

Flagpole

of themselves (sequences)

Hello [SeqFans](#),

a little word sequence not (yet?) in the OEIS:

```

1           O N E
7         S E V E N
3         T H R E E
6         S I X
5         F I V E
11        E L E V E N
8         E I G H T
9         N I N E
2         T W O
13        T H I R T E E N
4         F O U R
10        T E N
12        T W E L V E
16        S I X T E E N
15        F I F T E E N
26        T W E N T Y S I X
14        F O U R T E E N
18        E I G H T E E N
17        S E V E N T E E N
19        N I N E T E E N
20        T W E N T Y
111       O N E H U N D R E D E L E V E N
21        T W E N T Y O N E
25        T W E N T Y F I V E
22        T W E N T Y T W O
23        T W E N T Y T H R E E
24        T W E N T Y F O U R
28        T W E N T Y E I G H T
38        T H I R T Y E I G H T
30        T H I R T Y
27        T W E N T Y S E V E N
29        T W E N T Y N I N E
...

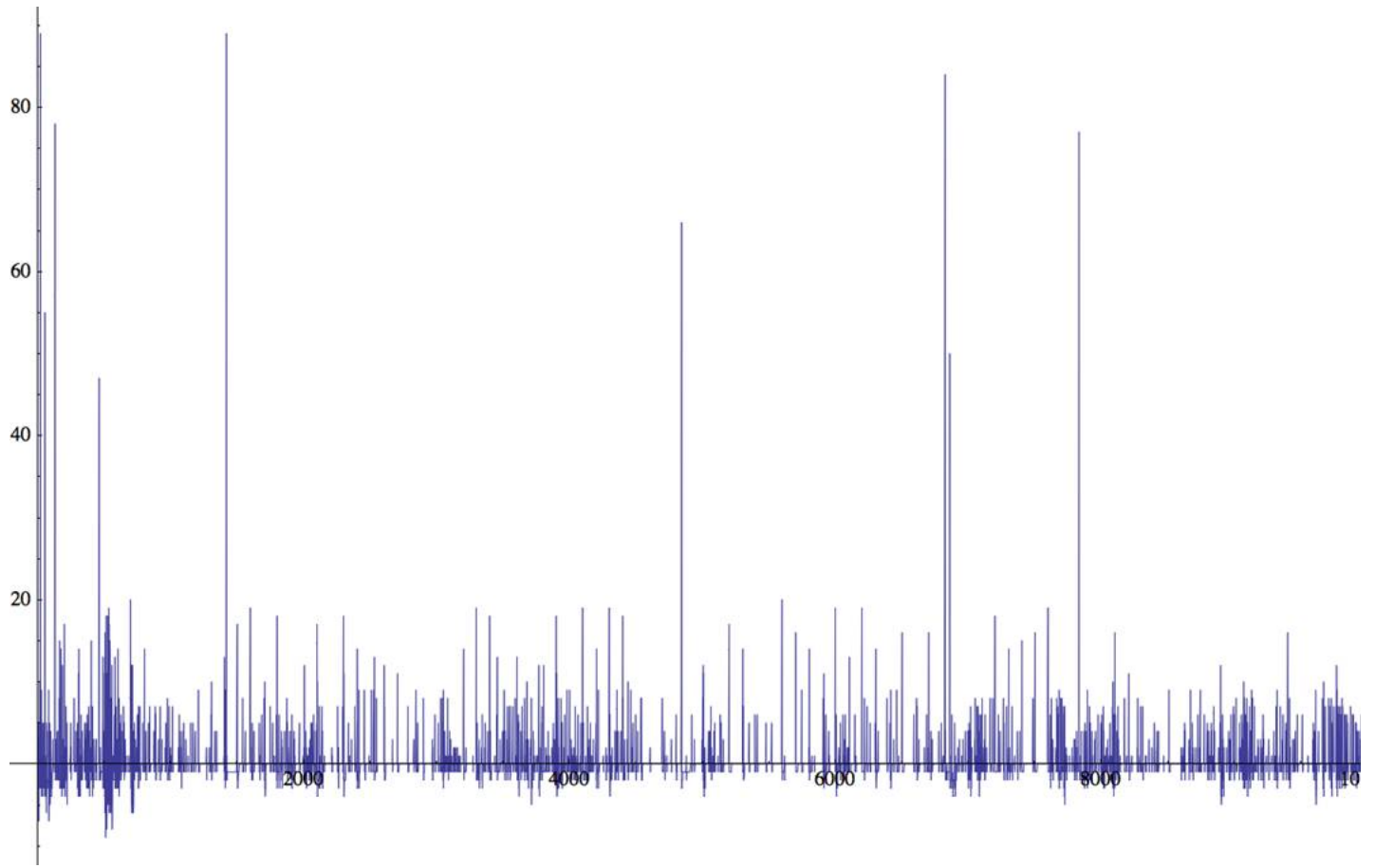
```

