ILLUSTRATION OF ALIQUOT SEQUENCE MERGERS

RICHARD J. MATHAR

Abstract. Aliquot sequences are illustrated by assembling members of prime families into a single graph, with arrows pointing to the next member of aliquot sequence.

1. Intro

An aliquot sequence is an iterative mapping of positive integers $n \to \sigma(n) - n$, from n to the sum of its divisors that are less than n [5, 1, 8, 7][6, Sec. B5]. These connections are tabulated in entry A001065 of the OEIS `www.oeis.org` [9], and can be investigated by the database in `www.factordb.com`.

The typical track of such a mapping ends in a prime number, and this prime number is followed by a 1, which is generally not mentioned explicitly. The length of the track is entry A098007 [9]. In other cases, the track may enter a cycle, see A098007 in the OEIS [9], even a cycle of length one. In any case, we may build families of the integers having common parts of their tracks (i.e., having the same termination, A115350 in the OEIS [9]).

The graphs in section 2 show some of these families, whereas section 3 considers the map $n \to \sigma^*(n)$.

2. Aliquot sequence Illustrations

The following plots up to Figure 72 show some of these mergers. Where they end in a known prime, that prime is marked with a `p`.

Obviously, each of the plots is only a part of a tree-type structure. Because I generated them by tracing a finite set of initial values, the "early" plots may appear to have more members, but this is an illusion generated from this statistical sampling artifact. In addition, I have not traced the sequences completely in all cases; this means the individual families are not complete up to the maximum number that appears in the individual plot!

The plots are created with the `dot` command of `www.graphviz.org`.

Date: March 21, 2014.
Key words and phrases. Aliquot Sequence, Divisors.

FIGURE 1. Prime family 3 [9, A127163]

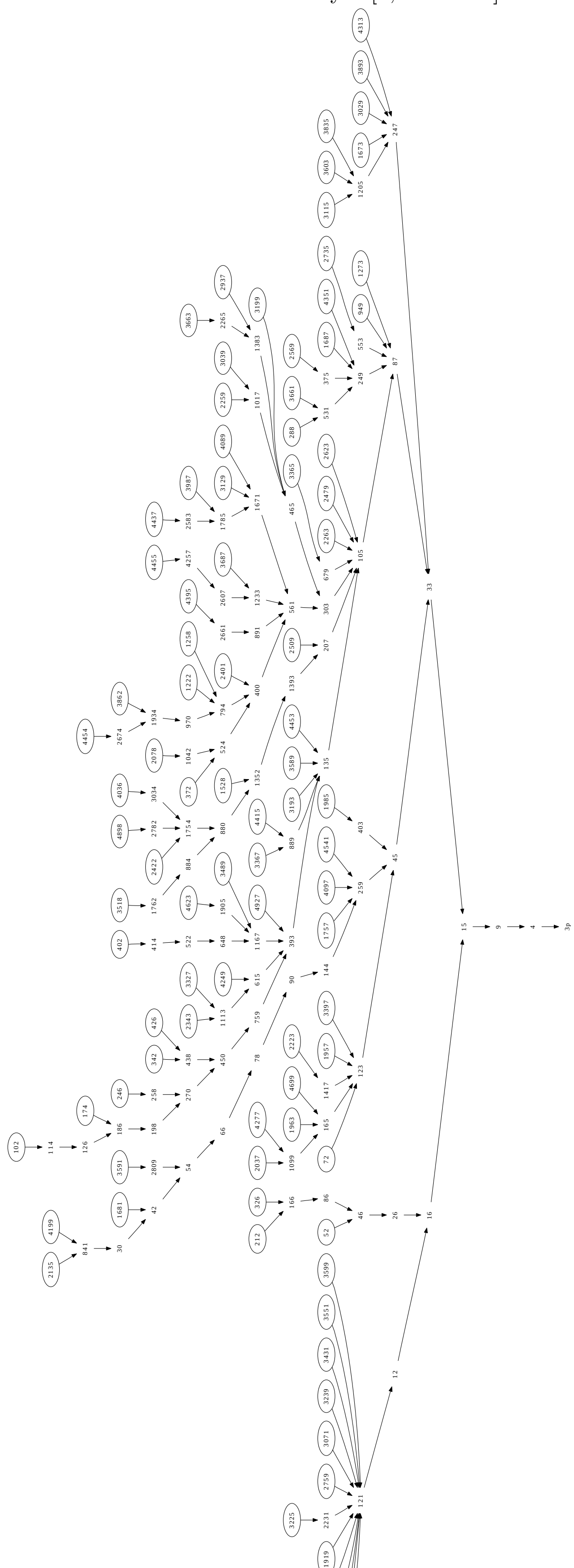
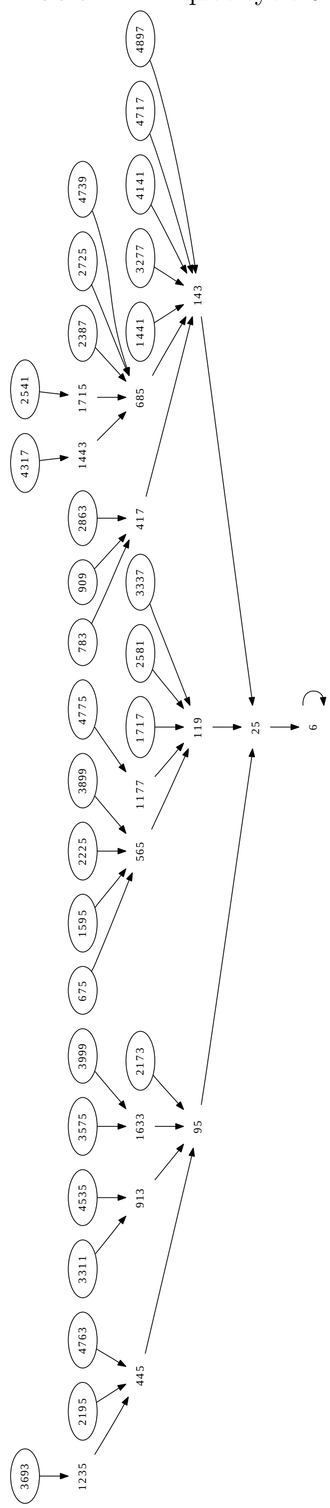


FIGURE 2. Aliquot cycle 6.



