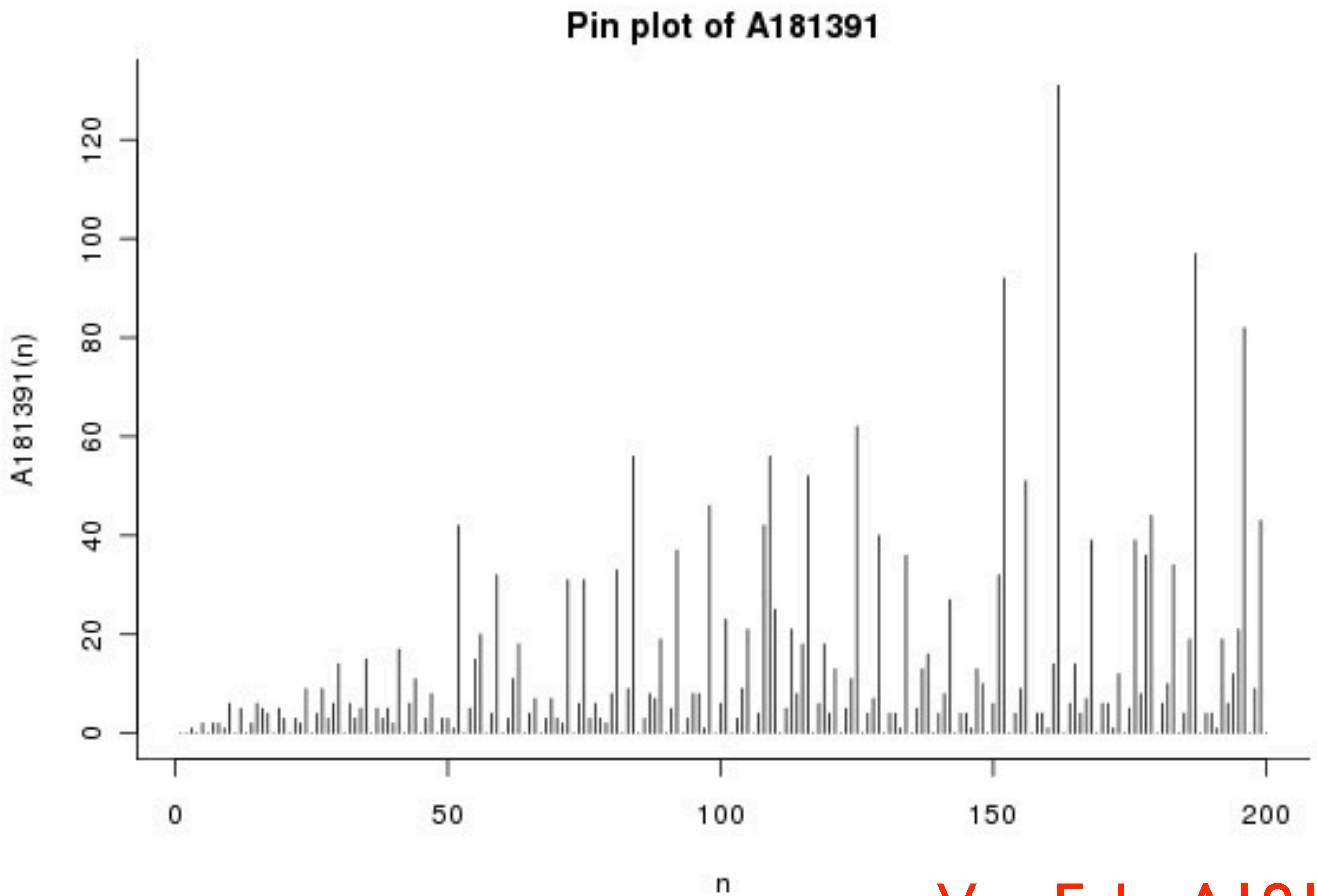
Jan Ritsema van Eck's Sequence

```
0, 0, 1, 0, 2, 0, 2, 2, 1, 6, 0, 5,
0, 2, 6, 5, 4, 0, 5, 3, 0, 3, 2, 9,
0, 4, 9, 3, 6, 14, 0, 6, 3, 5, 15, 0,
5, 3, 5, 2, 17, 0, 6, 11, 0, 3, 8, 0, ...
```

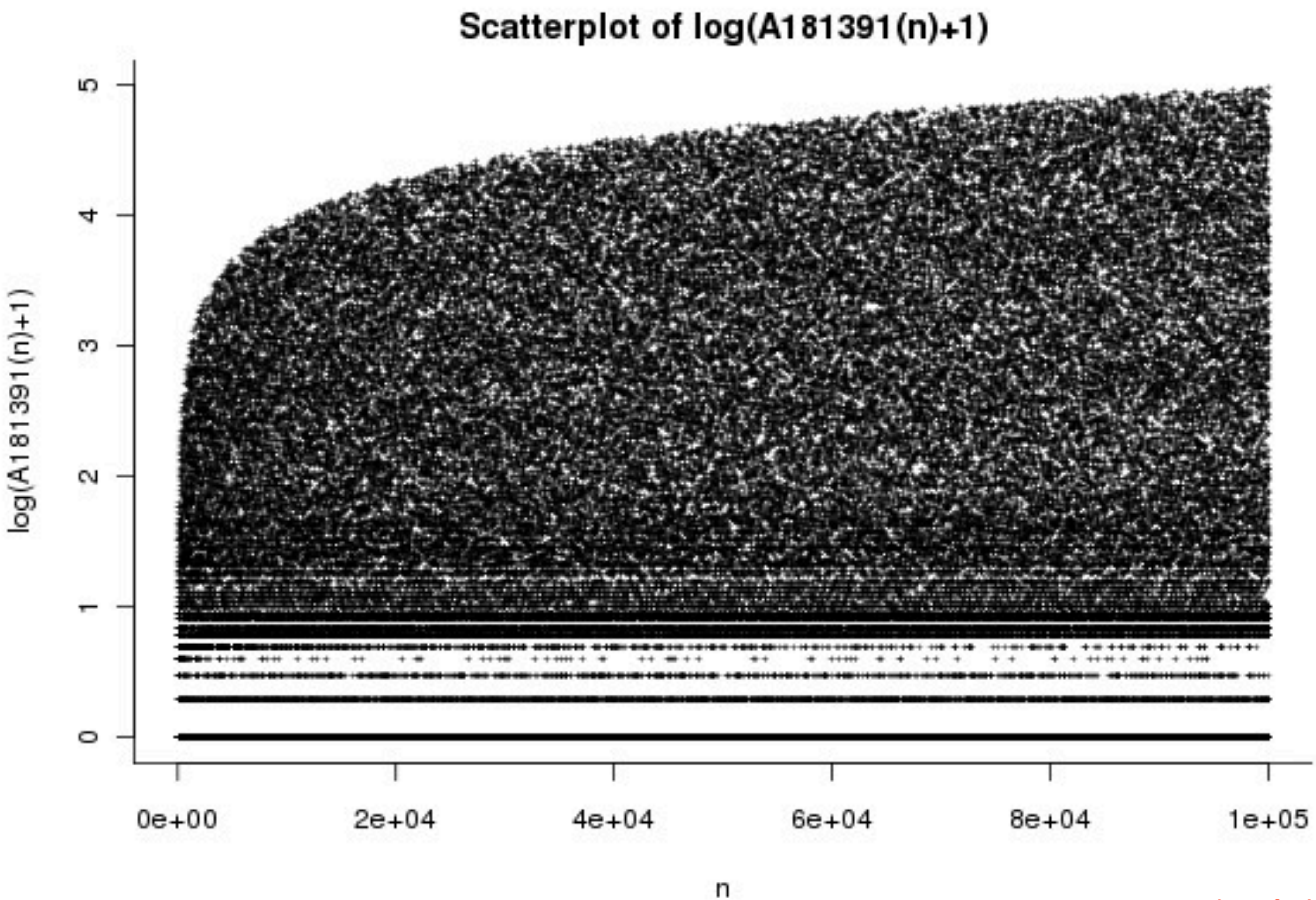
a(n): how far back did we last see a(n-1)?
or 0 if a(n-1) never appeared before.

