

Wolfdieter Lang, Oct 08 2015

a(n,k) tabf head (staircase) A263003.

Partition array for the products of the hook lengths of Ferrers (or Young) diagrams corresponding to a partitions of n, written in Abramowitz-Stegun (A-St) order. This is the array n!/A117506(n, k).

n\k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22 ...
1	1																					
2	2	2																				
3	6	3	6																			
4	24	8	12	8	24																	
5	120	30	24	20	24	30	120															
6	720	144	80	144	72	45	144	72	80	144	720											
7	5040	840	360	360	336	144	240	240	252	144	360	336	360	840	5040							
8	40320	5760	2016	1440	2880	1920	630	576	720	960	1152	448	720	576	2880	1152	630	1440	1920	2016	5760	40320
.																						
.																						
n\k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22 ...

The sequence of row lengths is A000041: [1, 2, 3, 5, 7, 11, 15, 22, 30, 42,...] (partition numbers).

For the ordering of this tabf array a(n,k) see Abramowitz-Stegun ref. pp. 831-2.

E.g. a(4,4) refers to the fourth partition of n = 4 in this ordering, namely (1^2,2) = [1,1,2], but here written with non-increasing part numbers as [2,1,1].

[2,1,1] corresponding to the Ferrers (Young diagram) $\begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \square & & \\ \hline \square & & \\ \hline \end{array}$.

The associated hook length diagram is $\begin{array}{c} 4 \ 1 \\ 2 \\ 1 \end{array}$, and $a(4, 4) = 4*1*2*1 = 8$.

The next rows are:

n=9: (30 partitions)
[362880, 45360, 13440, 7560, 8640, 12960, 3456, 2240, 4320, 3024, 2160, 8640, 6480, 1920, 1680, 1680, 2160, 4320, 5184, 1920, 3024, 2240, 8640, 6480, 3456, 7560, 12960, 13440, 45360, 362880];

n=10: (42 partitions)
[3628800, 403200, 103680, 48384, 40320, 86400, 100800, 22680, 11520, 12600, 16128, 8064, 14400, 17280, 43200, 10368, 6400, 12096, 6912, 4725, 17280, 12096, 14400, 28800, 8100, 6912, 6400, 8064, 12600, 86400, 28800, 10368, 16128, 11520, 40320, 43200, 22680, 48384, 100800, 103680, 403200, 3628800];

n=11: (56 partitions)
[39916800, 3991680, 907200, 362880, 241920, 302400, 887040, 172800, 72576, 57600, 120960, 103680, 40320, 40320, 60480, 86400, 332640, 67200, 32400, 34560, 36288, 17280, 30240, 33600, 48384, 30240, 86400, 190080, 43200, 25920, 48384, 25920, 17280, 60480, 34560, 40320, 120960, 158400, 43200, 36288, 32400, 40320, 57600, 302400, 190080, 67200, 103680, 72576, 241920, 332640, 172800, 362880, 887040, 907200, 3991680, 39916800];

n=12: (77 partitions)

[479001600, 43545600, 8870400, 3110400, 1741824, 1612800, 3628800, 8709120, 1496880, 537600, 340200, 414720, 777600, 248832, 179200, 362880, 290304, 226800, 1036800, 2903040, 506880, 201600, 155520, 322560, 230400, 85050, 82944, 115200, 161280, 248832, 107520, 181440, 161280, 1036800, 1451520, 277200, 129600, 136080, 134400, 62208, 107520, 115200, 136080, 82944, 226800, 322560, 362880, 1036800, 228096, 134400, 248832, 129600, 85050, 290304, 155520, 179200, 414720, 3628800, 1036800, 277200, 230400, 201600, 248832, 340200, 1612800, 1451520, 506880, 777600, 537600, 1741824, 2903040, 1496880, 3110400, 8709120, 8870400, 43545600, 479001600];

n=13: (101 partitions)

[6227020800, 518918400, 95800320, 29937600, 14515200, 10886400, 14515200, 94348800, 14515200, 4561920, 2419200, 2177280, 4838400, 6652800, 1814400, 1036800, 1209600, 1741824, 967680, 1814400, 2419200, 28304640, 4354560, 1478400, 907200, 1088640, 1710720, 518400, 362880, 725760, 544320, 414720, 1814400, 1555200, 518400, 483840, 537600, 725760, 1814400, 12579840, 2073600, 798336, 604800, 1244160, 844800, 302400, 290304, 388800, 537600, 691200, 290304, 483840, 414720, 2419200, 1244160, 725760, 1814400, 7862400, 1451520, 665280, 691200, 665280, 302400, 518400, 544320, 604800, 362880, 967680, 1088640, 1209600, 4838400, 6739200, 1451520, 844800, 1555200, 798336, 518400, 1741824, 907200, 1036800, 2177280, 14515200, 7862400, 2073600, 1710720, 1478400, 1814400, 2419200, 10886400, 12579840, 4354560, 6652800, 4561920, 14515200, 28304640, 14515200, 29937600, 94348800, 95800320, 518918400, 6227020800];

n=14: (135 partitions)