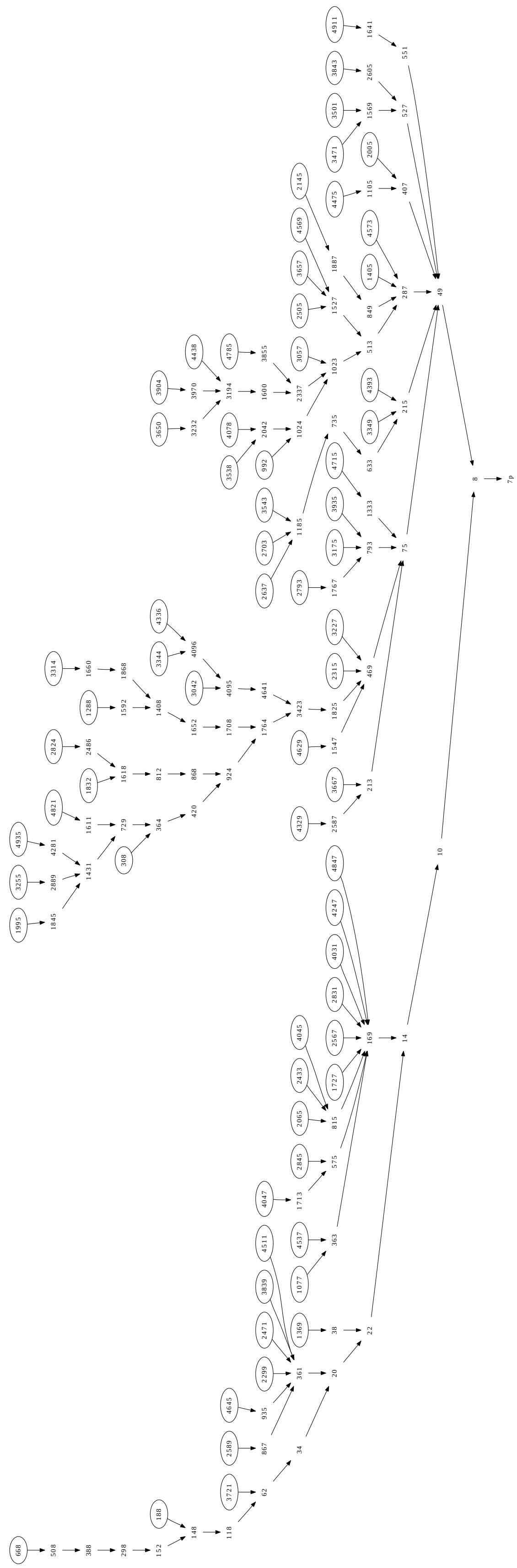
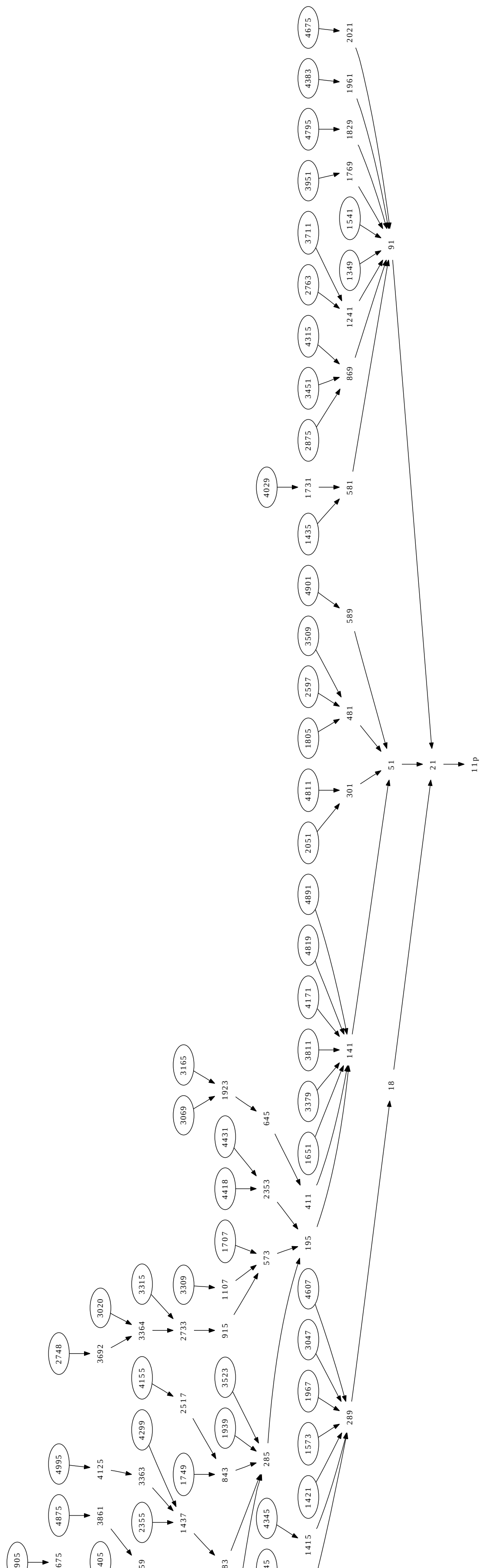
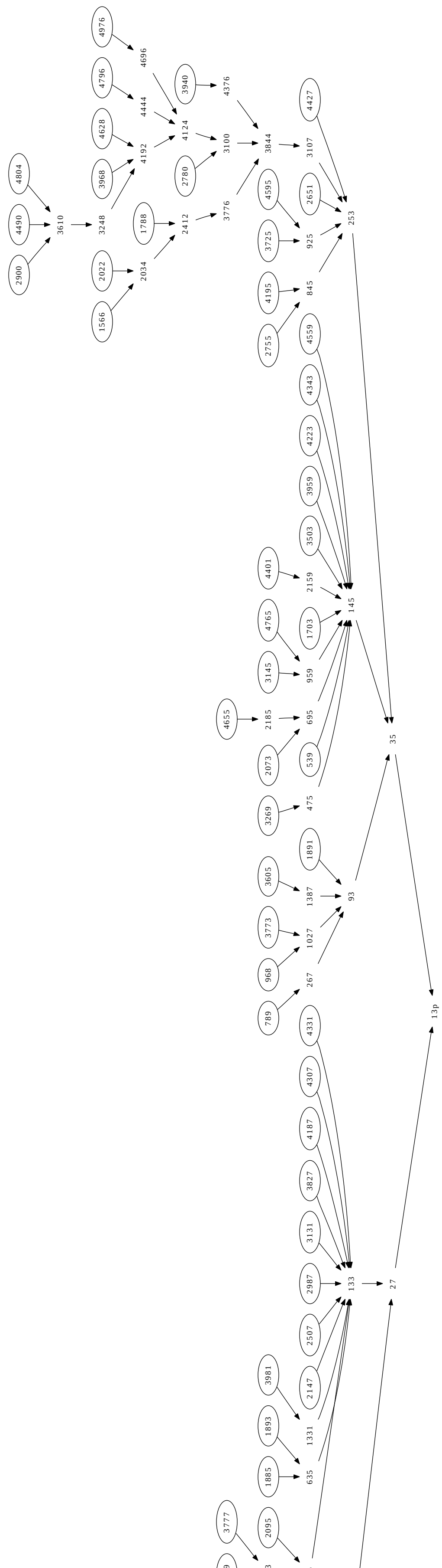


FIGURE 3. Prime family 7 [9, A127164].


4675
2021
4383
1961
4795
1829
3951
1769
1541
3711
91
1349
2763
1241
4315
869
3451
2875
4029
1731
581
1435
4901
589
3509
2597
481
1805
51
21
11p
4811
301
2051
4891
4819
4171
3811
141
3165
1923
3379
18
3069
645
1651
4431
4418
2353
411
1707
195
573
3315
3309
1107
4607
3020
3364
2733
915
2748
3692
3047
4155
3523
2517
1967
289
4299
1939
1573
4995
4125
3363
1749
843
285
1421
4875
3861
2355
1437
4345
1415