Van Eck: A181391

A181391 as a graph:



Van Eck: A181391



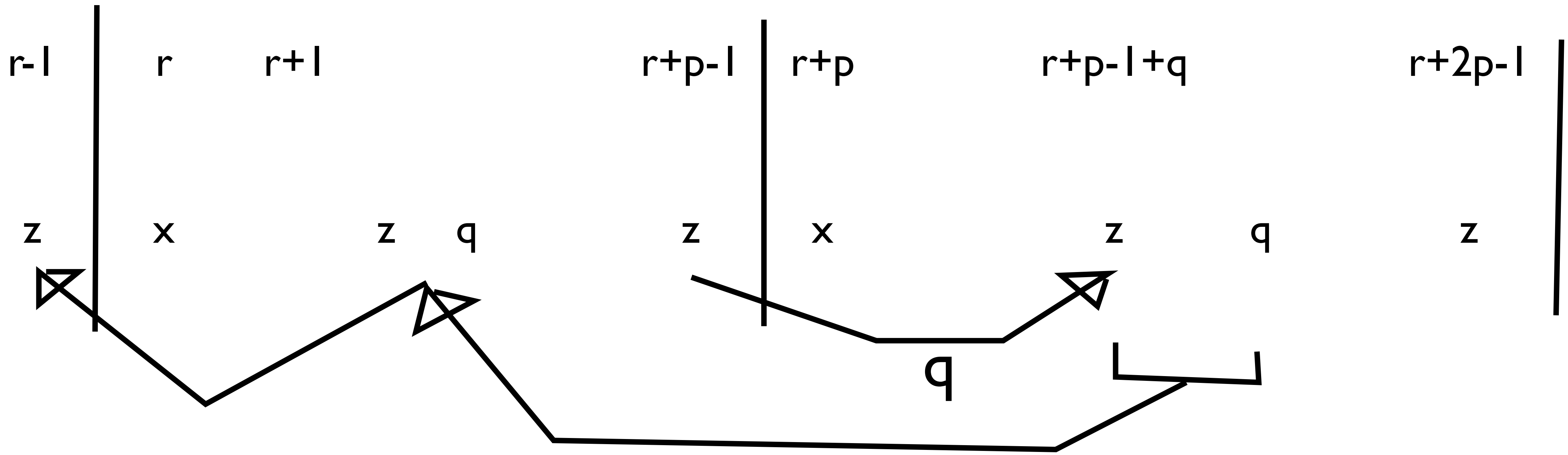
Van Eck: A181391

Thm. (Van Eck) There are infinitely many zeros.

Proof: (i) If not, no new terms, so bounded.
Let M = max term.
Any block of length M determines the sequence.
Only M^M blocks of length M.
So a block repeats.
So sequence becomes periodic.
Period contains no 0's.

Van Eck: A181391

Proof (ii). Suppose period has length p and starts at term r.



Therefore period really began at term r − 1.

....

Therefore period began at start of sequence.

But first term was 0, contradiction.

Van Eck: A181391

It seems that:

lim sup a(n) / n = 1

Gaps between 0's roughly log_10 n

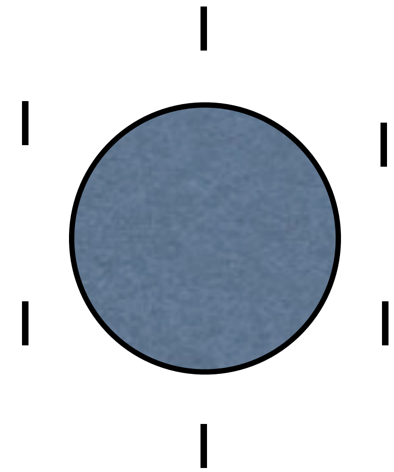
Every number eventually appears

Proofs are lacking!

