$972799^2_{10}=111001100000110101_5$

A recent *Notices* article [1] on Kurt Mahler quoted part of his 1988 letter to van der Poorten on basimal experimental mathematics. Specifically, Mahler found $20^2_{10}=400_{10}=1111_7=26^2_7$ and stated he could not find a base 5 example of a square containing only 1s and 0s.

Richard Guy [2] has classified similar open questions in base 10 as F24 Some decimal digital problems or F31 Miscellaneous digital problems. This question is related but expands the search to other bases where the probabilities differ.

My computer search yielded $972799^2_{10}=222112144^2_5=111001100000110101_5$. Web searches yield additional results: 22222111221444^2_5 and 100024441003001^2_5 [3]. Of course, 5^{2n} multiples of these numbers also work for an infinite set of examples.

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References

[1] J. M. Borwein, Y. Bugeaud, M. Coons, The Legacy of Kurt Mahler, *Notices of the AMS* **62** (2015), 526–531.

[2] R. K. Guy, *Unsolved Problem in Number Theory*, third edition, Springer Verlag, New York, 2004.

[3] mathoverflow.net/questions/22/can-n2-have-only-digits-0-and-1-other-than-n-10k

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Traditional Proofs vs. Structured Proofs

The authors of the article, "Investigating and Improving Undergraduate Proof Comprehension," on pp. 742–752 in the August 2015 issue of the *Notices*, deserve the highest praise for attempting to use scientific methodology to determine ways of improving proof comprehension by undergraduates. However, they seem not to have investigated a proof technique that, in my experience, is very helpful for making difficult proofs easier to understand. I am referring to what is sometimes called "structured proof."

A structured proof is like a structured computer program. At the top level there are a few steps that, provided that each of the steps is true, constitute a valid logical argument for the truth of the theorem or lemma in question. Each of the steps consists of a few substeps that, provided that each of the substeps is true, constitute a valid logical argument for the truth of the step in question. And so on, recursively, down to a sufficiently low level that each of the steps is known to be true by virtue of the knowledge assumed of any reader of the proof.

This structure can be applied to any proof, be it one page or ten pages or hundreds of pages long. The reader has only a few steps to comprehend at any time. He or she can choose the level of detail he or she wishes to deal with.

Traditional proofs require that the reader somehow understand the entire argument as he or she proceeds. This is typically difficult even if lemmas or theorems are invoked at various points in the proof. With structured proof, all proofs are "the same" (have the same structure) whereas with traditional proof, all proofs are "different" (do not have the same structure, as seen on the page).

Although there are papers in the literature, e.g., by Leslie Lamport, about structured proof, the technique has never become widely practiced, certainly not in any of the textbooks I am familiar with.

I strongly urge the authors of the above article to, (1) try structuring a few proofs, and then (2) try introducing the technique to some of their students.

I should mention that the technique is also very helpful when one is trying to create a proof of one's own.

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Reply to Schorer

We are very grateful to Peter Schorer for his kind comments on our article, and for suggesting we investigate structured proofs. In fact there has been some empirical research on the effectiveness of this presentation method. A group from Rutgers University (Fuller, Weber, Mejía-Ramos, Rhoads & Samkoff, 2014) investigated how presenting proofs in this structured format influences undergraduates' comprehension. They found mixed results: compared to those who read a traditional proof, the students in their study who read a structured proof were significantly better at identifying a good summary of the proof, but slightly (albeit nonsignificantly) worse at all other aspects of proof comprehension (on questions concerning justifications within the proof, transferring the ideas from the proof to another setting, and illustrating the ideas of the proof using examples). However, as far as we know this is the only empirical investigation of this topic. We agree that this is an area ripe for further investigation, and we would encourage those interested in collaborating on such an endeavour to make contact.

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References

E. Fuller, K. Weber, J. P. Mejía-Ramos, K. Rhoads, and A. Samkoff, Comprehending structured proofs, *International Journal for Studies in Mathematics Education*, 7(1), (2014), 1–32.

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