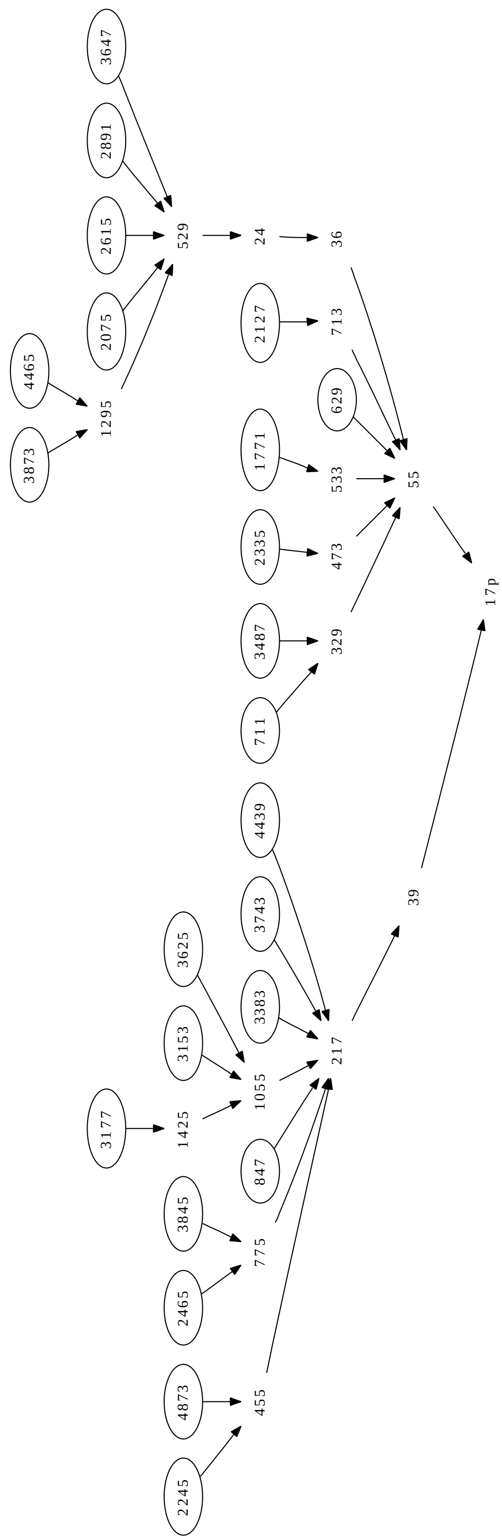


FIGURE 6. Prime family 17

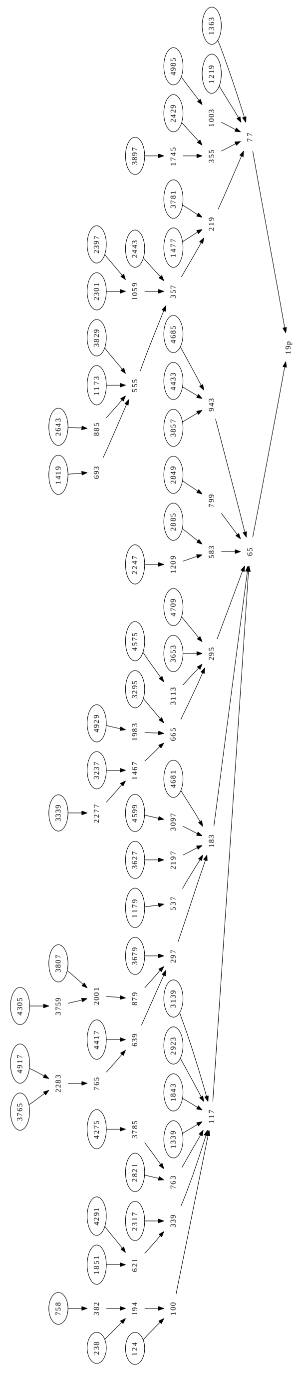


FIGURE 7. Prime family 19

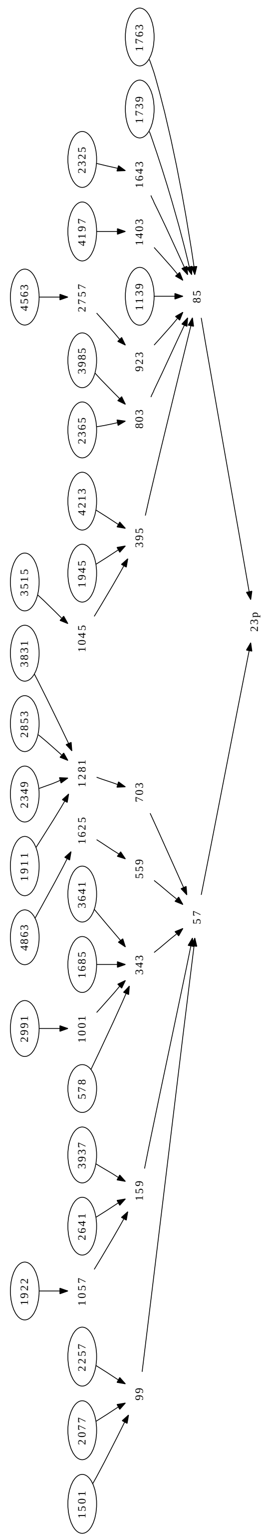


FIGURE 8. Prime family 23