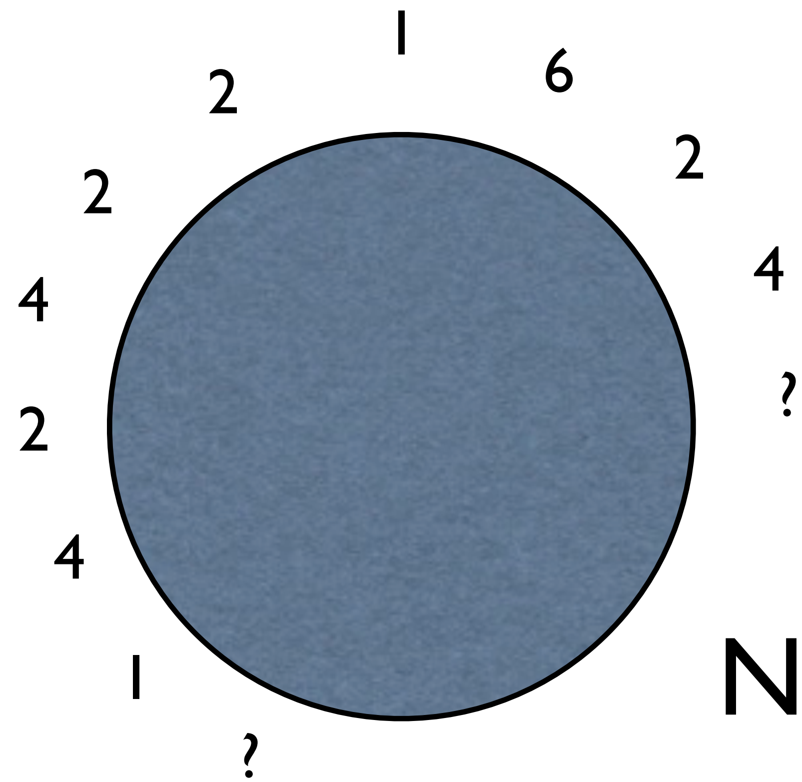
Van Eck: A181391

Van Eck: A181391

Conjecture:
There is no
nontrivial cycle



Trivial cycle



Nontrivial cycle ?

(David Applegate: Only trivial cycles of length up through 14)

Number Theory

- Sum of primes in sum of previous terms
- 3^n + 1 = square + square
- Yosemite graph
- Leroy Quet's prime-producing sequence
- 999999000000
- A memorable prime

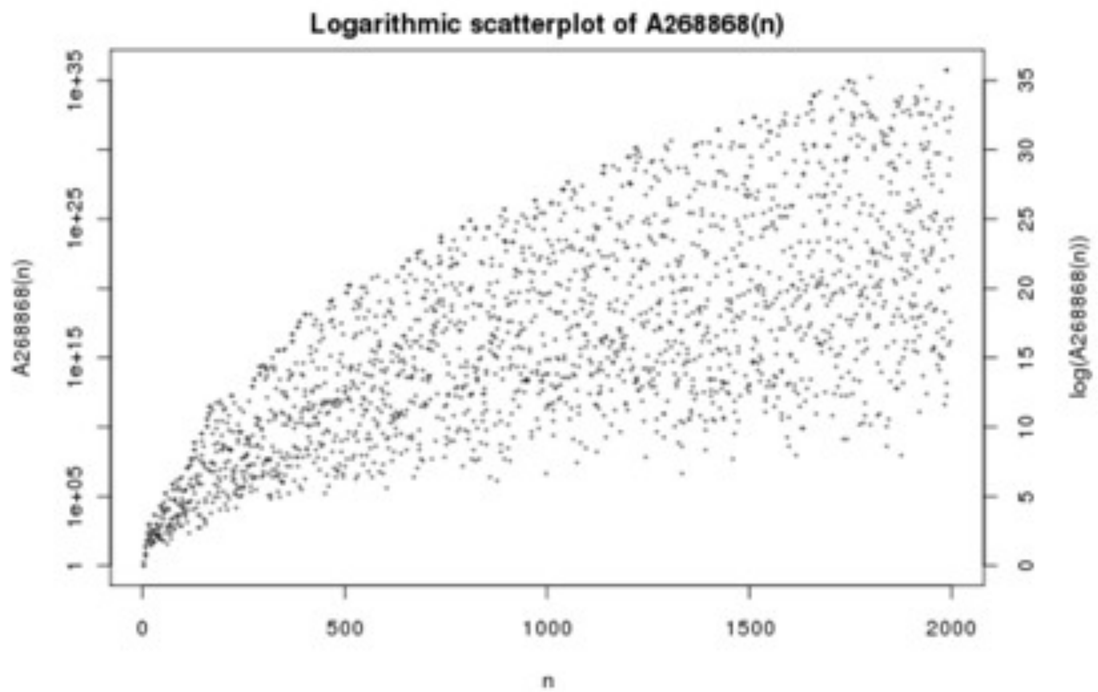
a(n) = sum of prime factors of sum of all previous terms

(with repetition, starting 1, 1)

1, 1, 2, 4, 6, 9, 23, 25, 71, 73, 48, 263, 265, 120, 911, 913, 552, 192, 85, 27, 35, 53, 296, 66, 455, 289, 48, 188, 5021, 5023, 159, 190, 379, 946, 900, 600, 97, 204, 118, 512, 87, 148, 3886, 23291, 23293, 71, 896, 11812, 60, 41359,

$$1+1+2+4+6 = 14 = 2 \times 7 \text{ gives } 2+7 = 9$$

A268868, David Sycamore, Feb 2016



Explain!

Generalize!

