OEIS A197032

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ABSTRACT. The constant 2.3532099... in sequence [1, A197032] is a root of a cubic polynomial.

The equation of a bundle of lines with inclination α that run through the point $(x, y) = (2, 1)$ is

$$y = \alpha(x - 2) + 1. \tag{1}$$

These lines intersect the horizontal axis at

$$\alpha(x - 2) + 1 = 0 \tag{2}$$

which has the solution

$$(x_1, y_1) = (2 - \frac{1}{\alpha}, 0). \tag{3}$$

These lines intersect the diagonal $y = x$ at

$$\alpha(x - 2) + 1 = x \tag{4}$$

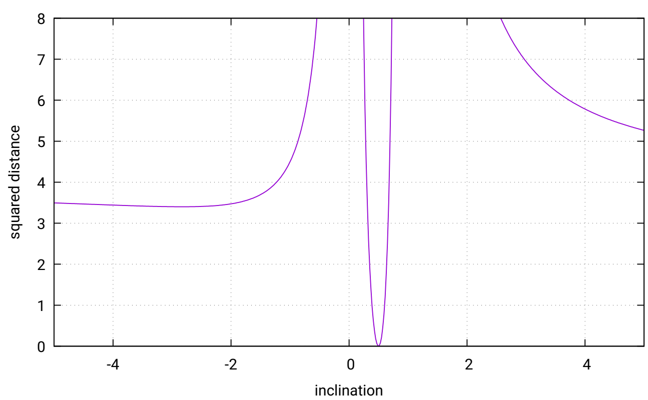
which has the solution

$$(x_2, y_2) = (2 + \frac{1}{\alpha - 1}, 2 + \frac{1}{\alpha - 1}). \tag{5}$$

The Euclidean distance between the intersections of the horizontal line and the diagonal is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \frac{|2\alpha - 1|\sqrt{1 + \alpha^2}}{|\alpha - 1|\,|\alpha|}. \tag{6}$$

The plot of the squared distance $d^2(\alpha)$ as a function of α looks as follows:



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The Philo Line is defined by the minimum of that curve at $\alpha \approx -2.831177\ldots$ [The global minimum at inclination $\alpha = 1/2$ does not define a triangle but means that the horizontal line, the diagonal and the line of the bundle all intersect at $(0,0)$.]

An arithmetic expression for the location of the minimum is obtained by setting the derivative $\partial d^2/\partial\alpha = 0$, so

$$-2\frac{(2\alpha-1)(\alpha^3+2\alpha^2-2\alpha+1)}{\alpha^3(\alpha-1)^3} = 0 \tag{7}$$

equivalent to the root of the polynomial

$$\alpha^3+2\alpha^2-2\alpha+1 = 0 \therefore \alpha_1 \approx -2.831177 \tag{8}$$

and the interception at

$$x_1 = 2 - \frac{1}{\alpha_1} \approx 2.3532099\ldots \tag{9}$$

Inverting (3) as $\alpha_1 = 1/(2-x_1)$ and plugging this into (8) one finds that x_1 is a root of the polynomial

$$x^3-4x^2+6x-5 = (x-1)^3-(x-1)^2+(x-1)-2. \tag{10}$$

This illustrates also the connection with [1, A357469].

References

[1] O. E. I. S. Foundation Inc., *The On-Line Encyclopedia Of Integer Sequences*, (2022), https://oeis.org/. MR 3822822

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