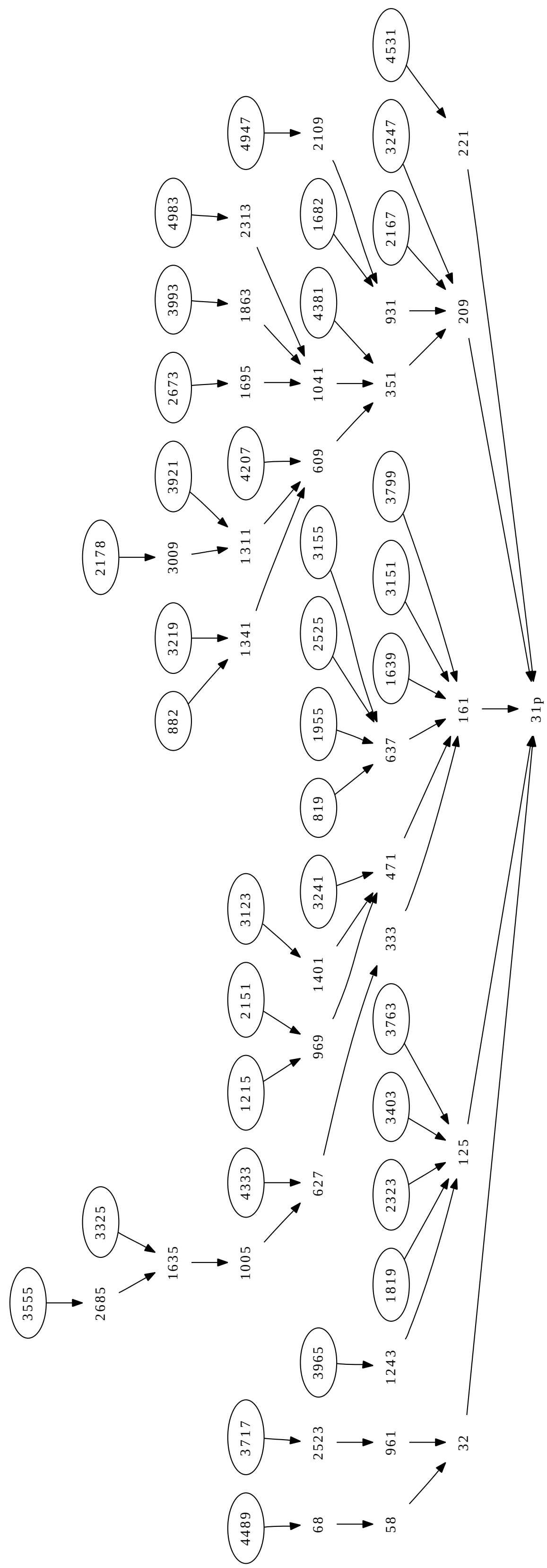


FIGURE 9. Prime family 31

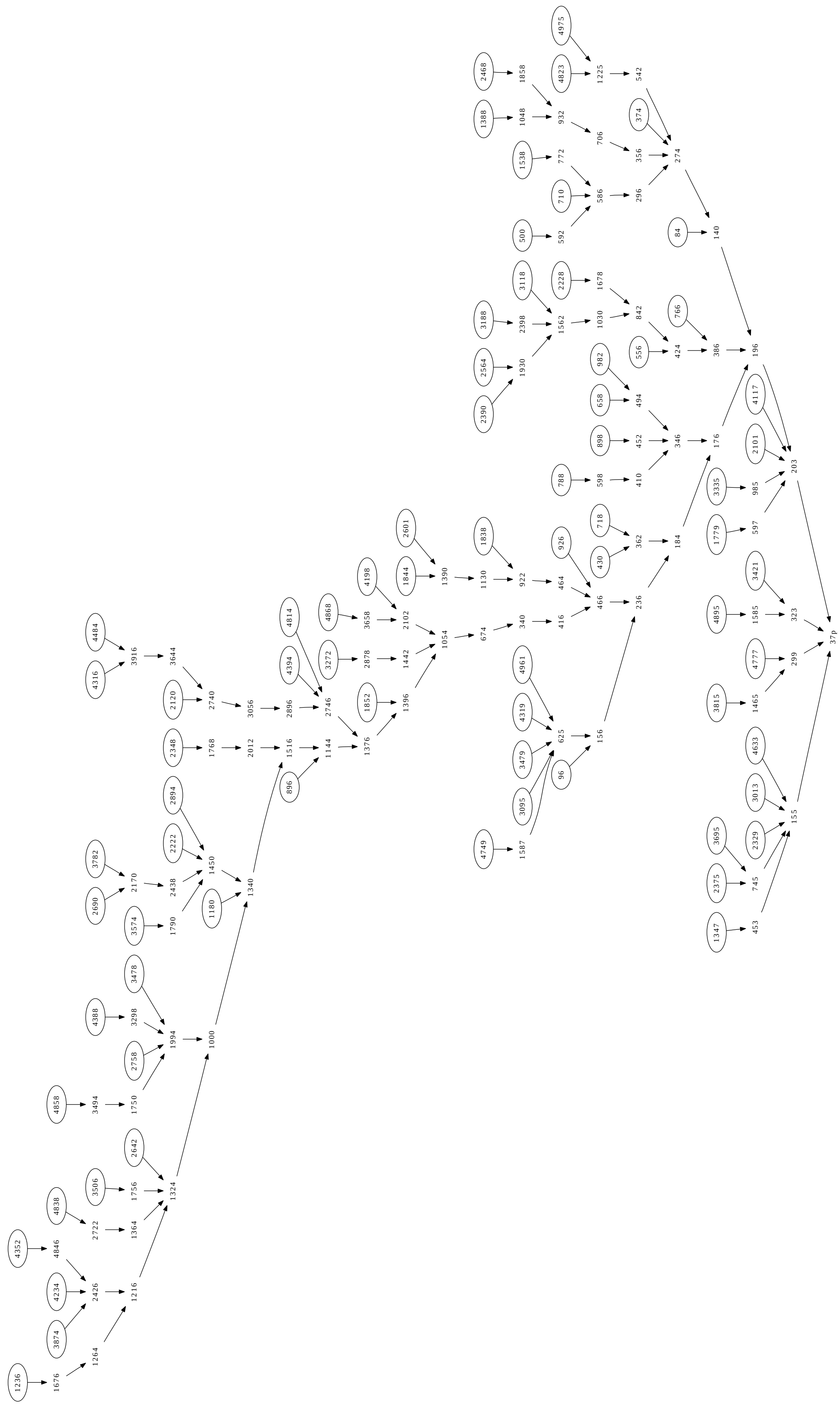
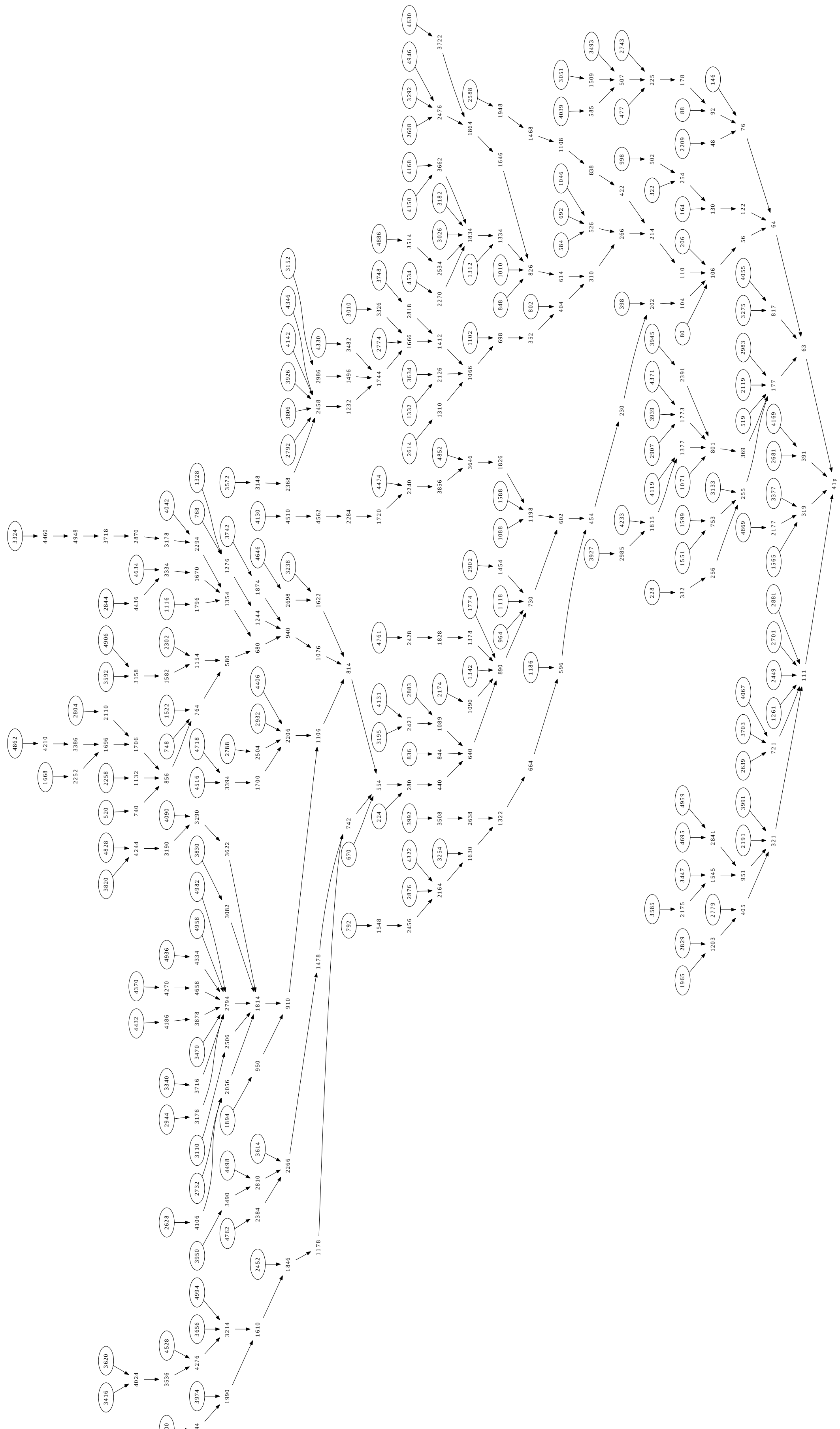
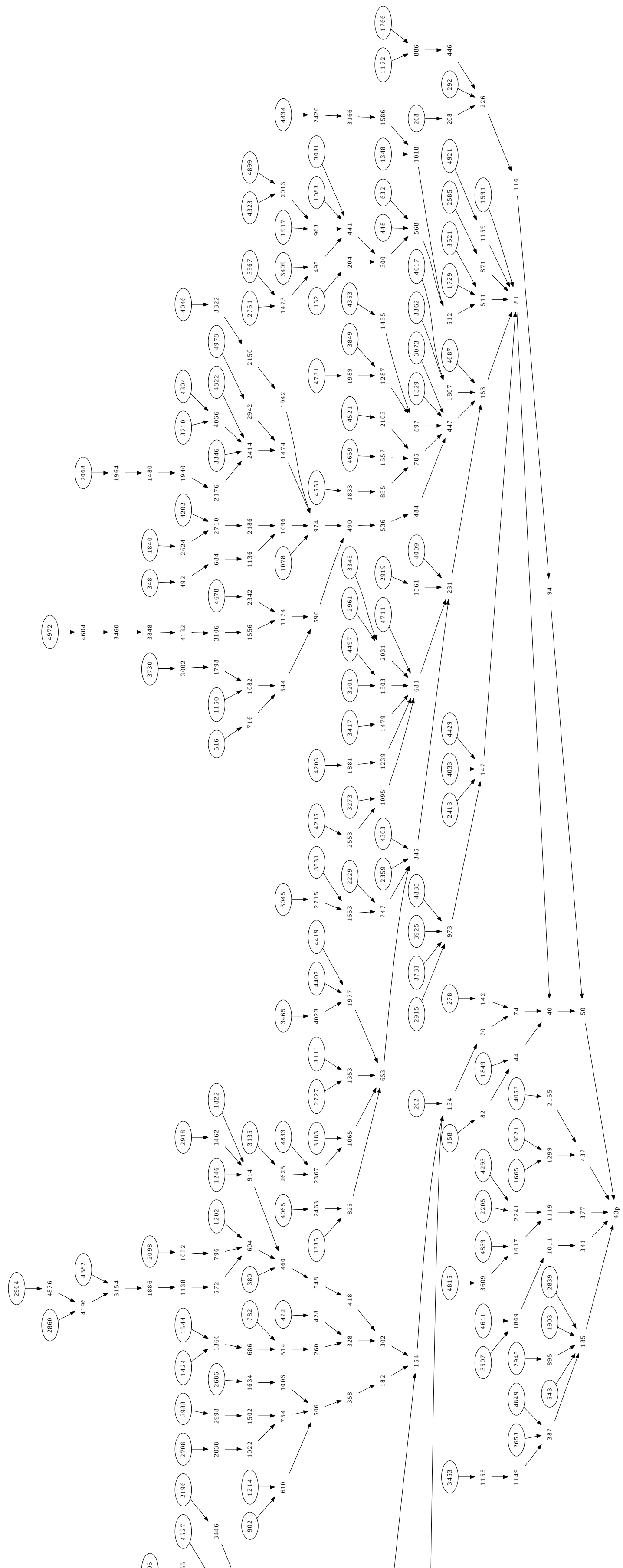


