

An array of naturals

(generated by a permutation of the prime numbers)

Hello SeqFans,

Jean-Marc Falcoz has computed 70 terms of the first line hereunder -- and the resulting array (see end of page).

The constraints were:

- 1) the first line of the array must be a permutation of the primes;
- 2) the array, seen as a whole, must be a permutation of the naturals;
- 3) any two neighboring integers have their absolute first difference written on the line below, between them.

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2 3 13 47 197 11 29 443 397 1321 4831 15559 211 5 19 41 293 113 971
...
1 10 34 150 186 18 414 46 924 3510 10728 15348 206 14 22 252 180 858 ...
9 24 116 36 168 396 368 878 2586 7218 4620 15142 192 8 230 72 678 ...
15 92 80 132 228 28 510 1708 4632 2598 10522 14950 184 222 158 606 ...
77 12 52 96 200 482 1198 2924 2034 7924 4428 14766 38 64 448 ...
65 40 44 104 282 716 1726 890 5890 3496 10338 14728 26 384 ...
25 4 60 178 434 1010 836 5000 2394 6842 4390 14702 358 ...
21 56 118 256 576 174 4164 2606 4448 2452 10312 14344 ...
35 62 138 320 402 3990 1558 1842 1996 7860 4032 ...
27 76 182 82 3588 2432 284 154 5864 3828 ...
49 106 100 3506 1156 2148 130 5710 2036 ...
57 6 3406 2350 992 2018 5580 3674 ...
51 3400 1056 1358 1026 3562 1906 ...
3349 2344 302 332 2536 1656 ...
1005 2042 30 2204 880 ...
1037 2012 2174 1324 ...
975 162 850 ...
813 688 ...
125 ...

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After 70 terms (first line), the smallest missing prime is 17 and the smallest missing non-prime is 39.

The first line is the lexicographically first one, as the building method forced the next prime to be the smallest available one, not yet present and not leading to a contradiction.

All odd non-primes are on the first descending diagonal from the left, and only there.

Many thanks to Jean-Marc Falcoz for his computer skills, to Maximilian Hasler and Alexandre Wajnberg (for their support).

This is now <http://oeis.org/A203985> and <http://oeis.org/A204137>

É.

[French explanation]:

- 1) la première ligne est une permutation des nombres premiers;
- 2) la différence absolue entre deux nombres voisins du tableau figure à la ligne du dessous, entre eux;
- 3) l'ensemble du tableau est une permutation des naturels.

Tableau de _70_ lignes (justifié à gauche) :