Odd numbers n such that 3^n + 1 is sum of 2 squares

5, 13, 65, 149, 281, 409, 421, 449, 461, 577

$$3^5 + 1 = 244 = 10^2 \ + \ 12^2$$

Found by Keenan Curtis, u/grad, Wake Forest U.
Only 10 terms known

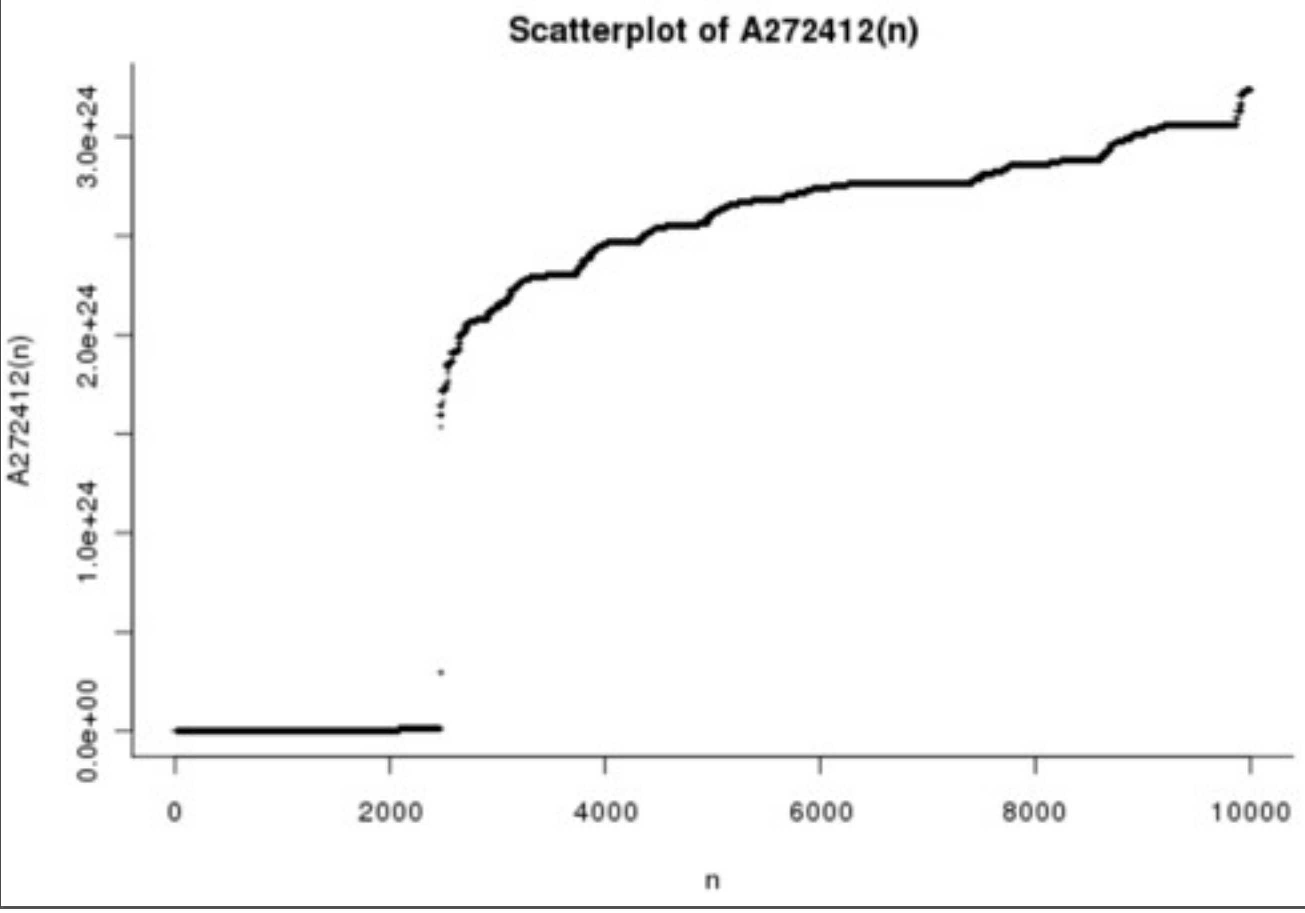
A272069 = A404 intersect A34472

April 19 2016

Yosemite Graph?? (A272412)

Numbers n such that sum of divisors (A203(n)) is a Fibonacci number (in A45)

Random combination of 2 sequences, except look at the graph:



Altug Alkan, Apr 29 2016

Have 10000 terms but need a lot more

Hostadter’s Q-sequence
A5185

Leroy Quet’s Prime-generating sequence
A134204

Franklin Adams-Watters
A166133

“About your cat, Mr. Schrödinger—I have good news and bad news.”

(The New Yorker, March 2015)

A134204

Leroy Quet's Prime-Producing Sequence

n

0	1	2	3	4	5	6	7	8	9	10
2	3	5	7	13	17	19	23	41	31	29

p q

q = smallest missing prime such that n divides p + q

10 divides 31 + 29

$$p + q = kn$$

$$q = -p + kn$$

Dirichlet: OK unless p divides n

Does the sequence exist?

800 000 000 terms exist

999999000000

Max Alekseyev, A261206, Aug 11 2015

If $\lceil n^{1/k} \rceil \mid n$ for all k then n ≤ 999999000000 (conj.)

1, 2, 4, 6, 12, 36, 132, 144, 156, 900, 3600, 4032, 7140, 18360, 44100, 46440, 4062240, 9147600, 999999000000

No more terms below 10^16

999999000000 (cont.)

Th. 1

$$\lceil \sqrt{n} \rceil \mid n \Leftrightarrow n = \left\lfloor \frac{M}{2} \right\rfloor \left\lceil \frac{M}{2} \right\rceil \quad \text{for some } M$$

(the quarter-squares, A002620)

Pf.

$$\lceil \sqrt{n} \rceil = m+1 \Leftrightarrow m^2+1 \le n \le (m+1)^2$$

$$\text{Say } n = m^2+1+i$$

$$\text{So } i = m-1 \text{ or } 2m, \quad n = m(m+1) \text{ or } (m+1)^2$$

$$M = 2m+1 \text{ or } 2m+2$$

Example:

$$999999000000 = \left\lfloor \frac{1999999}{2} \right\rfloor \left\lceil \frac{1999999}{2} \right\rceil$$

999999000000 (cont.)

Th. 2

$$\lceil n^{1/3} \rceil \mid n \iff n = m^3 + 1 + \lambda(m+1), \ 0 \le \lambda \le 3m$$

for some m (A261011)

Example: With $m = 9999, \lambda = 29897,$

$$m^3 + 1 + \lambda(m+1) = 999999000000$$

If both Th 1 and Th 2 apply, get A261417 :

1, 2, 4, 6, 9, 12, 36, 56, 64, 90, 100, 110, 132, 144, 156, 210, 400, 576, 702, 729, 870, ...

And so on ?

A Memorable Prime

12345678910987654321

When is 123...n-1 n n-1...321 prime?

It is a square: $11\ldots1^2$ for $n \leq 9$.

Prime for n=10, 2446 (Shyam Gupta, PRP only), ...

Or, in base b, when is 123...b-1 b b-1...321 prime?