FIGURE 10. Prime family 37





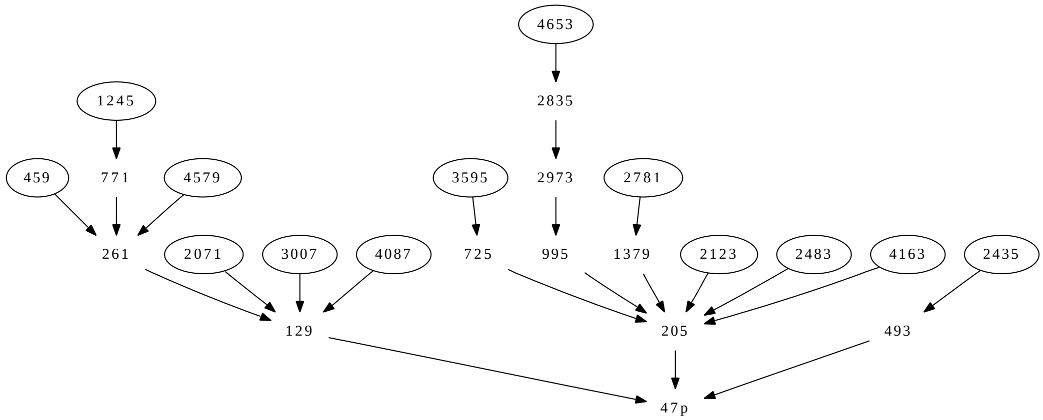


Figure 13. Prime family 47

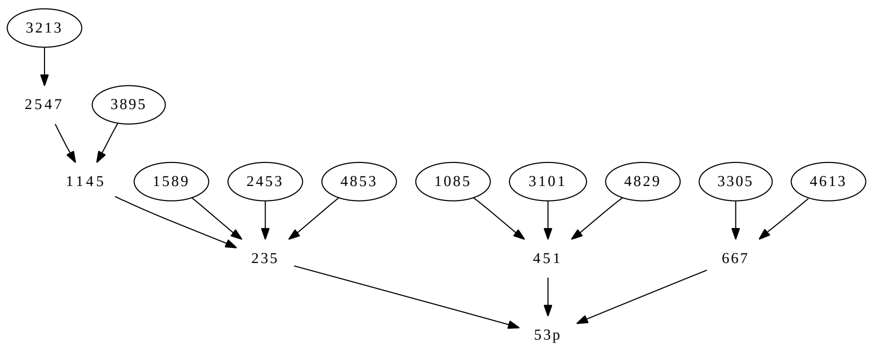


Figure 14. Prime family 53

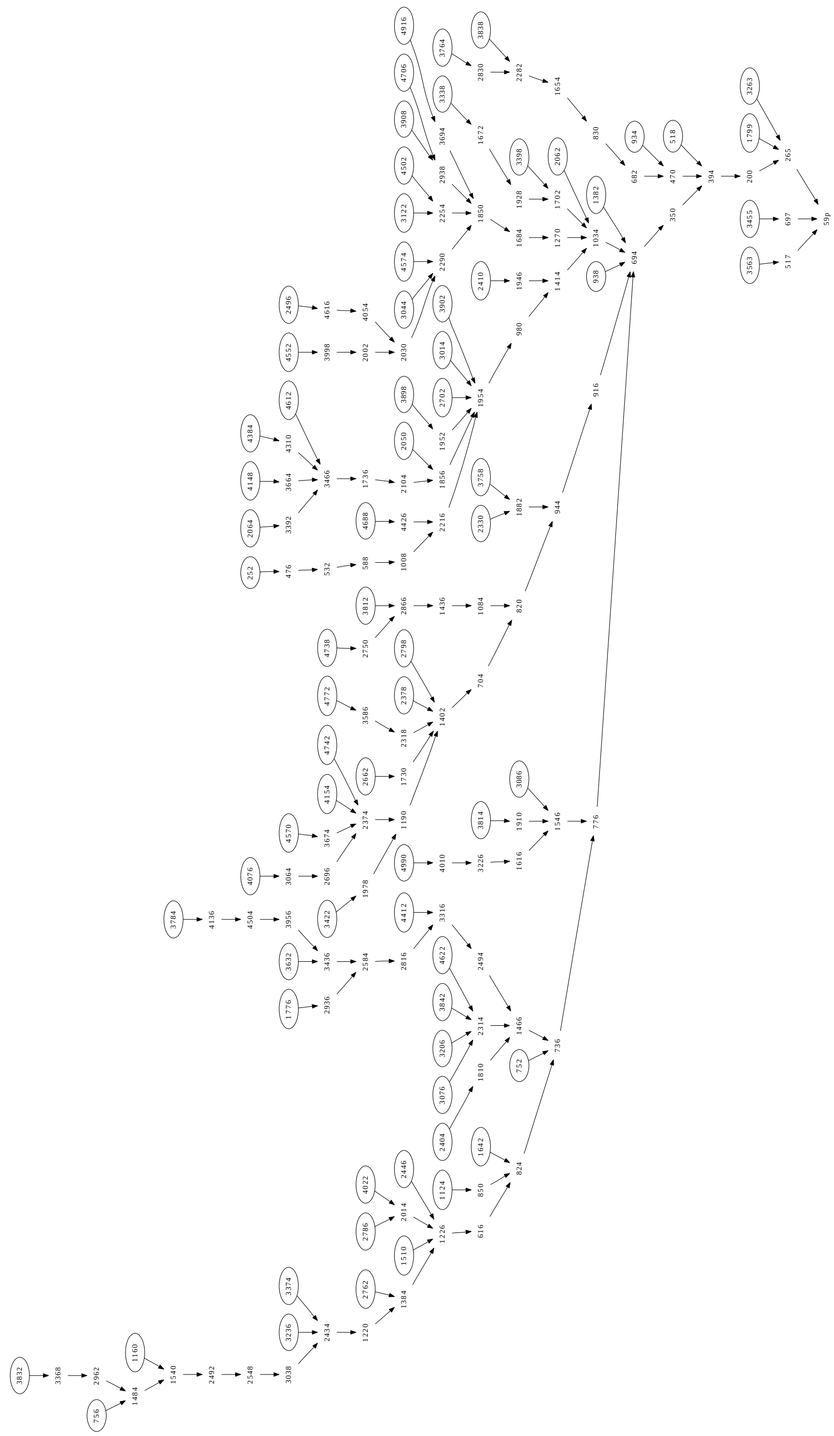


FIGURE 15. Prime family 59

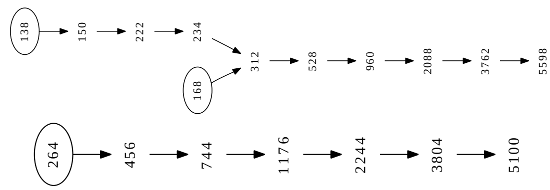


FIGURE 16. Other branches of prime family 59

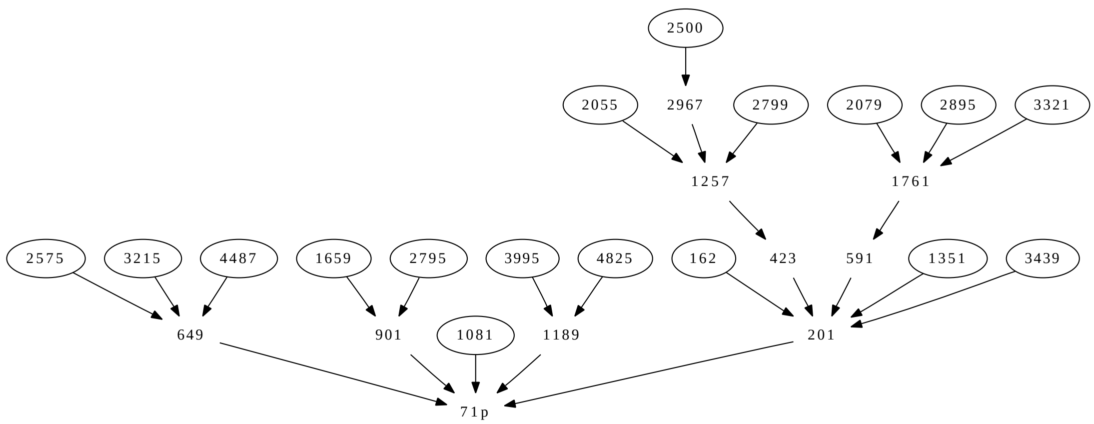


FIGURE 17. Prime family 71

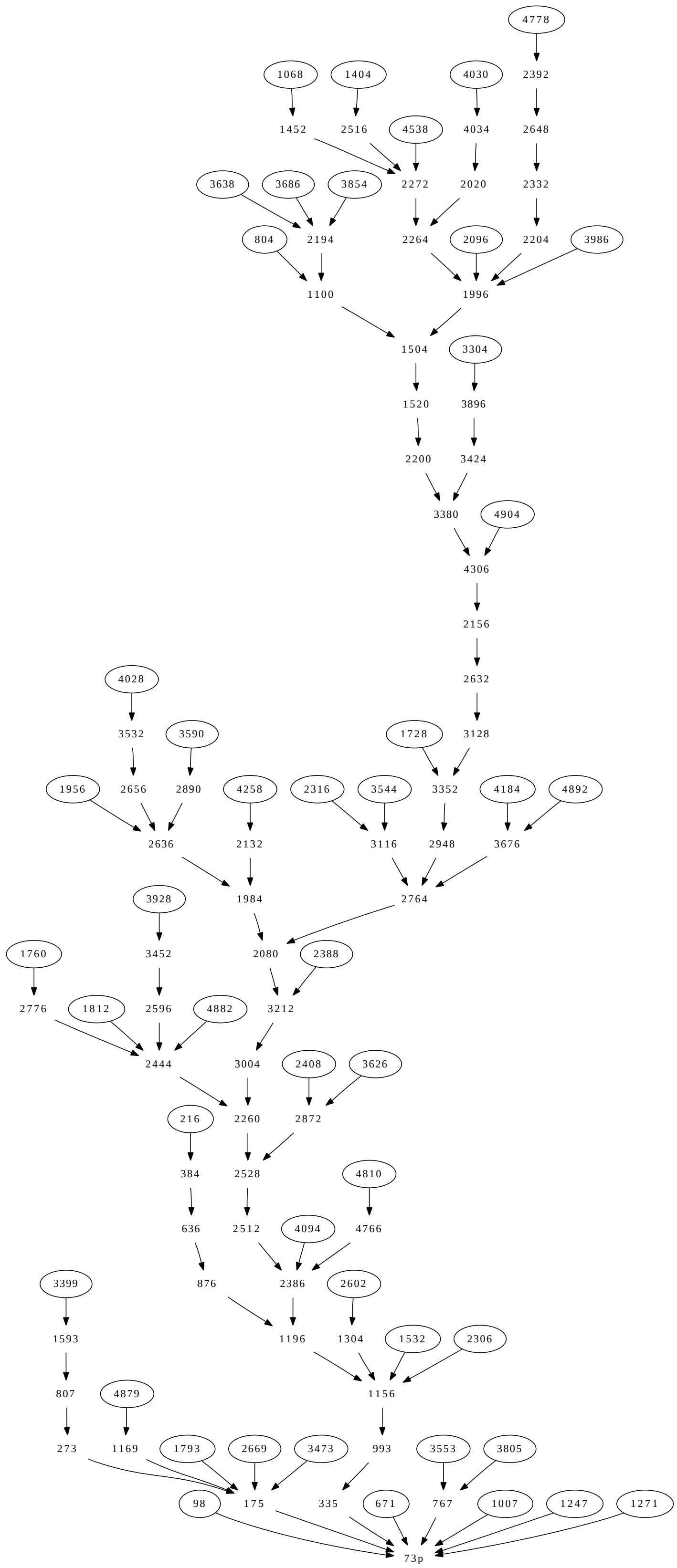


FIGURE 18. Prime family 73