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{2,3,13,47,197,11,29,443,397,1321,4831,15559,211,5,19,41,293,113,971,419,2687,1087,1709,5851,23629,83,17341,65171,268969,:
{1,10,34,150,186,18,414,46,924,3510,10728,15348,206,14,22,252,180,858,552,2268,1600,622,4142,17778,23546,17258,47830,2037:
{9,24,116,36,168,396,368,878,2586,7218,4620,15142,192,8,230,72,678,306,1716,668,978,3520,13636,5768,6288,30572,155968,444:
{15,92,80,132,228,28,510,1708,4632,2598,10522,14950,184,222,158,606,372,1410,1048,310,2542,10116,7868,520,24284,125396,11:
{77,12,52,96,200,482,1198,2924,2034,7924,4428,14766,38,64,448,234,1038,362,738,2232,7574,2248,7348,23764,101112,13846,265:
{65,40,44,104,282,716,1726,890,5890,3496,10338,14728,26,384,214,804,676,376,1494,5342,5326,5100,16416,77348,87266,251824,:
{25,4,60,178,434,1010,836,5000,2394,6842,4390,14702,358,170,590,128,300,1118,3848,16,226,11316,60932,9918,164558,14092,39:
{21,56,118,256,576,174,4164,2606,4448,2452,10312,14344,188,420,462,172,818,2730,3832,210,11090,49616,51014,154640,150466,:
{35,62,138,320,402,3990,1558,1842,1996,7860,4032,14156,232,42,290,646,1912,1102,3622,10880,38526,1398,103626,4174,140332,:
{27,76,182,82,3588,2432,284,154,5864,3828,10124,13924,190,248,356,1266,810,2520,7258,27646,37128,102228,99452,136158,1211:
{49,106,100,3506,1156,2148,130,5710,2036,6296,3800,13734,58,108,910,456,1710,4738,20388,9482,65100,2776,36706,14974,11844:
{57,6,3406,2350,992,2018,5580,3674,4260,2496,9934,13676,50,802,454,1254,3028,15650,10906,55618,62324,33930,21732,103472,5:
{51,3400,1056,1358,1026,3562,1906,586,1764,7438,3742,13626,752,348,800,1774,12622,4744,44712,6706,28394,12198,81740,43184:
{3349,2344,302,332,2536,1656,1320,1178,5674,3696,9884,12874,404,452,974,10848,7878,39968,38006,21688,16196,69542,350100,3:
{1005,2042,30,2204,880,336,142,4496,1978,6188,2990,12470,48,522,9874,2970,32090,1962,16318,5492,53346,280558,34586,134594:
{1037,2012,2174,1324,544,194,4354,2518,4210,3198,9480,12422,474,9352,6904,29120,30128,14356,10826,47854,227212,245972,100:
{975,162,850,780,350,4160,1836,1692,1012,6282,2942,11948,8878,2448,22216,1008,15572,3530,37028,179358,18760,145964,94796,:
{813,688,70,430,3810,2324,144,680,5270,3340,9006,3070,6430,19768,21208,14764,12242,33498,142330,160598,127204,51168,35063:
{125,618,360,3380,1486,2180,536,4590,1930,5666,5936,3360,13338,1440,6444,2522,21256,108832,18268,33394,76036,299464,26536:
{493,258,3020,1894,694,1644,4054,2660,3736,270,2576,9978,11898,5004,3922,18734,87576,90564,15126,42642,223428,272928,1310:
{235,2762,1126,1200,950,2410,1394,1076,3466,2306,7402,1920,6894,1082,14812,68842,2988,75438,27516,180786,49500,141878,742:
{2527,1636,74,250,1460,1016,318,2390,1160,5096,5482,4974,5812,13730,54030,65854,72450,47922,153270,131286,92378,67588,101:
{891,1562,176,1210,444,698,2072,1230,3936,386,508,838,7918,40300,11824,6596,24528,105348,21984,38908,24790,947194,14292,3:
{671,1386,1034,766,254,1374,842,2706,3550,122,330,7080,32382,28476,5228,17932,80820,83364,16924,14118,922404,932902,33302:
{715,352,268,512,1120,532,1864,844,3428,208,6750,25302,3906,23248,12704,62888,2544,66440,2806,908286,10498,599882,277588,:
{363,84,244,608,588,1332,1020,2584,3220,6542,18552,21396,19342,10544,50184,60344,63896,63634,905480,897788,589384,322294,:
{279,160,364,20,744,312,1564,636,3322,12010,2844,2054,8798,39640,10160,3552,262,841846,7692,308404,267090,75482,173004,54:
{119,204,344,724,432,1252,928,2686,8688,9166,790,6744,30842,29480,6608,3290,841584,834154,300712,41314,191608,97522,37451:
{85,140,380,292,820,324,1758,6002,478,8376,5954,24098,1362,22872,3318,838294,7430,533442,259398,150294,94086,276990,13268:
{55,240,88,528,496,1434,4244,5524,7898,2422,18144,22736,21510,19554,834976,830864,526012,274044,109104,56208,182904,14430:
{185,152,440,32,938,2810,1280,2374,5476,15722,4592,1226,1956,815422,4112,304852,251968,164940,52896,126696,38602,7170,118:
{33,288,408,906,1872,1530,1094,3102,10246,11130,3366,730,813466,811310,300740,52884,87028,112044,73800,88094,31432,111288,

