CHORD DIAGRAMS WITH DIRECTED CHORDS A260847

RICHARD J. MATHAR

Abstract. Sequence [1, A260847] counts chord diagrams where chords are directed edges (arcs). Illustrations of all diagrams up to 8 vertices (4 chords) are provided here.

1. Overview

The standard chord diagrams are cubic graphs with two types of edges (i) the frame, a Hamiltonian cycle through the vertices, (ii) chords that connect two distinct vertices of the frame such that the degree of each vertex is 3.

The variant shown in [1, A260847] counts diagrams where the chords are directed edges (arcs). These arcs are painted green in our illustrations; the frame of the cycle is the circle with black edges.

Remark 1. *We refer to the case* [1, A054499] *where rotations and flips of a graph do not create a different graph. So this is not about the refined set of diagrams of* [1, A260296] *where flips/mirror images are considered distinct.*

Each of the graphs on V nodes (with $V/2$ chords) can be described as a permutation of the V numbers $[-V/2, -V/2+1, \ldots -1, 1, 2, \ldots, V/2]$ which serves as labels around (say ccw) the Hamiltonian circle. Positive and negative numbers are attached to the tails and heads of the arcs, and the permutation describes the order encountering these tails and heads walking along the Hamiltonian circle.

Example 1. *For 3 chords and 6 vertics, the permutation [1,3,-2,-1,-3,2] indicates that walking ccw one meets the tail of the first chord, the tail of the third chord, the head of the second chord, the head of the first chord, the head of the third chord, and the tail of the second chord in that order.*

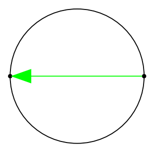
Remark 2. *One may think of a canonical list-format which identifies a chord diagram with a permutation: The permutation is cyclically shifted until some fixed number is the first entry (because rotations of the graph do not count as distinct diagrams). If the reversed permutation (after shifting) is smaller, the reversed permutation is the canonical format (because the flipped diagram is not considered different). Amongst all permutations of the* $V/2$ *positive entries, the negative entries permuted co-jointly, the smallest permutation is retained (because the graph is not labeled). This recipe of applying the product of (i) the group of order 2 for the flips, (ii) the symmetric group of order* $V/2$ *for the relabeling (iii) the cyclic group of order* V *for the shifts was actually used to filter duplicates in the symmetric group of order* V *of all permutations.*

Date: September 11, 2024.

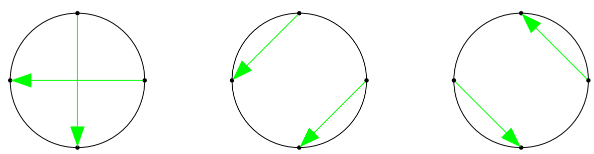
2020 *Mathematics Subject Classification.* Primary 05C30; Secondary 05C75.

Key words and phrases. Graph Enumeration, Cubic Graphs, chord diagrams.