[87178291200, 6706022400, 1132185600, 319334400, 136857600, 87091200, 87091200, 203212800, 1117670400, 155675520, 43545600, 19958400, 14515200, 19051200, 63866880, 15206400, 7257600, 6220800, 13547520, 12441600, 5443200, 5806080, 9676800, 14515200, 304819200, 41932800, 12441600, 6386688, 5644800, 12441600, 14515200, 3742200, 2073600, 2381400, 3225600, 1741824, 3225600, 4147200, 11404800, 3110400, 2073600, 4064256, 2488320, 1814400, 7257600, 5806080, 7257600, 121927680, 17690400, 5806080, 3492720, 4147200, 6220800, 1824768, 1254400, 2488320, 1814400, 1360800, 5806080, 4276800, 1382400, 1270080, 1360800, 1814400, 4147200, 6220800, 2488320, 4064256, 3225600, 14515200, 67737600, 10782720, 4064256, 3041280, 6220800, 4147200, 1455300, 1382400, 1814400, 2488320, 3041280, 1254400, 2073600, 1741824, 9676800, 4147200, 2381400, 5806080, 12441600, 13547520, 50803200, 9172800, 4147200, 4276800, 4064256, 1824768, 3110400, 3225600, 3492720, 2073600, 5443200, 5644800, 6220800, 19051200, 203212800, 50803200, 10782720, 6220800, 11404800, 5806080, 3742200, 12441600, 6386688, 7257600, 14515200, 87091200, 67737600, 17690400, 14515200, 12441600, 15206400, 19958400, 87091200, 121927680, 41932800, 63866880, 43545600, 136857600, 304819200, 155675520, 319334400, 1117670400, 1132185600, 6706022400, 87178291200];

n=15: (176 partitions)

[1307674368000, 93405312000, 14529715200, 3736212480, 1437004800, 798336000, 653184000, 914457600, 14370048000, 1828915200, 461260800, 186624000, 114960384, 112896000, 261273600, 679311360, 143700480, 59136000, 40824000, 52254720, 102643200, 37324800, 29030400, 60963840, 52254720, 43545600, 217728000, 3592512000, 447068160, 117936000, 52254720, 37255680, 48384000, 138378240, 31104000, 14370048, 12096000, 26127360, 22809600, 9676800, 10160640, 16329600, 24192000, 95800320, 22176000, 11664000, 13063680, 14515200, 7464960, 13547520, 16128000, 26127360, 17418240, 54432000, 1306368000, 169344000, 48522240, 24385536, 21288960, 46656000, 52416000, 13063680, 7096320, 8064000, 10644480, 5644800, 10368000, 13063680, 31104000, 8211456, 5376000, 10450944, 6220800, 4465125, 17418240, 13063680, 16128000, 38016000, 11664000, 10450944, 10368000, 13547520, 24192000, 217728000, 653184000, 91445760, 29352960, 17418240, 20528640, 30326400, 8709120, 5913600, 11664000, 8382528, 6220800, 26127360, 18662400, 5913600, 5376000, 5644800, 7464960, 16329600, 20528640, 8064000, 13063680, 10160640, 43545600, 46656000, 26127360, 60963840, 435456000, 67737600, 25159680, 18662400, 38016000, 25159680, 8709120, 8211456, 10644480, 14515200, 17418240, 7096320, 11664000, 9676800, 52254720, 21288960, 12096000, 29030400, 48384000, 52254720, 261273600, 381024000, 67737600, 30326400, 31104000, 29352960, 13063680, 22176000, 22809600, 24385536, 14370048, 37324800, 37255680, 40824000, 112896000, 914457600, 435456000, 91445760, 52416000, 95800320, 48522240, 31104000, 102643200, 52254720, 59136000, 114960384, 653184000, 653184000, 169344000, 138378240, 117936000, 143700480, 186624000, 798336000, 1306368000, 447068160, 679311360, 461260800, 1437004800, 3592512000, 1828915200, 3736212480, 14370048000, 14529715200, 93405312000, 1307674368000].

etc.

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The row sums give A263004:

[1, 4, 15, 76, 368, 2365, 14892, 116236, 966064, 9256889, 96638496, 1129309316, 14261533248, 196315312964, 2900635720869, 45926240752560, 773725147192412, 13831256551416480, 261227089570409028, 5198858467673903360].

e.o.f.