Prime for b =

2, 3, 4, 6, 9, 10, 16, 40, 104, 8840 (PRP)

(David Broadhurst, Aug 2015, A260343)

Counting Problems

- Sequences with no final repeats
- Lines in the plane; or in general position
- Points in {0,1}^n with no right angles
- Alex Meiburg's A260273

Sequences with no final repeats

Number of binary sequences, length n, not of form

$$XY^k, k > 1$$

Good: 00001, 11001 Bad: 00000, 00011, 00101

2, 2, 4, 6, 12, 20, 40, 74, 148, 286, ... (A122536)

$$a(2n+1) = 2a(2n), \; a(2n) = 2a(2n-1) - b(n)$$

where b(n) = number of robust sequences S

[SS without initial symbol has no final repeats]

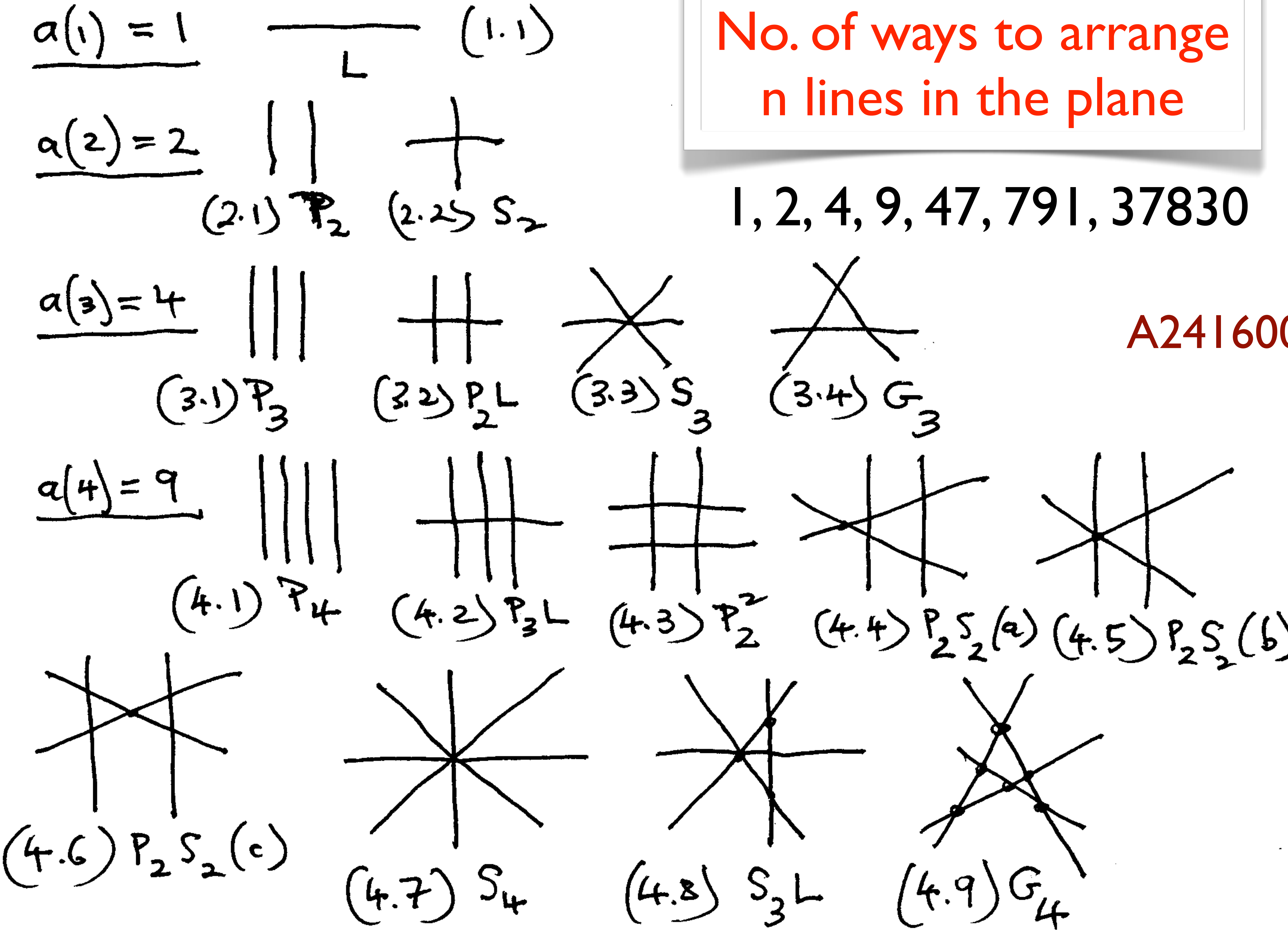
S = 32232 is not robust: SS = 32232 32232

Have 200 terms. Conj. a(n) / 2↑n → 0.27004339...

No. of ways to arrange n lines in the plane

1, 2, 4, 9, 47, 791, 37830

A241600



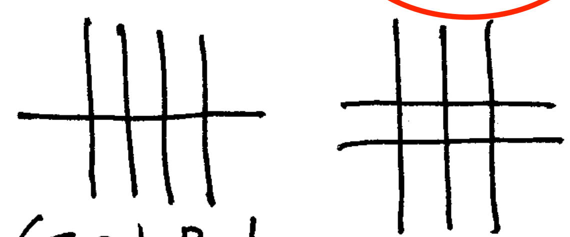
$a(5) = 47$. Summary: ②

A241600 (cont.)

$P_5 : 1$, $P_4 L : 1$, $P_3 P_2 : 1$, $P_3 S_2 : 4$, $P_2^2 L : 6$,

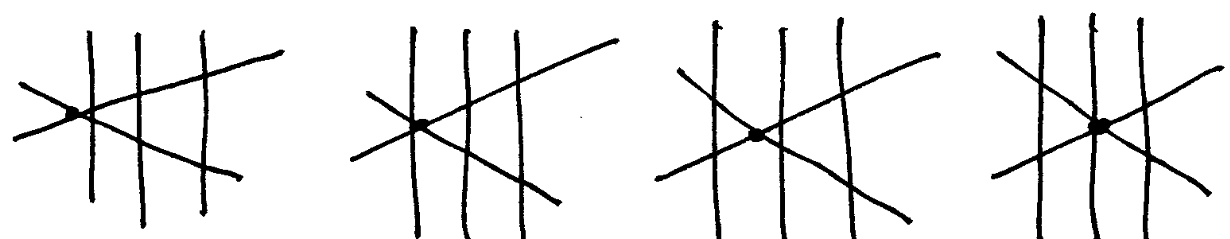
$P_2 G_3 : 14$, $P_2 S_3 : 3$, $S_5 : 1$, $S_4 L : 1$, ~~$S_3^2 : 6$, $S_5 L : 1$.~~

$S_3^2 : 3$, $S_3 S_2 : 6$, $G_5 : 6$.