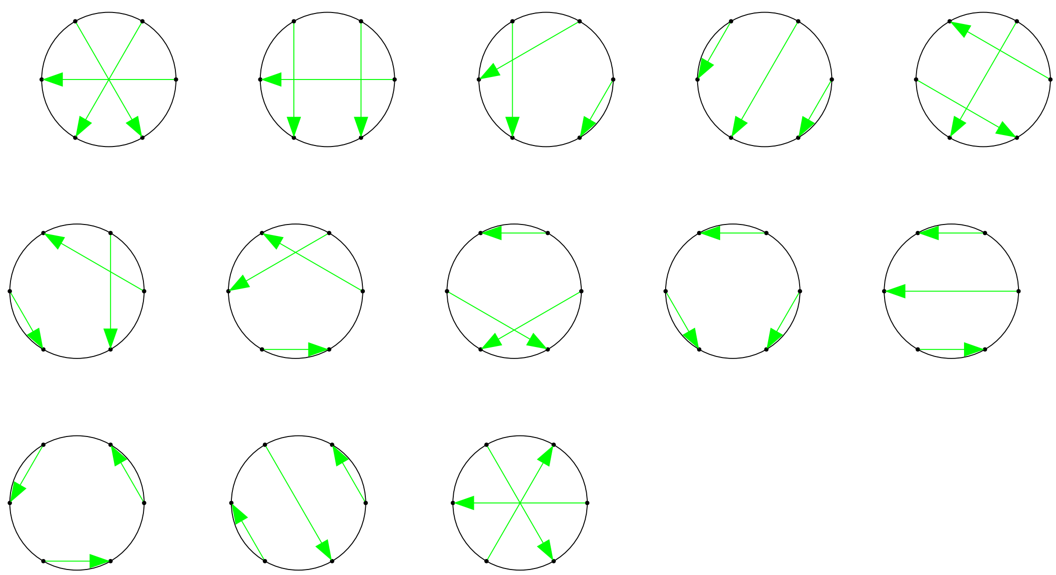
2. One graph on 2 nodes



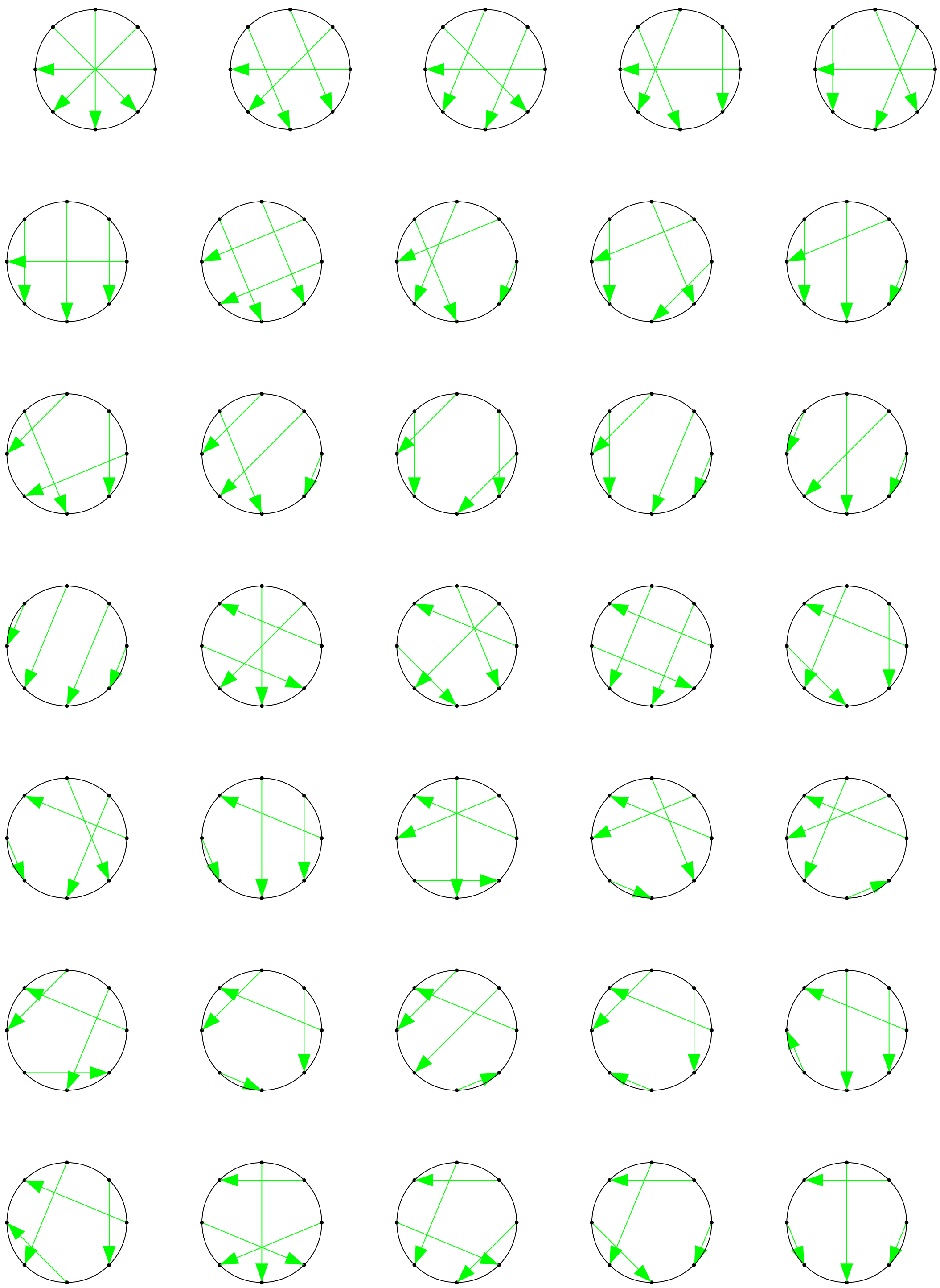
3. Three graphs on 4 nodes