Explanation: the (yellow) "single letter column" (the "flagpole"), spells vertically O N E S E V E N ... which is the sequence itself. In extending the sequence, always try to use the smallest Natural not yet present in the sequence.

Is the sequence a permutation of the Naturals?

[**Hans Havermann**]:

The terms of this sequence are pretty close to a straight line. In fact, of the first 10000 terms, I count 5832 where $a(n) = n$, beginning with $n = 1, 3, 5, 15, 18, 28, 30, 45, 65, 113, \dots$, and the variances from this line for the rest is (in this range) relatively small:



[Eric Angelini]:

Below is the same idea as above -- but with integers instead (the yellow "digit column b" spells the digits of the sequence):

Digit column: a b c

	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	1	0
2	0	
1	2	
3	0	
	1	1
	2	1
1	3	
4	0	
	1	4
	1	5
	2	2
	1	6
	1	7
2	3	
2	4	
5	0	
	1	8
3	4	
	1	9
2	5	
	2	6
	2	7
	.	
	.	
	.	

The above sequence 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 12, 30, 11, 21, 13, 40, 14, 15,

22,16,17,23,24,50,18,34,19,25,26,27,... is for sure a permutation of the Naturals.

And if this "flagpole" idea is worth entering the OEIS, then one could add a lot of such sequences:

Fla*Prime would start: 2,3,5,7,11,13,17,23,19,37,29,31,41,59,37,47,...

Fla*Fibo would start: 0,1,1,2,3,5,8,13,34,233,144,21,46368,121393,...

Fla*Even would start: 2,4,6,8,10,20,12,30,14,22,32,40,16,24,26,28,...

[Claudio Meller]:

Fla in Spanish: 1, 5, 2, 4, 6, 9, 8, 11, 10, 12, 3, 13, 15, 14, 7, 23, 18, 16, 17, 19, 22, 20, 24, 25, 26,

[Éric Angelini]:

Fla in French: 1, 5, 25, 3, 9, 4, 20, 6, 11, 21, 7, 35, 8, 15, 14, 13, 23, 12, 10, 16, 19, 2, 18, 29, 24, 22, 34, 17, 30, 27, 26, 28, 31, 80, 32, 33, 36, 42, 43, 37, 71, 38, ...

