A Recurring Digital Invariant variant

[le titre est (c) Mensanator sur rec.puzzles]

L'idée est la suivante :

a) choisir un nombre N
b) appeler « k » la quantité de chiffres qui composent N
c) élever tous les chiffres de N à la puissance k et additionner les résultats
d) appeler ce nouveau nombre N et retourner à l'instruction (b)

Exemple :

```
a) 14 = N
b) k = 2
c) 1^2 + 4^2 = 17
d) 17 = N
e) k = 2
f) 1^2 + 7^2 = 50
g) 50 = N
... etc.
```

On se propose d'étudier les boucles, les points fixes éventuels, etc.

Tout avait commencé (à la mi-février 2009) par une lecture (le lien bleu ci-dessous) et un double message aux listes SeqFans et rec.puzzles.

Hello SeqFans,

http://mathworld.wolfram.com/RecurringDigitalInvariant.html

... what if k = "length of the considered integer"?
(k = 2 for the integer 14, for instance)

Starting with said 14:

```
 14 -> 1^2 + 4^2 = 17
 17 -> 1^2 + 7^2 = 50
 50 -> 5^2 + 0^2 = 25
 25 -> 2^2 + 5^2 = 29
 29 -> 2^2 + 9^2 = 85
 85 -> 8^2 + 5^2 = 89
 89 -> 8^2 + 9^2 = 145
145 -> 1^3 + 4^3 + 5^3 = 190
190 -> 1^3 + 9^3 + 0^3 = 730
730 -> 7^3 + 3^3 + 0^3 = 370
370 -> 3^3 + 7^3 + 0^3 = 370 (fixed point)
```

First fixed points:
S = 1, 2, 3, 4, 5, 6, 7, 8, 9, ... , 370, ...

Any more?
Best,
É.

[Réponse de **Mensanator** sur *rec.puzzles*] :

```
Oh, a Recurring Digital Invariant variant, eh?

> Any more?

Lots.
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 370, 217, 153, 352, 371, 136, 586,
886898, 1009, 160, 244, 76438, 853, 259, 736, 862, 664, 2929, 407,
496, 845130, 3283, 8208, 6514, 6562, 50062, 23131558, 1634, 2178,
124618, 13154, 4274, 59536, 3233, 7154, 4394, 9474]
```

[Python]:

```
import gmpy
inv_hist =[]
for n in xrange(10000):
  hist = []
  while n not in hist:
    hist.append(n)
    s = gmpy.digits(n)
    p = len(s)
    n = 0
    for d in s:
      n += int(d)**p
  if n not in inv_hist:
    inv_hist.append(n)
print inv_hist
```

[Réponse de **Hans** Havermann sur *SeqFans*] :

```
Envoyé : mercredi 18 février 2009 2:00
À : Sequence Fanatics Discussion list
Objet : [seqfan] Re: Recurring Digital Invariant

Eric Angelini:
> ... what if k = "length of the considered integer"?

If I am doing this correctly, here are the first 34 cycles (by size
of smallest precursor).

The format for each is:
```