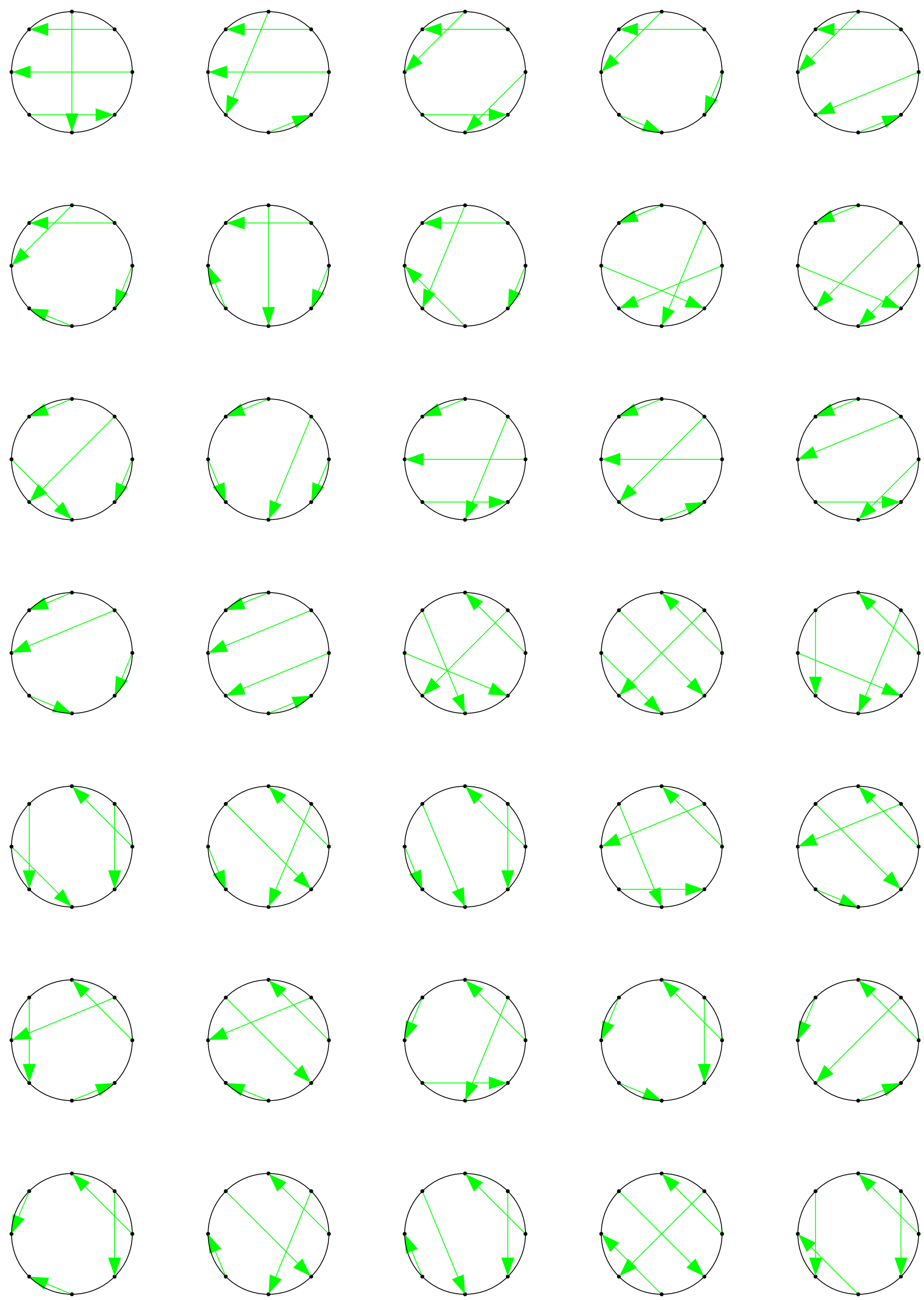


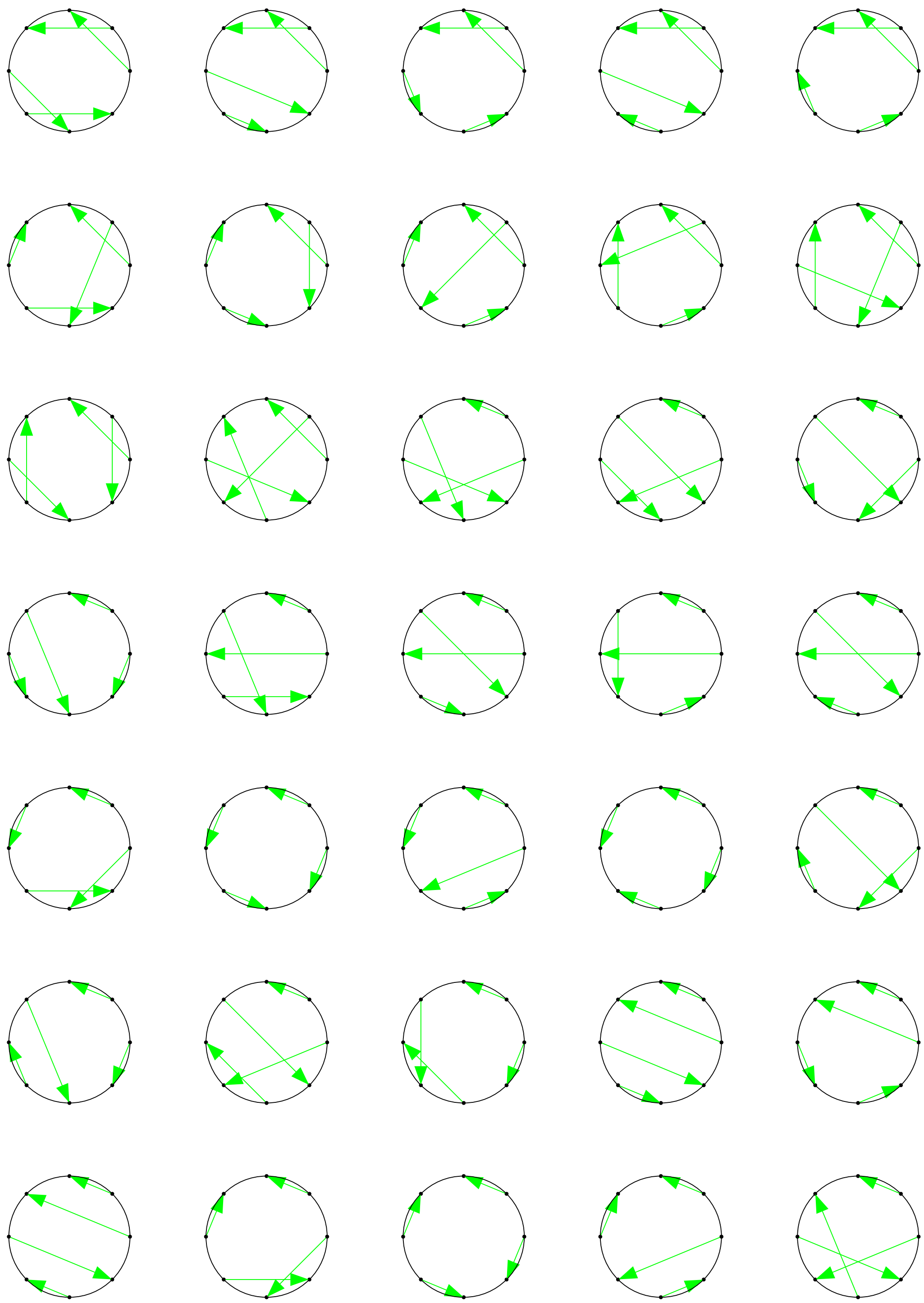
4. 13 graphs on 6 nodes

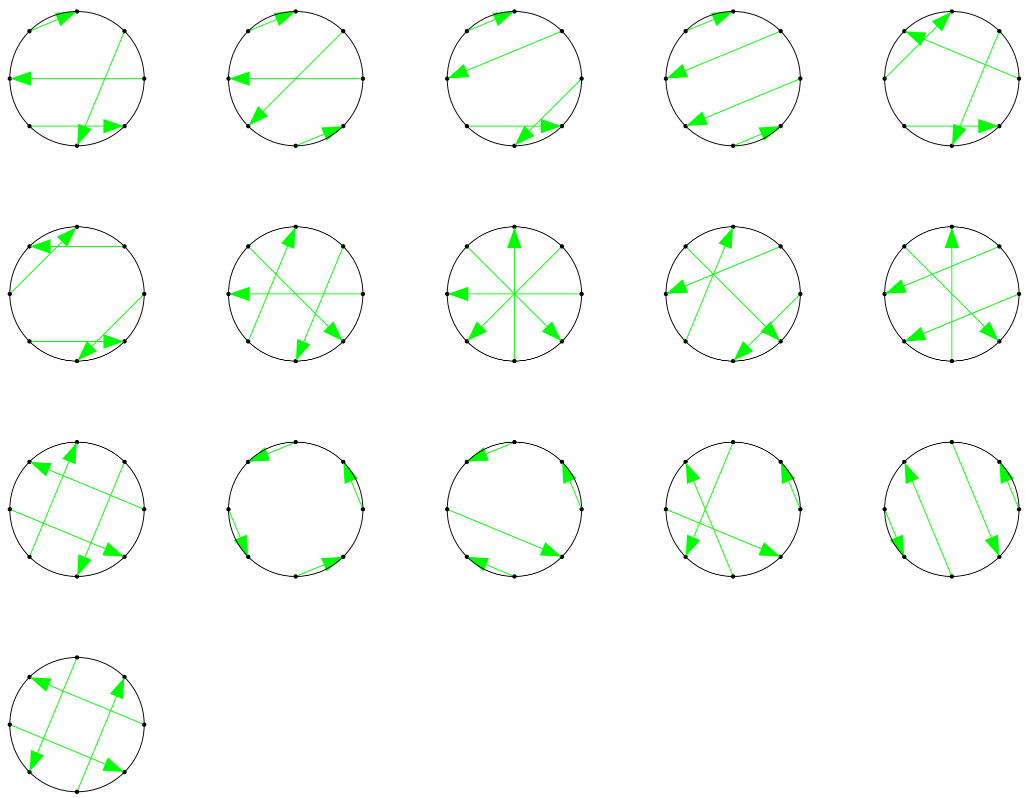


5. 121 graphs on 8 nodes









References

1. O. E. I. S. Foundation Inc., *The On-Line Encyclopedia Of Integer Sequences*, (2024), https://oeis.org/. MR 3822822

URL: `https://www.mpia-hd.mpg.de/~mathar`

Max-Planck Institute for Astronomy, Königstuhl 17, 69117 Heidelberg, Germany