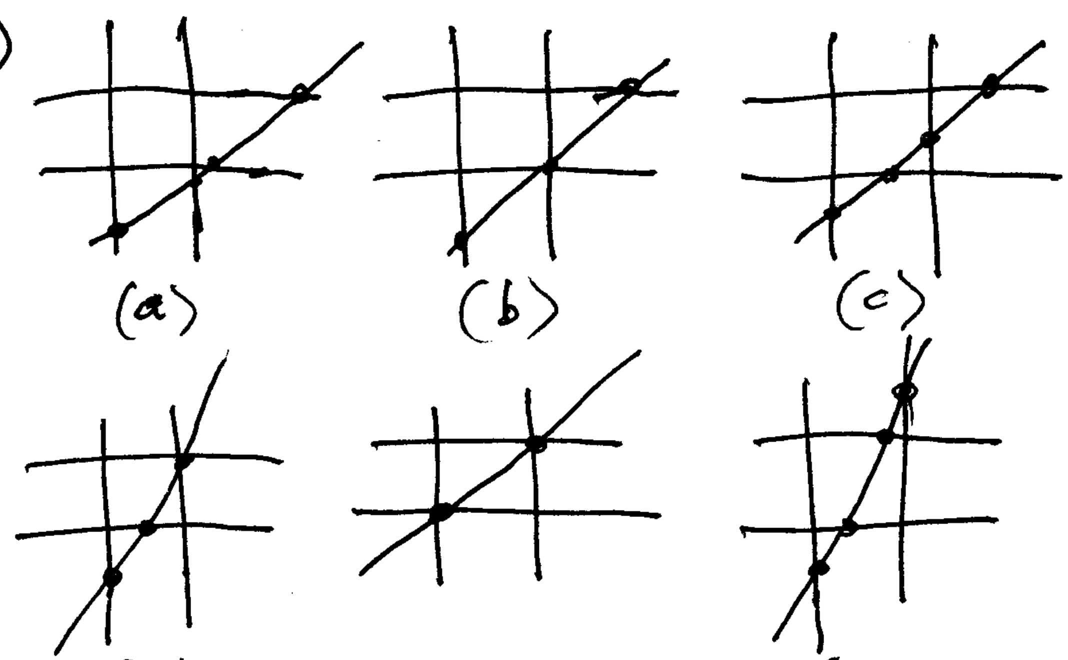
(5.1) P_5 (5.2) $P_4 L$ (5.3) $P_3 P_2$

(5.4) – (5.7) $P_3 S_2$:



(a) (b) (c) (d)

(5.8) – (5.13) $P_2^2 L$:



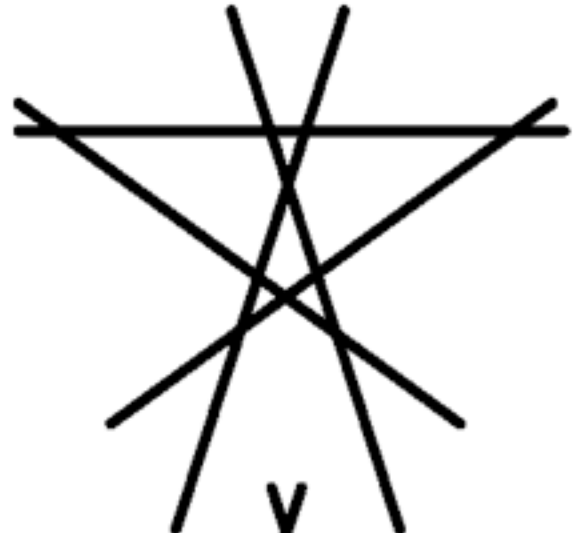
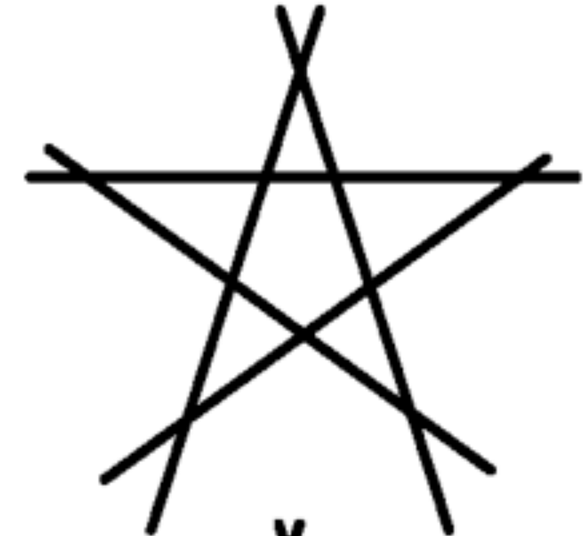
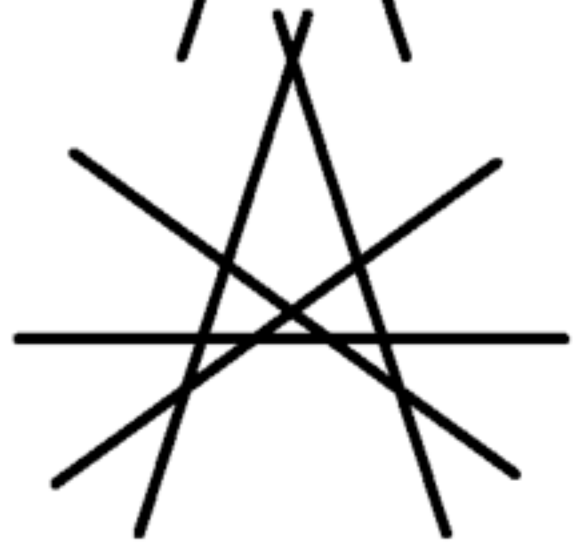
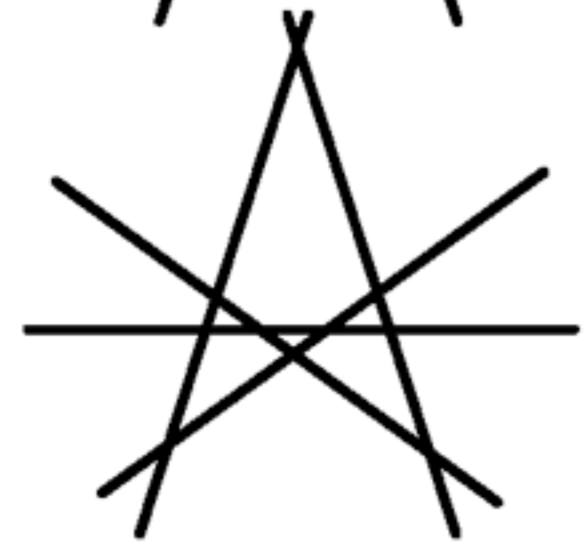
(a) (b) (c) (d) (e) (f)