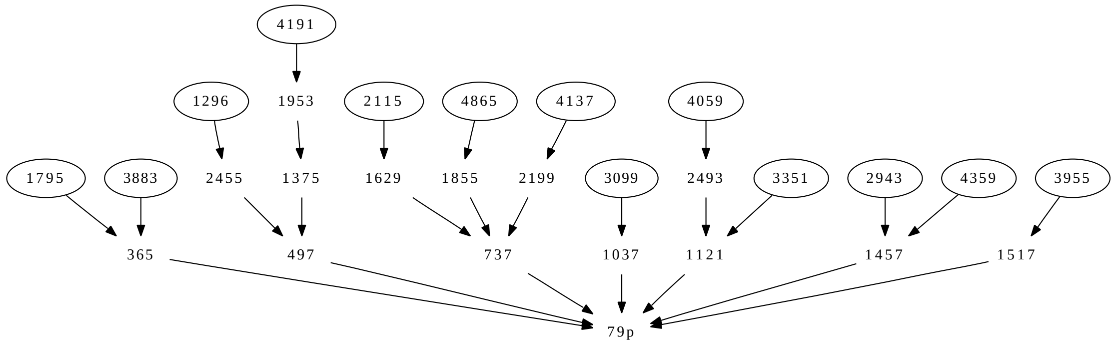


FIGURE 19. Prime family 79

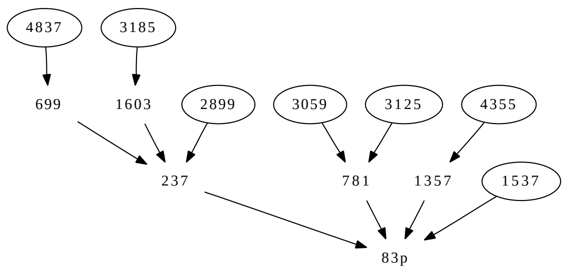


FIGURE 20. Prime family 83

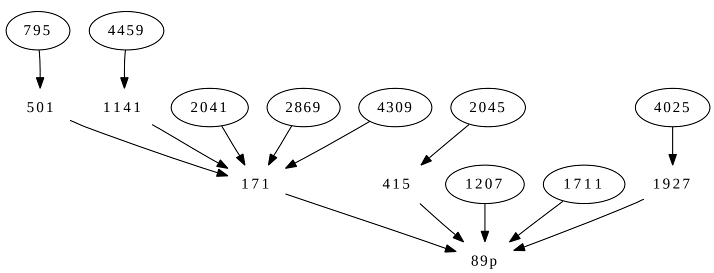


FIGURE 21. Prime family 89

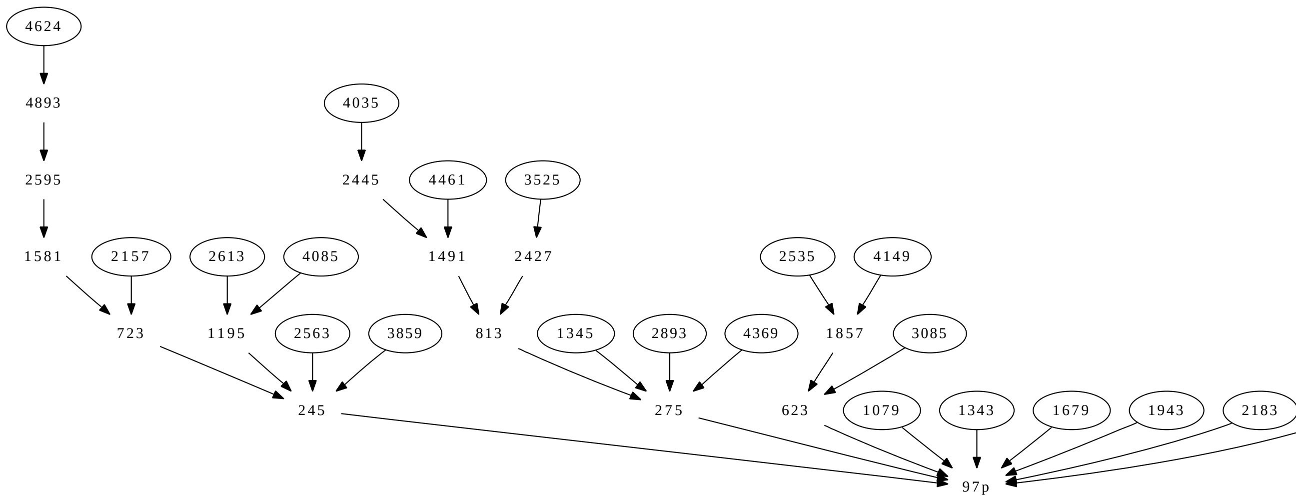


Figure 22. Prime family 97

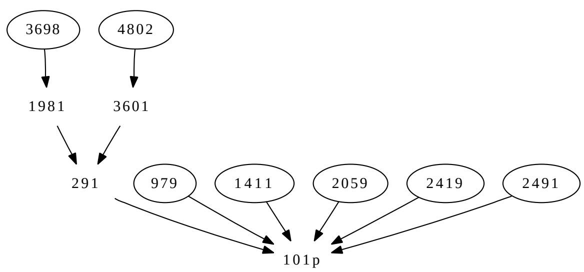


Figure 23. Prime family 101

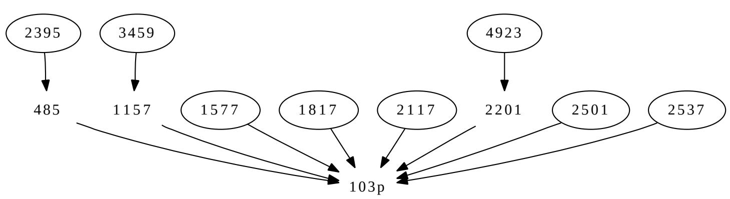


Figure 24. Prime family 103

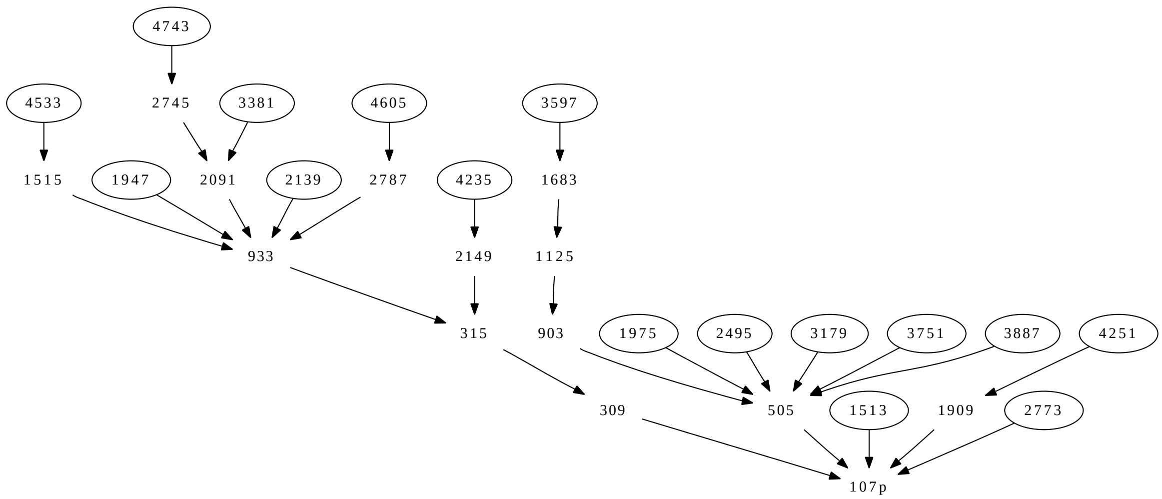


Figure 25. Prime family 107