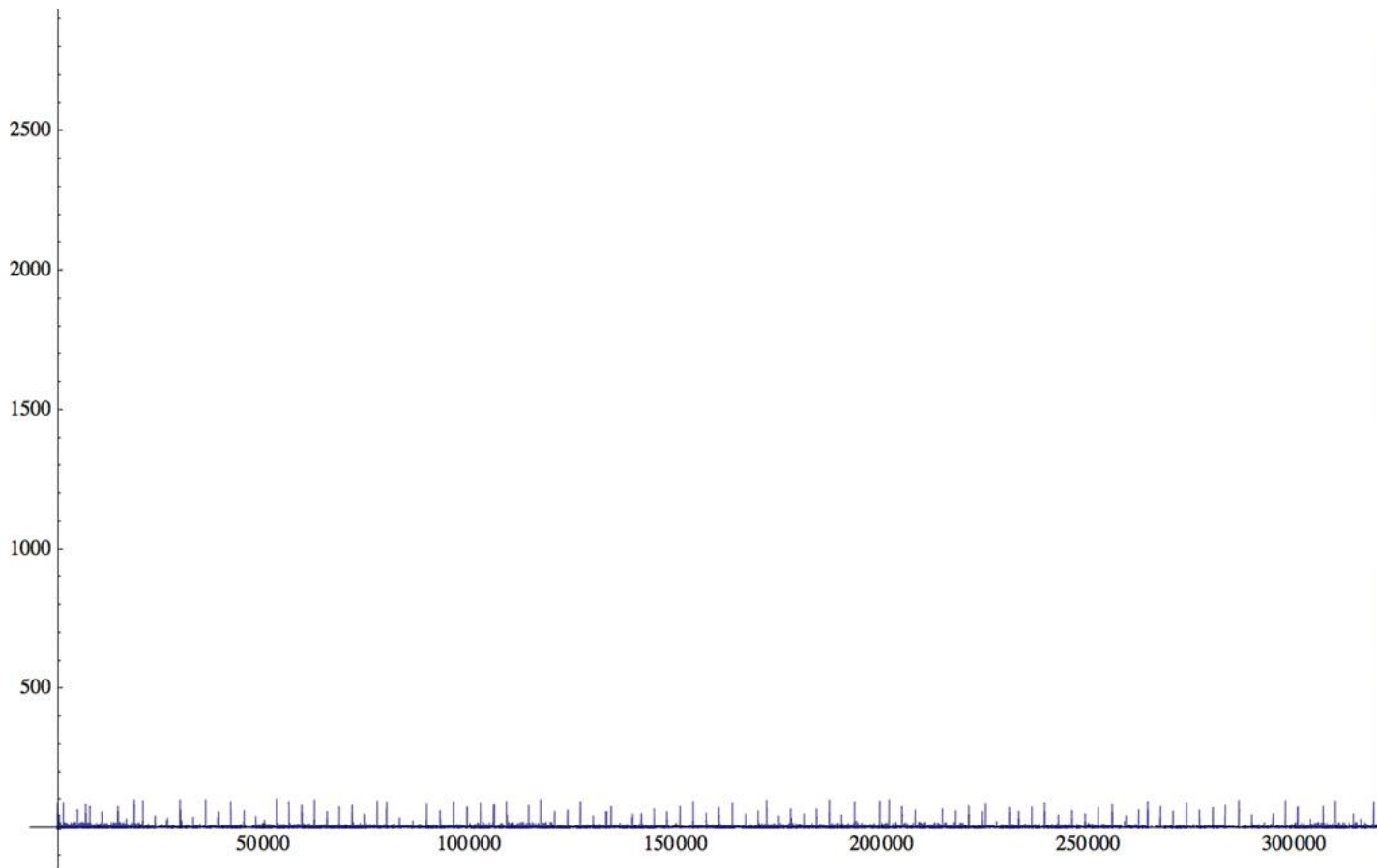
```

      U N
    C I N Q
  V I N G T-C I N Q
    T R O I S
      N E U F
      Q U A T R E
      V I N G T
      S I X
      O N Z E
    V I N G T E T U N
    S E P T
  T R E N T E-C I N Q
    H U I T
    Q U I N Z E
      Q U A T O R Z E
      T R E I Z E
  V I N G T-T R O I S
    D O U Z E
    D I X
      S E I Z E
    D I X-N E U F
      D E U X
    D I X-H U I T
  V I N G T-N E U F
    V I N G T-Q U A T R E
  V I N G T-D E U X
  T R E N T E-Q U A T R E
    D I X-S E P T
      T R E N T E
    V I N G T-S E P T
      V I N G T-S I X
      V I N G T-H U I T
    T R E N T E E T U N
  Q U A T R E-V I N G T S
      T R E N T E-D E U X
  T R E N T E-T R O I S
    T R E N T E-S I X
  Q U A R A N T E-D E U X
  Q U A R A N T E-T R O I S
    T R E N T E-S E P T
  S O I X A N T E E T O N Z E
    T R E N T E-H U I T
    ...

```

[Hans Havermann]:

I had a hunch that the linear behaviour that I charted might have been a small-numbers-only phenomenon so I decided to calculate one million terms to see if the small variances continued. I'm only just past 324000 terms (so far) when I decided to have a look:



It appears that a correspondence search for the letter 'L' is responsible for the increasing positive variances starting with term 321680:

```

321680 + 31   three hundred twenty-one thousand seven hundred eleven
321859 + 52   three hundred twenty-one thousand nine hundred eleven
321873 + 39   three hundred twenty-one thousand nine hundred twelve
321890 + 121  three hundred twenty-two thousand eleven
321909 + 103  three hundred twenty-two thousand twelve
321927 + 184  three hundred twenty-two thousand one hundred eleven
321945 + 167  three hundred twenty-two thousand one hundred twelve
321962 + 249  three hundred twenty-two thousand two hundred eleven
321983 + 229  three hundred twenty-two thousand two hundred twelve
322002 + 309  three hundred twenty-two thousand three hundred eleven
322021 + 291  three hundred twenty-two thousand three hundred twelve
322039 + 372  three hundred twenty-two thousand four hundred eleven
322056 + 356  three hundred twenty-two thousand four hundred twelve
322078 + 433  three hundred twenty-two thousand five hundred eleven
322092 + 420  three hundred twenty-two thousand five hundred twelve
322098 + 513  three hundred twenty-two thousand six hundred eleven
322114 + 498  three hundred twenty-two thousand six hundred twelve
322118 + 593  three hundred twenty-two thousand seven hundred eleven
322140 + 572  three hundred twenty-two thousand seven hundred twelve
322161 + 650  three hundred twenty-two thousand eight hundred eleven
322183 + 629  three hundred twenty-two thousand eight hundred twelve
322208 + 703  three hundred twenty-two thousand nine hundred eleven
322231 + 681  three hundred twenty-two thousand nine hundred twelve
322251 + 760  three hundred twenty-three thousand eleven
322275 + 737  three hundred twenty-three thousand twelve
322297 + 814  three hundred twenty-three thousand one hundred eleven
322320 + 792  three hundred twenty-three thousand one hundred twelve
322343 + 868  three hundred twenty-three thousand two hundred eleven
322367 + 845  three hundred twenty-three thousand two hundred twelve
322390 + 921  three hundred twenty-three thousand three hundred eleven
322413 + 899  three hundred twenty-three thousand three hundred twelve
322438 + 973  three hundred twenty-three thousand four hundred eleven
322463 + 949  three hundred twenty-three thousand four hundred twelve
322487 + 1024 three hundred twenty-three thousand five hundred eleven
322507 + 1005 three hundred twenty-three thousand five hundred twelve
322530 + 1081 three hundred twenty-three thousand six hundred eleven
322553 + 1059 three hundred twenty-three thousand six hundred twelve
322578 + 1133 three hundred twenty-three thousand seven hundred eleven
322602 + 1110 three hundred twenty-three thousand seven hundred twelve
322624 + 1187 three hundred twenty-three thousand eight hundred eleven
322648 + 1164 three hundred twenty-three thousand eight hundred twelve
322673 + 1238 three hundred twenty-three thousand nine hundred eleven
322698 + 1214 three hundred twenty-three thousand nine hundred twelve
322722 + 1289 three hundred twenty-four thousand eleven
322741 + 1271 three hundred twenty-four thousand twelve