index {smallest precursor, cycle length, {the cycle itself with the smallest element of the cycle first}}:

```
 1 {     1,    1, {1}}
 2 {     2,    1, {2}}
 3 {     3,    1, {3}}
 4 {     4,    1, {4}}
 5 {     5,    1, {5}}
 6 {     6,    1, {6}}
 7 {     7,    1, {7}}
 8 {     8,    1, {8}}
 9 {     9,    1, {9}}
10 {    14,    1, {370}}
11 {    59,    3, {160, 217, 352}}
12 {   108,    1, {153}}
13 {   119,    1, {371}}
14 {   136,    2, {136, 244}}
15 {   138,   10, {259, 862, 736, 586, 853, 664, 496, 1009, 6562,
```

```
3233}}
16 {   147,   14, {18829, 124618, 312962, 578955, 958109, 1340652,
376761, 329340, 537059, 681069, 886898, 1626673, 1665667, 2021413}}
17 {   177,    2, {58618, 76438}}
18 {   389,    6, {2929, 13154, 4394, 7154, 3283, 4274}}
19 {   407,    1, {407}}
20 {   559,    3, {282595, 824963, 845130}}
21 {   709,    1, {8208}}
22 {   999,    2, {2178, 6514}}
23 {  1118,    4, {10933, 59536, 73318, 50062}}
24 {  1157,   12, {5908997, 17347727, 23131558, 17571846, 30442597,
49340036, 44870531, 23070276, 13216291, 44733413, 5981093, 11743403}}
25 {  1346,    1, {1634}}
26 {  4479,    1, {9474}}
27 { 11227,    1, {54748}}
28 { 12399,    1, {32164049651}}
29 { 22779,    1, {92727}}
30 { 30489,    1, {93084}}
31 {100666,   12, {1680387, 5299971, 15250704, 6611844, 2689794,
12783081, 39326052, 45130596, 45579685, 68505765, 27073124,
11602212}}
32 {127779,    1, {548834}}
33 {577999,    1, {4210818}}
34 {677779,    3, {2767918, 8807272, 5841646}}
35 {1000259,   1, {9926315}}
36 {1001458,   6, {2191663, 5345158, 2350099, 9646378, 8282107,
5018104}}
37 {1007889,   1, {9800817}}
38 {1035889,   2, {8139850, 9057586}}
39 {1124577,   1, {1741725}}
40 {1188888,   1, {24678051}}
41 {2055779,   2, {2755907, 6586433}}
42 {2566699,   1, {472335975}}
43 {4888888,  10, {180450907, 564207094, 440329717, 468672187,
369560719, 837322786, 359260756, 451855933, 527799103, 857521513}}
44 {10135679,  1, {24678050}}
45 {10146899,  1, {146511208}}
46 {10233389,  1, {88593477}}
47 {10266888,  7, {1139785743, 5136409024, 3559173428, 4863700423,
1418899523, 9131926726, 7377037502}}
48 {14489999,  3, {180975193, 951385123, 525584347}}
49 {14788889,  1, {912985153}}
50 {20248999,  1, {534494836}}
51 {155999999, 2, {277668893, 756738746}}
```

Any number < 10^9 will fall into one of these 51 cycles.

Magnifiques travaux, **Mensanator** et **Hans** !

On peut tirer plusieurs suites à partir de cette idée (les trois dernières ont été calculées par **Hans**) :

Suite S(1) des nombres qui cyclent sur eux-mêmes (nombres « **narcissiques** ») ; on regarde les « 1 » de la 3^{e} colonne ci-dessus :

S(**1**) = 1,2,3,4,5,6,7,8,9,153,370,371,407,1634,8208,9474,54748,92727,93084,548834, ... [A005188]. Cette suite ne comporte que 88 termes.

Suite S(2) des plus petits nombres qui entrent dans un cycle encore inconnu (c'est la 2e colonne ci-dessus) :

S(**2**) = 1, 2, 3, 4, 5, 6, 7, 8, 9, 14, 59, 108, 119, 136, 138, 147, 177, 389, 407, 559, 709, 999, 1118, 1157, 1346, 4479, 11227, 12399, 22779, 30489, 100666, 127779, 577999, 677779, 1000259, 1001458, 1007889, 1035889, 1124577, 1188888, 2055779, 2566699, 4888888, 10135679, 10146899, 10233389, 10266888, 14489999, 14788889, 20248999, 155999999, ...

Suite S(3) des nombres qui font partie d'un cycle (on classe par ordre croissant les résultats de la 4e colonne) :