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{255,120,498,966,342,436,2008,7144,884,7764,2636,812736,2156,510570,247856,34144,25016,38244,14294,56662,79856,77386,4824,
{135,378,468,624,94,1572,5136,6260,6880,5128,810100,810580,508414,262714,213712,9128,13228,23950,42368,23194,2470,72562,11
{243,90,156,530,1478,3564,1124,620,1752,804972,480,302166,245700,49002,204584,4100,10722,18418,19174,20724,70092,1805018,
{153,66,374,948,2086,2440,504,1132,803220,804492,301686,56466,196698,155582,200484,6622,7696,756,1550,49368,1734926,16633
{87,308,574,1138,354,1936,628,802088,1272,502806,245220,140232,41116,44902,193862,1074,6940,794,47818,1685558,71562,30584
{221,266,564,784,1582,1308,801460,800816,501534,257586,104988,99116,3786,148960,192788,5866,6146,47024,1637740,1613996,23
{45,298,220,798,274,800152,644,299282,243948,152598,5872,95330,145174,43828,186922,280,40878,1590716,23744,1379716,18340,
{253,78,578,524,799878,799508,298638,55334,91350,146726,89458,49844,101346,143094,186642,40598,1549838,1566972,1355972,13
{175,500,54,799354,370,500870,243304,36016,55376,57268,39614,51502,41748,43548,146044,1509240,17134,211000,5404,446016,67
{325,446,799300,798984,500500,257566,207288,19360,1892,17654,11888,9754,1800,102496,1363196,1492106,193866,205596,440612,
{121,798854,316,298484,242934,50278,187928,17468,15762,5766,2134,7954,100696,1260700,128910,1298240,11730,235016,61798,35
{798733,798538,298168,55550,192656,137650,170460,1706,9996,3632,5820,92742,1160004,1131790,1169330,1286510,223286,173218,
{195,500370,242618,137106,55006,32810,168754,8290,6364,2188,86922,1067262,28214,37540,117180,1063224,50068,120602,49484,6
{500175,257752,105512,82100,22196,135944,160464,1926,4176,84734,980340,1039048,9326,79640,946044,1013156,70534,71118,5848
{242423,152240,23412,59904,113748,24520,158538,2250,80558,895606,58708,1029722,70314,866404,67112,942622,584,513766,19159
{90183,128828,36492,53844,89228,134018,156288,78308,815048,836898,971014,959408,796090,799292,875510,942038,513182,322174,
{38645,92336,17352,35384,44790,22270,77980,736740,21850,134116,11606,163318,3202,76218,66528,428856,191008,244258,28084,1
{53691,74984,18032,9406,22520,55710,658760,714890,112266,122510,151712,160116,73016,9690,362328,237848,53250,216174,13316
{21293,56952,8626,13114,33190,603050,56130,602624,10244,29202,8404,87100,63326,352638,124480,184598,162924,83008,131466,1
{35659,48326,4488,20076,569860,546920,546494,592380,18958,20798,78696,23774,289312,228158,60118,21674,79916,48458,66982}
{12667,43838,15588,549784,22940,426,45886,573422,1840,57898,54922,265538,61154,168040,38444,58242,31458,18524}
{31171,28250,534196,526844,22514,45460,527536,571582,56058,2976,210616,204384,106886,129596,19798,26784,12934}
{2921,505946,7352,504330,22946,482076,44046,515524,53082,207640,6232,97498,22710,109798,6986,13850}
{503025,498594,496978,481384,459130,438030,471478,462442,154558,201408,91266,74788,87088,102812,6864}
{4431,1616,15594,22254,21100,33448,9036,307884,46850,110142,16478,12300,15724,95948}
{2815,13978,6660,1154,12348,24412,298848,261034,63292,93664,4178,3424,80224}
{11163,7318,5506,11194,12064,274436,37814,197742,30372,89486,754,76800}
{3845,1812,5688,870,262372,236622,159928,167370,59114,88732,76046}
{2033,3876,4818,261502,25750,76694,7442,108256,29618,12686}
{1843,942,256684,235752,50944,69252,100814,78638,16932}
{901,255742,20932,184808,18308,31562,22176,61706}
{254841,234810,163876,166500,13254,9386,39530}
{20031,70934,2624,153246,3868,30144}
{50903,68310,150622,149378,26276}
{17407,82312,1244,123102}
{64905,81068,121858}
{16163,40790}
{24627}
}