```

322764 + 1347 three hundred twenty-four thousand one hundred eleven
 322788 + 1324 three hundred twenty-four thousand one hundred twelve
 322810 + 1401 three hundred twenty-four thousand two hundred eleven
 322834 + 1378 three hundred twenty-four thousand two hundred twelve
 322857 + 1454 three hundred twenty-four thousand three hundred eleven
 322879 + 1433 three hundred twenty-four thousand three hundred twelve
 322901 + 1510 three hundred twenty-four thousand four hundred eleven
 322925 + 1487 three hundred twenty-four thousand four hundred twelve
 322948 + 1563 three hundred twenty-four thousand five hundred eleven
 322967 + 1545 three hundred twenty-four thousand five hundred twelve
 322989 + 1622 three hundred twenty-four thousand six hundred eleven
 323013 + 1599 three hundred twenty-four thousand six hundred twelve
 323035 + 1676 three hundred twenty-four thousand seven hundred eleven
 323058 + 1654 three hundred twenty-four thousand seven hundred twelve
 323081 + 1730 three hundred twenty-four thousand eight hundred eleven
 323103 + 1709 three hundred twenty-four thousand eight hundred twelve
 323127 + 1784 three hundred twenty-four thousand nine hundred eleven
 323151 + 1761 three hundred twenty-four thousand nine hundred twelve
 323174 + 1837 three hundred twenty-five thousand eleven
 323193 + 1819 three hundred twenty-five thousand twelve
 323215 + 1896 three hundred twenty-five thousand one hundred eleven
 323239 + 1873 three hundred twenty-five thousand one hundred twelve
 323262 + 1949 three hundred twenty-five thousand two hundred eleven
 323285 + 1927 three hundred twenty-five thousand two hundred twelve
 323307 + 2004 three hundred twenty-five thousand three hundred eleven
 323331 + 1981 three hundred twenty-five thousand three hundred twelve
 323355 + 2056 three hundred twenty-five thousand four hundred eleven
 323378 + 2034 three hundred twenty-five thousand four hundred twelve
 323399 + 2112 three hundred twenty-five thousand five hundred eleven
 323423 + 2089 three hundred twenty-five thousand five hundred twelve
 323447 + 2164 three hundred twenty-five thousand six hundred eleven
 323473 + 2139 three hundred twenty-five thousand six hundred twelve
 323498 + 2213 three hundred twenty-five thousand seven hundred eleven
 323523 + 2189 three hundred twenty-five thousand seven hundred twelve
 323547 + 2264 three hundred twenty-five thousand eight hundred eleven
 323573 + 2239 three hundred twenty-five thousand eight hundred twelve
 323599 + 2312 three hundred twenty-five thousand nine hundred eleven
 323624 + 2288 three hundred twenty-five thousand nine hundred twelve
 323649 + 2362 three hundred twenty-six thousand eleven
 323669 + 2343 three hundred twenty-six thousand twelve
 323692 + 2419 three hundred twenty-six thousand one hundred eleven
 323715 + 2397 three hundred twenty-six thousand one hundred twelve
 323738 + 2473 three hundred twenty-six thousand two hundred eleven
 323762 + 2450 three hundred twenty-six thousand two hundred twelve
 323786 + 2525 three hundred twenty-six thousand three hundred eleven
 323811 + 2501 three hundred twenty-six thousand three hundred twelve
 323836 + 2575 three hundred twenty-six thousand four hundred eleven
 323861 + 2551 three hundred twenty-six thousand four hundred twelve
 323885 + 2626 three hundred twenty-six thousand five hundred eleven
 323905 + 2607 three hundred twenty-six thousand five hundred twelve
 323928 + 2683 three hundred twenty-six thousand six hundred eleven
 323951 + 2661 three hundred twenty-six thousand six hundred twelve
 323975 + 2736 three hundred twenty-six thousand seven hundred eleven
 323999 + 2713 three hundred twenty-six thousand seven hundred twelve

[I've placed a text file of terms 321500 - 324000 (so that you can better appreciate this emergent behaviour) [here](#). It will be interesting to see how this evolves.]

...

I've taken this file down, because I will replace it with something much better. I'm going to do an actual flagpole down to this region, and beyond. In order to keep file-sizes relatively small, I'm breaking the pole up into 50000-term pieces, but even with that the first file is 7 MB:

<http://chesswanks.com/num/flagpole/000050.txt>

Depending on your browser, you might need to shrink the font-size to keep things from wrapping. There's lots of white space (which adds to the file-size) but I'm thinking of extending the flagpole (eventually) into the millions and I expect a lot of that white space to disappear in those larger regions. So why keep the white space in the smaller regions? I want smooth annealing: If you copy/paste (part of) one file into another, I still want everything to line up.

You can see which files are available by refreshing:

<http://chesswanks.com/num/flagpole>

The program that is actually calculating the terms has really slowed down in this (I'm currently at 340000) region where we encounter the L-spur (spur = railroad terminology). That's because my program starts looking at the smallest available number and increments by ones from there. For every L that it now encounters, it has to go many thousands of numbers into the search, which is not very efficient. The good news is that I fully expect things to get back on track (more railroad terminology) once the pole's "eleven-" and "twelve-" thousands are used up and we approach the 411- and 412-thousand Ls for search finds. In other words, I'm guessing that the increasingly large $a(n) = n$ variances generated by the letter L will stabilize at 400000, or so, and disappear soon thereafter.