A90338

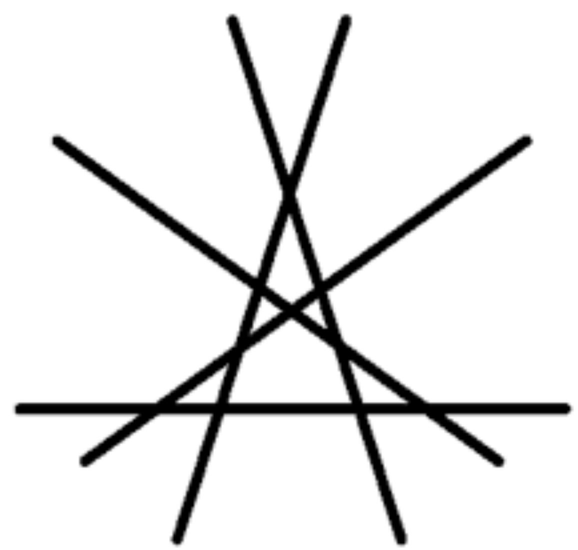
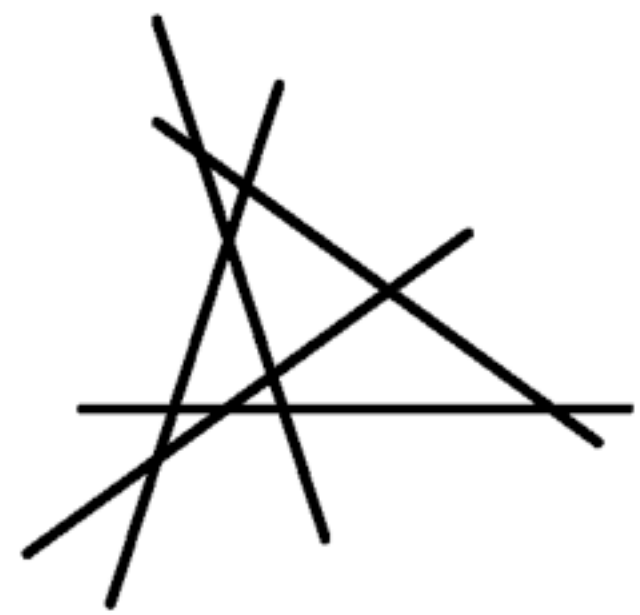
A subset: n lines in general position

1, 1, 1, 1, 6, 43, 922, 38609

Wild and Reeves, 2004

a(5)=6

5 lines in general position: 6 ways

Points in {0,1}^n with no right angles

a(n) = max no of points in {0,1}^n such that all angles PQR are less than 90 degs.

A89676, Classic problem, only 10 terms known!

1, 2, 2, 4, 5, 6, 8, 9, 10, 16

a(3) = 4: {000, 011, 101, 110}.
a(4) = 5: {0011, 0101, 0110, 1000, 1111}.
a(5) = 6: {00000 00011 00101 01001 10001 11110}

NEED MORE TERMS!

Prompted by Prof. Jeff Kahn's lecture on The Probabilistic Method, March 28 2016

Alex Meiburg's A260273

Alex Meiburg's A260273

Define M(n): E.g. n = 57 = 111001

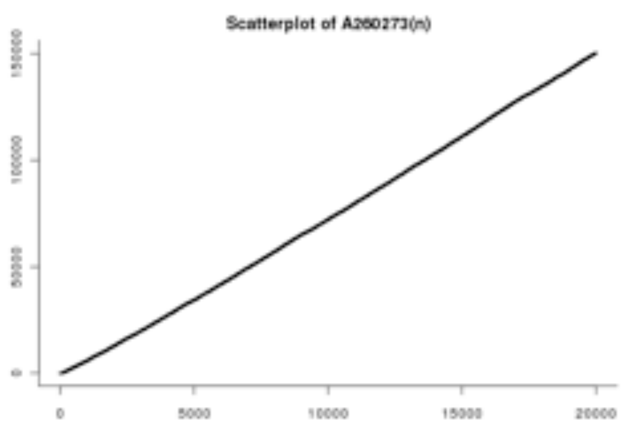
Can see 0, 1, 10, 11, 100 but not 101 so M(57)=5

M(n) = smallest missing number in binary exp. of n (A261922)

M'(n) = smallest missing positive number in binary exp. of n

a(1)=1; a(n+1) = a(n) + M'(a(n))