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Plus petit premier manquant : 17
Plus petit non-premier manquant : 39

Jean-Marc:

J'ai essayé de commencer le tableau autrement, juste pour voir.
J'ai commencé arbitrairement par {2,11}, et après environ 3 heures de calcul (à cause du
66^e nb premier de la première ligne, qui vaut plus de 13000000), j'ai aussi un tableau
de 83 lignes avec un plus petit premier manquant égal à 13 et un plus petit composite
manquant valant 69, ce qui me semble ne pas infirmer la conjecture qu'on obtient tous
les naturels.

Tableau (bis) de 83 lignes (justifié à gauche) :

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{2,11,3,37,19,113,71,293,331,821,2789,7723,2707,14983,2459,947,5471,43,24359,1523,74861,15683,51431,190261,3739,11971,537
{9,8,34,18,94,42,222,38,490,1968,4934,5016,12276,12524,1512,4524,5428,24316,22836,73338,59178,35748,138830,186522,8232,52
{1,26,16,76,52,180,184,452,1478,2966,82,7260,248,11012,3012,904,18888,1480,50502,14160,23430,103082,47692,178290,517566,1
{25,10,60,24,128,4,268,1026,1488,2884,7178,7012,10764,8000,2108,17984,17408,49022,36342,9270,79652,55390,130598,339276,10
{15,50,36,104,124,264,758,462,1396,4294,166,3752,2764,5892,15876,576,31614,12680,27072,70382,24262,75208,208678,755178,19
{35,14,68,20,140,494,296,934,2898,4128,3586,988,3128,9984,15300,31038,18934,14392,43310,46120,50946,133470,546500,735442,
{21,54,48,120,354,198,638,1964,1230,542,2598,2140,6856,5316,15738,12104,4542,28918,2810,4826,82524,413030,188942,300426,9
{33,6,72,234,156,440,1326,734,688,2056,458,4716,1540,10422,3634,7562,24376,26108,2016,77698,330506,224088,111484,665970,4
{27,66,162,78,284,886,592,46,1368,1598,4258,3176,8882,6788,3928,16814,1732,24092,75682,252808,106418,112604,554486,237620,
{39,96,84,206,602,294,546,1322,230,2660,1082,5706,2094,2860,12886,15082,22360,51590,177126,146390,6186,441882,316866,1150
{57,12,122,396,308,252,776,1092,2430,1578,4624,3612,766,10026,2196,7278,29230,125536,30736,140204,435696,125016,201778,62
{45,110,274,88,56,524,316,1338,852,3046,1012,2846,9260,7830,5082,21952,96306,94800,109468,295492,310680,76762,139388,6844
{65,164,186,32,468,208,1022,486,2194,2034,1834,6414,1430,2748,16870,74354,1506,14668,186024,15188,233918,62626,545046,116
{99,22,154,436,260,814,536,1708,160,200,4580,4984,1318,14122,57484,72848,13162,171356,170836,218730,171292,482420,428512,
{77,132,282,176,554,278,1172,1548,40,4380,404,3666,12804,43362,15364,59686,158194,520,47894,47438,311128,53908,80568,5685
{55,150,106,378,276,894,376,1508,4340,3976,3262,9138,30558,27998,44322,98508,157674,47374,456,263690,257220,26660,23718,5
{95,44,272,102,618,518,1132,2832,364,714,5876,21420,2560,16324,54186,59166,110300,46918,263234,6470,230560,2942,530016,20
{51,228,170,516,100,614,1700,2468,350,5162,15544,18860,13764,37862,4980,51134,63382,216316,256764,224094,227618,527074,32
{177,58,346,416,514,1086,768,2118,4812,10382,3316,5096,24098,32882,46154,12248,152934,40448,32674,3528,299456,204886,2922
{119,288,70,98,572,318,1350,2694,5570,7066,1780,19002,8784,13272,33906,140686,112486,7774,29146,295928,94570,87406,196656,