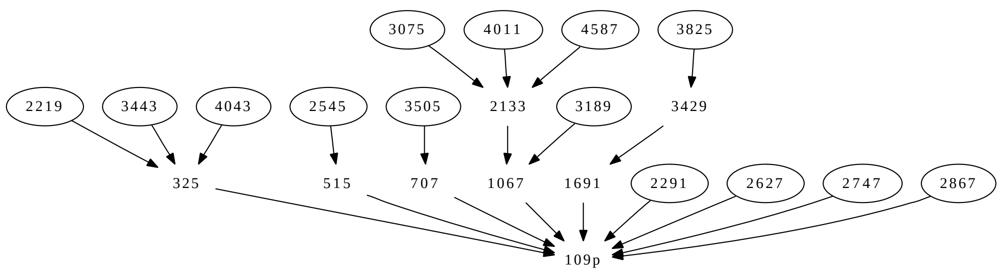


Figure 26. Prime family 109

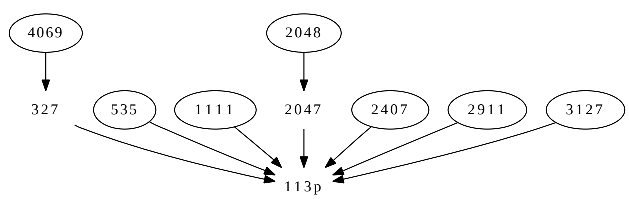


Figure 27. Prime family 113

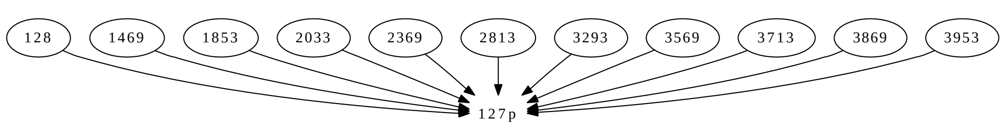


Figure 28. Prime family 127

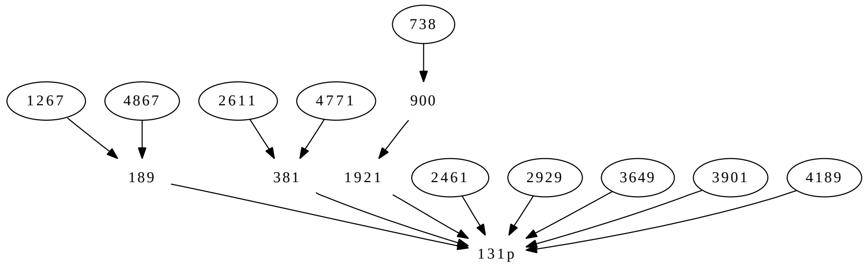


Figure 29. Prime family 131

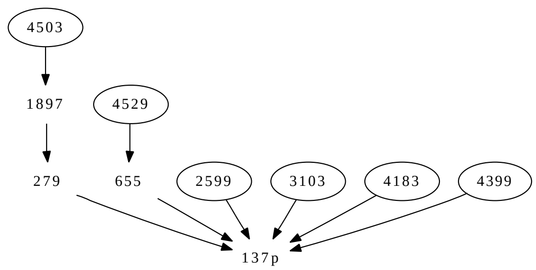


Figure 30. Prime family 137

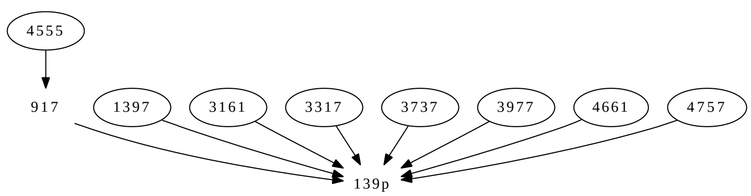


Figure 31. Prime family 139

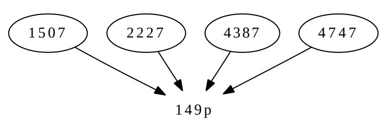


Figure 32. Prime family 149