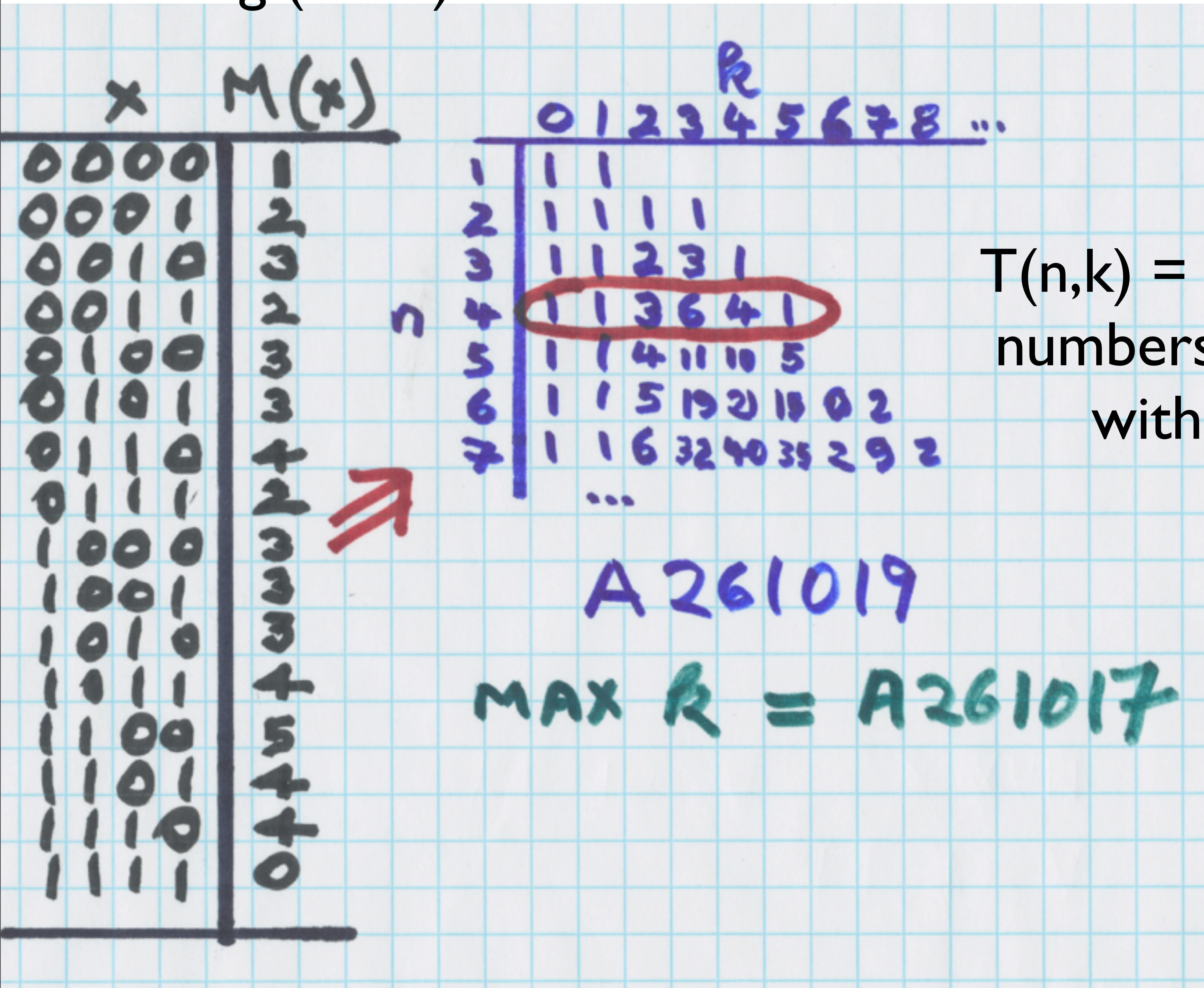
1, 3, 5, 8, 11, 15, 17, 20, 23, 27, 31, 33, 36, ...



Conjecture (Meiburg):

$$a(n) \sim \frac{n}{2} \log_2(n)$$

Meiburg (cont.)



T(n,k) = no. of binary numbers of length n with M(x)=k

Meiburg (cont.)

Sum k*T(n,k) = A261016:

1, 6, 18, 46, 107, 241, 535, 1178, 2569, 5546, 11859, 25156, 53058, ...

Divide by 2^n: average step size in Meiburg's sequence

What is this sequence? Have 58 terms from Hiroaki Yamanouchi.

$$a(n) \approx 2^n \left(\frac{n}{4} + 4.3\right) \quad ??$$

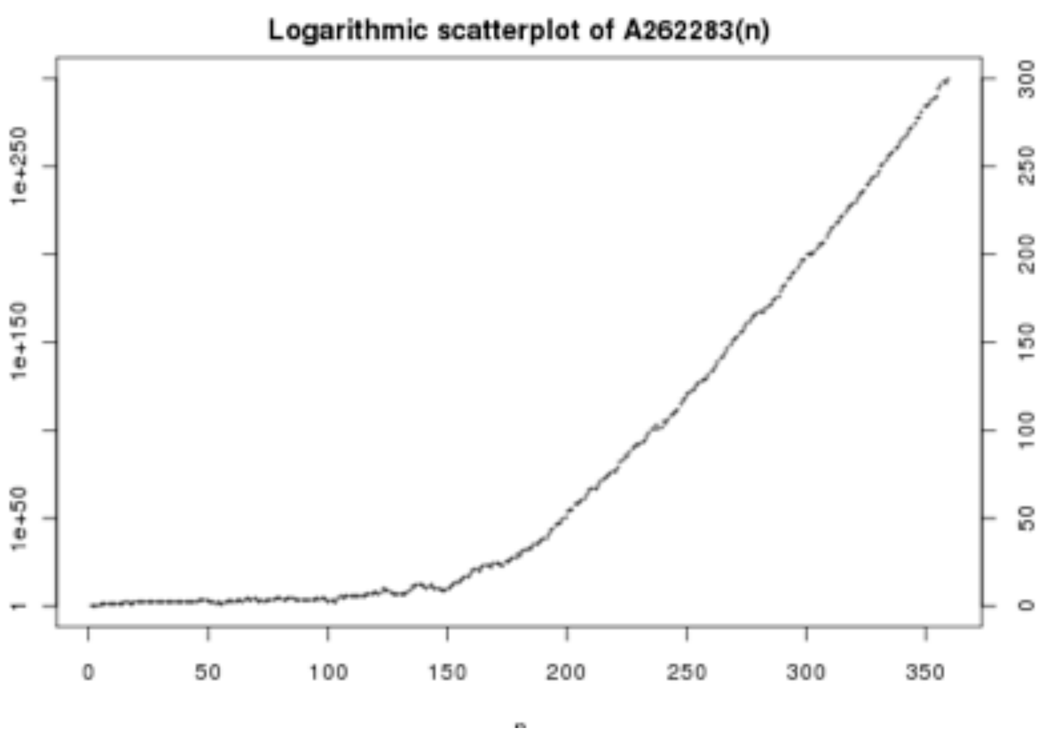
Need analysis of A261019, A261016 and related sequences!

Smallest Prime Beginning With the “igits” of Previous Prime

A262283

2, 3, 5, 7, 11, 13, 31, 17, 71, 19, 97, 73, 37, 79, 907, 701, 101, ...

s = digits of a(n) without leading digit,
a(n+1) = smallest missing prime beginning with s.



Show 23 etc never appear!

A. Murthy, F. Adams-Watters, A. Heinz, R. Zumkeller, NJAS

(Binary analog A262374 etc)

Circulant determinant equals number

Generalize: $$\begin{vmatrix} 2 & 4 & 7 \\ 7 & 2 & 4 \\ 4 & 7 & 2 \end{vmatrix} = 247.$$

N. I. Belukhov, 2011.

247, 370, 378, 407, 481, 518, 592, 629, 1360, 3075, 26027, ...

47 terms are known (A219324).

OEIS.org

We need editors!

Send me email:
njasloane@gmail.com