{169,218,28,474,254,1032,1344,2876,1496,5286,17222,10218,4488,20634,106780,28200,104712,21372,266782,201358,7164,109250,5
{49,190,446,220,778,312,1532,1380,3790,11936,7004,5730,16146,86146,78580,76512,83340,245410,65424,194194,102086,141396,35
{141,256,226,558,466,1220,152,2410,8146,4932,1274,10416,70000,7566,2068,6828,162070,179986,128770,92108,311820,56818,7365
{115,30,332,92,754,1068,2258,5736,3214,3658,9142,59584,62434,5498,4760,155242,17916,51216,36662,219712,255002,16832,41884,
{85,302,240,662,314,1190,3478,2522,444,5484,50442,2850,56936,738,150482,137326,33300,14554,183050,35290,238170,25052,4430
{217,62,422,348,876,2288,956,2078,5040,44958,47592,54086,56198,149744,13156,104026,18746,168496,147760,202880,213118,4179
{155,360,74,528,1412,1332,1122,2962,39918,2634,6494,2112,93546,136588,90870,85280,149750,20736,55120,10238,204844,119836,
{205,286,454,884,80,210,1840,36956,37284,3860,4382,91434,43042,45718,5590,64470,129014,34384,44882,194606,85008,151210,17
{81,168,430,804,130,1630,35116,328,33424,522,87052,48392,2676,40128,58880,64544,94630,10498,149724,109598,66202,24660,542
{87,262,374,674,1500,33486,34788,33096,32902,86530,38660,45716,37452,18752,5664,30086,84132,139226,40126,43396,41542,5173
{175,112,300,826,31986,1302,1692,194,53628,47870,7056,8264,18700,13088,24422,54046,55094,99100,3270,1854,475804,139890,14
{63,188,526,31160,30684,390,1498,53434,5758,40814,1208,10436,5612,11334,29624,1048,44006,95830,1416,473950,335914,4744,29
{125,338,30634,476,30294,1108,51936,47676,35056,39606,9228,4824,5722,18290,28576,42958,51824,94414,472534,138036,331170,2
{213,30296,30158,29818,29186,50828,4260,12620,4550,30378,4404,898,12568,10286,14382,8866,42590,378120,334498,193134,30690
{30083,138,340,632,21642,46568,8360,8070,25828,25974,3506,11670,2282,4096,5516,33724,335530,43622,141364,113770,13074,216
{29945,202,292,21010,24926,38208,290,17758,146,22468,8164,9388,1814,1420,28208,301806,291908,97742,27594,100696,8612,2123
{29743,90,20718,3916,13282,37918,17468,17612,22322,14304,1224,7574,394,26788,273598,9898,194166,70148,73102,92084,203714,

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{29653,20628,16802,9366,24636,20450,144,4710,8018,13080,6350,7180,26394,246810,263700,184268,124018,2954,18982,111630,192.
{9025,3826,7436,15270,4186,20306,4566,3308,5062,6730,830,19214,220416,16890,79432,60250,121064,16028,92648,80856,333730,8.
{5199,3610,7834,11084,16120,15740,1258,1754,1668,5900,18384,201202,203526,62542,19182,60814,105036,76620,11792,252874,244.
{1589,4224,3250,5036,380,14482,496,86,4232,12484,182818,2324,140984,43360,41632,44222,28416,64828,241082,8814,50270,17790.
{2635,974,1786,4656,14102,13986,410,4146,8252,170334,180494,138660,97624,1728,2590,15806,36412,176254,232268,41456,32480.
{1661,812,2870,9446,116,13576,3736,4106,162082,10160,41834,41036,95896,862,13216,20606,139842,56014,190812,8976,147558,16.
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{1209,4518,2754,4130,3620,9470,157606,6054,120248,30876,54062,40174,82680,4964,111846,35408,50970,47038,43254,7072,16552.
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Plus petit premier manquant : 13
Plus petit non premier manquant : 69

Merci à tous !
É.