S(**3**) = 1, 2, 3, 4, 5, 6, 7, 8, 9, 136, 153, 160, 217, 244, 259, 352, 370, 371, 407, 496, 586, 664, 736, 853, 862, 1009, 1634, 2178, 2929, 3233, 3283, 4274, 4394, 6514, 6562, 7154, 8208, 9474, 10933, 13154, 18829, 50062, 54748, 58618, 59536, 73318, 76438, 92727, 93084, 124618, 282595, 312962, 329340, 376761, 537059, 548834, 578955, 681069, 824963, 845130, 886898, 958109, 1340652, 1626673, 1665667, 1680387, 1741725, 2021413, 2191663, 2350099, 2689794, 2755907, 2767918, 4210818, 5018104, 5299971, 5345158, 5841646, 5908997, 5981093, 6586433, 6611844, 8139850, 8282107, 8807272, 9057586, 9646378, 9800817, 9926315, 11602212, 11743403, 12783081, 13216291, 15250704, 17347727, 17571846, 23070276, 23131558, 24678050, 24678051, 27073124, 30442597, 39326052, 44733413, 44870531, 45130596, 45579685, 49340036, 68505765, 88593477, 146511208, 180450907, 180975193, 277668893, 359260756, 369560719, 440329717, 451855933, 468672187, 472335975, 525584347, 527799103, 534494836, 564207094, 756738746, 837322786, 857521513, 912985153, 951385123, ...

Suite S(4) des nombres qui font partie d'un cycle sans être « narcissiques » ; **Hans** Havermann les appelle « pseudo-altruistes » :

S(**4**) = 136, 160, 217, 244, 259, 352, 496, 586, 664, 736, 853, 862, 1009, 2178, 2929, 3233, 3283, 4274, 4394, 6514, 6562, 7154, 10933, 13154, 18829, 50062, 58618, 59536, 73318, 76438, 124618, 282595, 312962, 329340, 376761, 537059, 578955, 681069, 824963, 845130, 886898, 958109, 1340652, 1626673, 1665667, 1680387, 2021413, 2191663, 2350099, 2689794, 2755907, 2767918, 5018104, 5299971, 5345158, 5841646, 5908997, 5981093, 6586433, 6611844, 8139850, 8282107, 8807272, 9057586, 9646378, 11602212, 11743403, 12783081, 13216291, 15250704, 17347727, 17571846, 23070276, 23131558, 27073124, 30442597, 39326052, 44733413, 44870531, 45130596, 45579685, 49340036, 68505765, 180450907, 180975193, 277668893, 359260756, 369560719, 440329717, 451855933, 468672187, 525584347, 527799103, 564207094, 756738746, 837322786, 857521513, 951385123, 1139785743, 1418899523, 3559173428, 4863700423, 5136409024, 7377037502, 9131926726, 59906808718, 66814785298, 71352591397, 90920874919, 99312318232, 136095696124, 571650873350, 1113928853354, 1128275756843, 1308860468429, 3396705890823, 3643890762383, 3654709782417, 3656948275943, 3764461348892, 3764592377975, 4217390478269, 5486860104254, 5650346085989, 5759076689801, 5840462013812, 6213095485028, 6294418483143, 6405584099531, 22955961974580, 24318257549352, 27510477911590, 27971919071792, 28794385423806, 32357226447319, 36834169210461, 47800729611562, 73803590128032, 94220062144011, 255349823145519, 321411732579837, 447090882837630, 1988938580054728, 2276352319249162, 2419253396913226, 2766744975063429, 3745072497367240, 3814368015105159, 4314122390900936, 4840861420987271, 5146957705687367, 5561890395668808, 5564859798630665, 18963633035544997, 21697619891079652, 21897923093961655, 21914086555935085, 25950934023321628,

33637808638944484, 35624633319183334, 35876461872431926,
36306344090162179, 37878721692554416, 37909523382771553,
38160589126493611, 52551389500766905, 69228536582676925,
69477330558375418, ...

Hans termine son courrier avec cette question pertinente :

> Knowing that the number of cycles of length 1 is finite, a question remains:
> *Is the number of cycles of ALL possible lengths also finite?*

Qui trouvera par ailleurs ne fut-ce qu'un cycle de longueur 5 ? En existe-t-il ?

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Depuis cet appel à l'aide, un cycle de longueur 5 a été trouvé indépendamment par **Hans** Havermann puis **Jean-Paul** Davalan :

3656948275943 5759076689801 6405584099531 5650346085989 6213095485028

« Ces nombres ont 13 chiffres. Il n'existe pas d'autre 5-cycle contenant un nombre de 17 chiffres ou moins », précise **Jean-Paul**.

Merci à tous,
(*à suivre*)
É.

(cette page du site de Harvey Heinz nous fut précieuse)

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Retour à la page d'accueil du site.