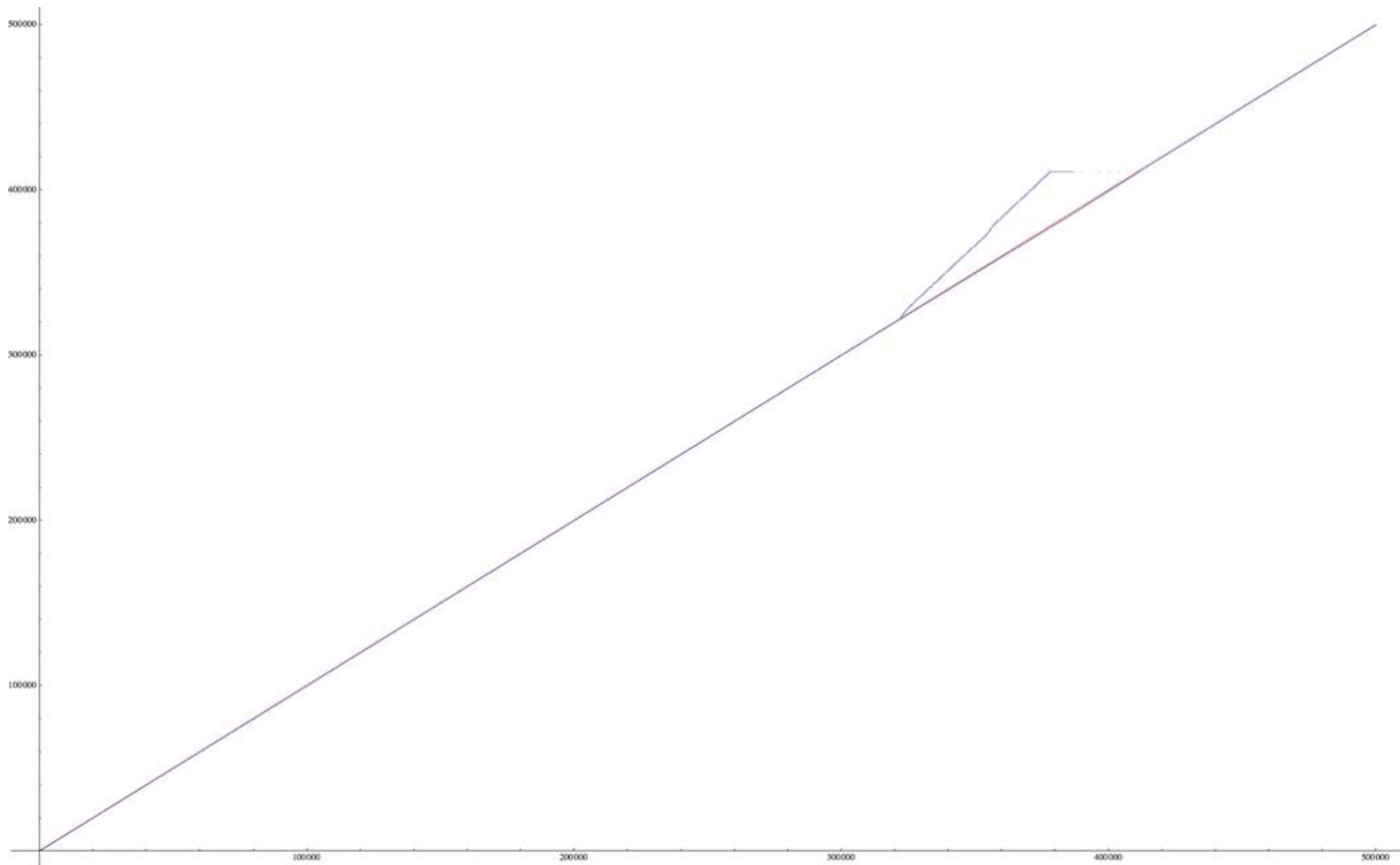
[Hans Havermann] (August 24th update):

OK. I misjudged how fast the program would run beyond 386700. I'm up to 600000.

