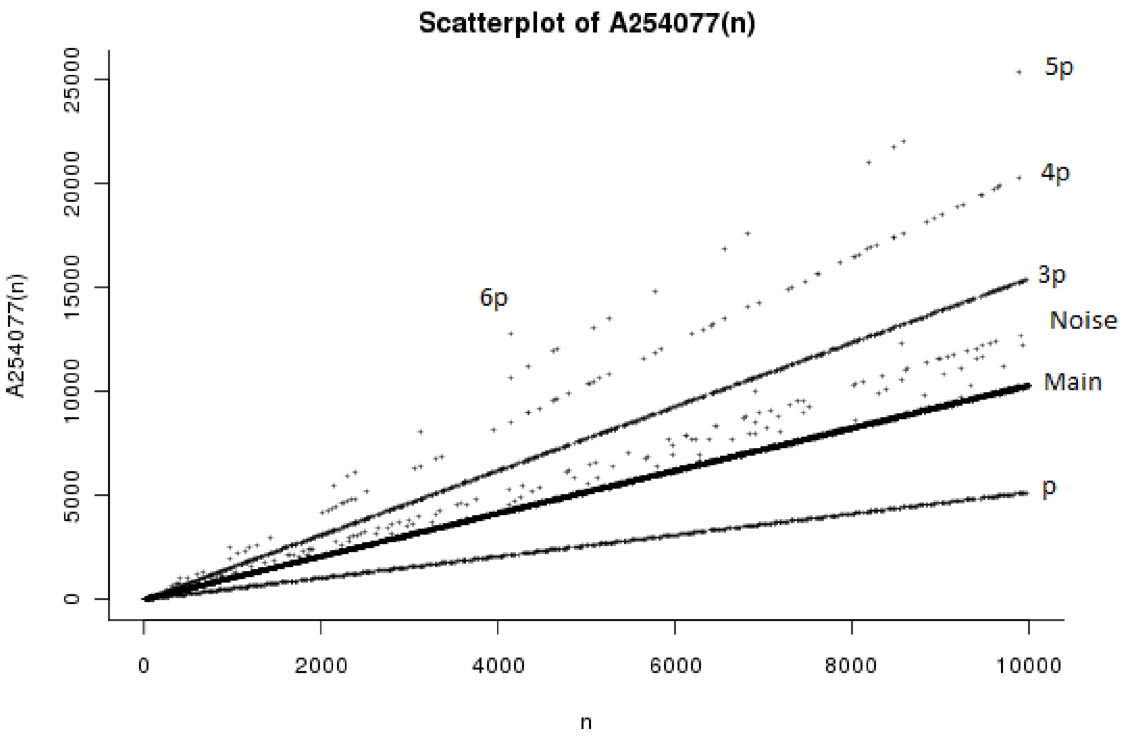
A254077 - Observations

John Mason – 3 May 2015

This document discusses some observations of the sequence A254077. I have included the scatterplot from the OEIS site in annotated form. (Just another example of how useful the OEIS graph mechanism is!)



The scatterplot shows clearly specific features of the sequence. The main, thickest line, corresponds to a(n) when it has values very close to n[1]. Below that we have the primes which are left behind, as each prime p occurs in position (approximately) 2p. Symmetrically above is the line corresponding to a(n)=3p, then just above 4p, a faint 5p, and just one 6p.

These lines, corresponding to the multiples of primes, depend on the locations of the primes in the sequence. In fact, according to observation of the first 50 million terms, all primes occur in standard sub-sequences. If a(n)=p, prime, then a(n-2)=2p, and a(n+2)=3p. Under certain conditions[2], the sub-sequence may continue such that a(n+4)=4p, less often a(n+6)=5p, and, just once in the scatterplot, a(n+8)=6p.

Sub-sequences through to 6p occur between around 5 and 10 times per million terms. Just 2 sub-sequences reach 7p in the first 50 million terms. The first of these has values:

a(22534884) = 22910614 = 2 * 11455307
a(22534886) = 11455307
a(22534888) = 34365921 = 3 * 11455307
a(22534890) = 45821228 = 4 * 11455307
a(22534892) = 57276535 = 5 * 11455307
a(22534894) = 68731842 = 6 * 11455307
a(22534896) = 80187149 = 7 * 11455307

It is difficult to imagine if longer sub-sequences exist, and how far you would have to go to find them.

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[1] Over the first 10000 terms, considering only the main line, a(n)/n averages to 1.03, and the same ratio reaches 1.016 in the interval a(49990000)-a(50000000)

[2] Theorem: if a(n-2)=mp for some prime p, and m divides a(n-1), then a(n), if it exists, is a multiple of p. See OEIS page A254077

What's that noise?

Apart from the principle lines visible in the scatterplot, there is some noise between the main line and the 3p line. In many cases these points correspond to sub-sequences further on down. That is, low multiples of relatively large primes that are not attached to the sub-sequences that originate when 2p and p enter the sequence.

For example:

a(4113) = 4244 = 4 * 1061 is in the main line

a(4115) = 5305 = 5 * 1061 is half way between the main line and the 3p line

Another example:

a(6901) = 7135 = 5 * 1427 is in the main line

a(6903) = 8562 = 6 * 1427 is between the main line and the 3p line

a(6905) = 9989 = 7 * 1427 is between the main line and the 3p line

The 5p points, like a(4115) form a discernible line, more populated than the 5p line previously discussed and which is captioned in the annotated scatterplot.

These observations lead one to suspect that within the 3p line there are some values that are really 6p; that is, in the main line there is a(n)=4p, just above is a(n+2)=5p, and in the 3p line there a(n+4)=6p.
In fact this happens, as illustrated by the example:

a(6110) = 6316 = 4 * 1579

a(6112) = 7895 = 5 * 1579

a(6114) = 9474 = 6 * 1579

Conclusion

The sequence shows strong patterns. Whether or not this fact can help in demonstrating the various conjectures in the OEIS page remains to be seen.