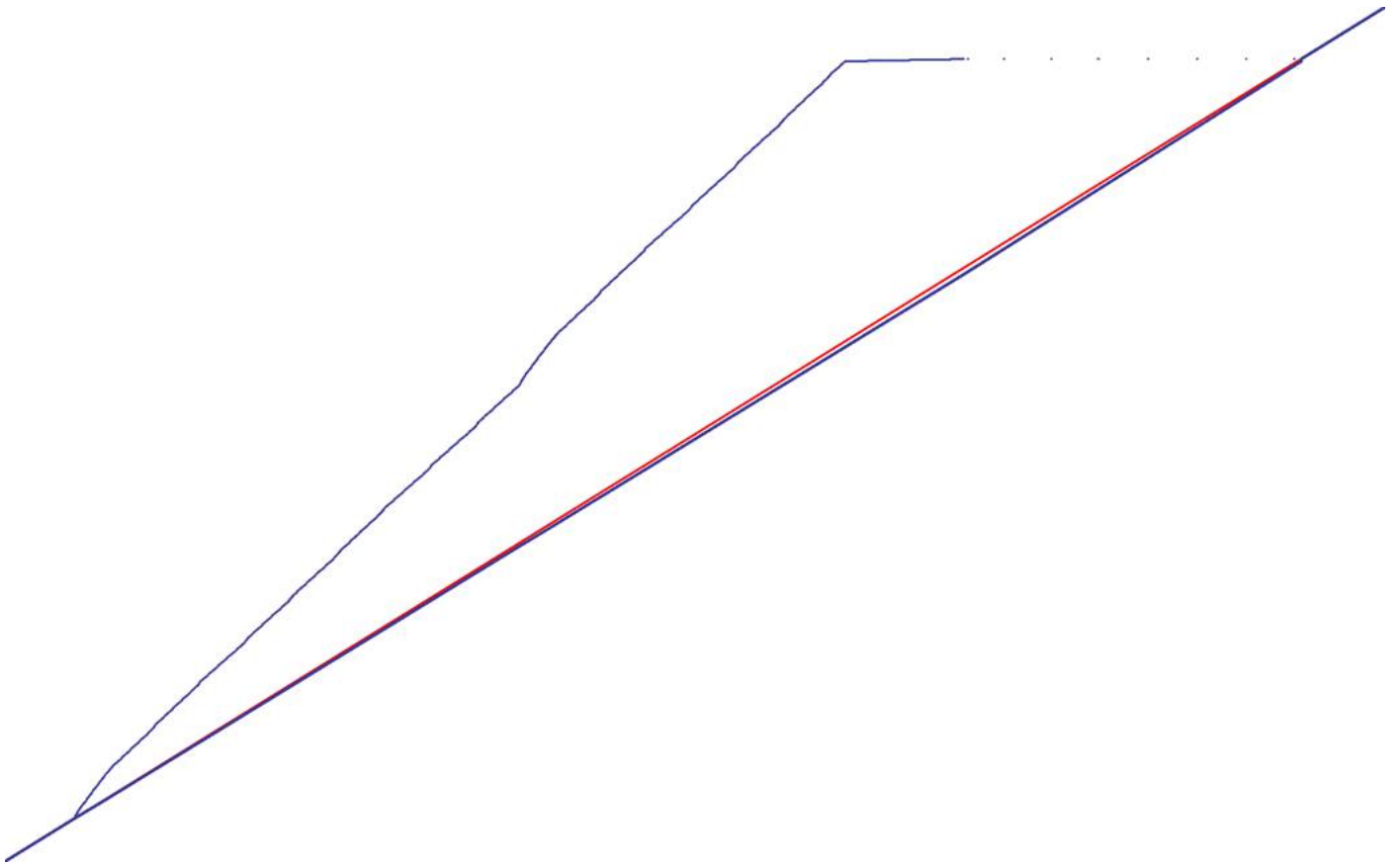
<http://chesswanks.com/num/flagpole>

The next zero variance $[a(n) = n]$ after the one at 321947 is 411273. I've been thinking about the supply side of specific letters. I had suggested that W might create a spur approaching 700000, because of demand, but I no longer think that. The letters E, F, G, H, I, N, O, R, S, T, U, V, W, and X occur every ten number-words, so there is always plentiful local supply for these, regardless of demand. Y less so, but still plentiful enough. L less than Y but cannot ever be a problem after we reach the *-illion* numbers. D is easily satisfied once we reach *-hundred-*, and A, once we reach *-thousand-* numbers. (Think of how we will supply demand for A after we reach 10^6 .) That leaves M after we reach 10^9 (subsequently, B, Q, P, and C, but that is far beyond our scope). Every demand for M after 10^9 will have to travel to $10^9 + 10^6$ for its supply, so this should create the next **significant** spur. I don't think it will look like the L-spur because it will lack the gradual approach to the plateau.

I'm attaching a graph of the flagpole sequence, to 500000, superimposed on a red $a(n) = n$ line. Most of the red line is not visible, because the flagpole sequence is so close to it. I'm also attaching a blow-up of the L-spur region.



Blow up:



Many thanks to you, **Hans** and **Claudio**!
Best,
É.

(August 14th 2012, Brussels, Belgium)