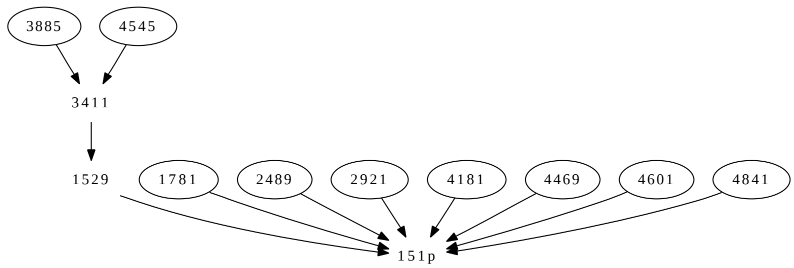


FIGURE 33. Prime family 151

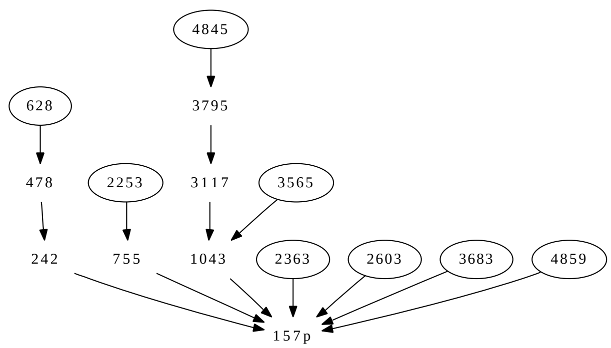


FIGURE 34. Prime family 157

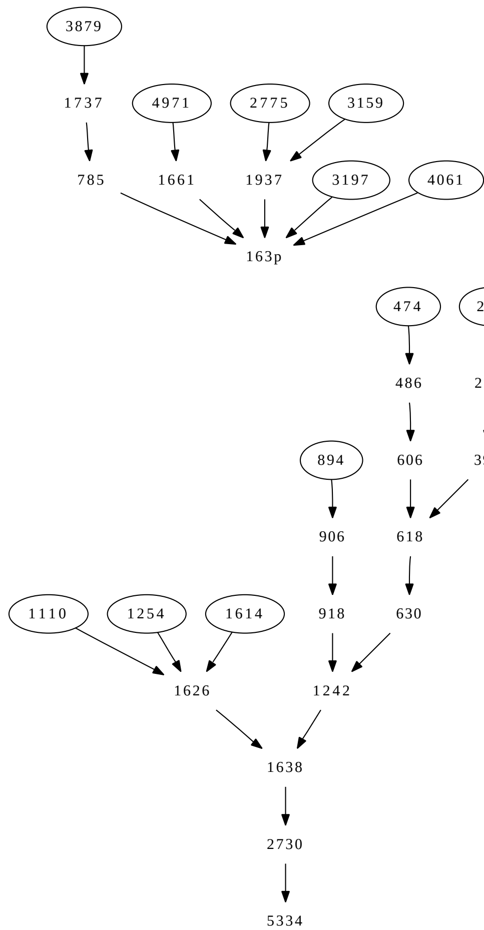


Figure 35. Prime family 163

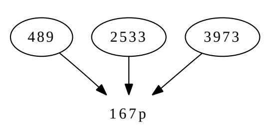


Figure 36. Prime family 167

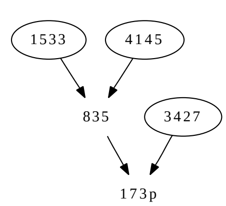


Figure 37. Prime family 173

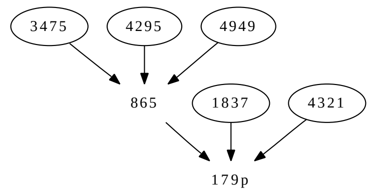


Figure 38. Prime family 179

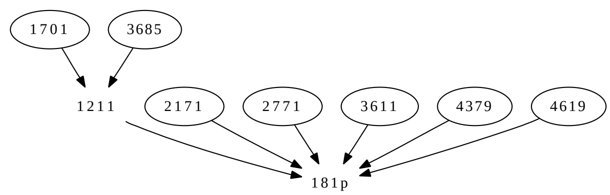


Figure 39. Prime family 181

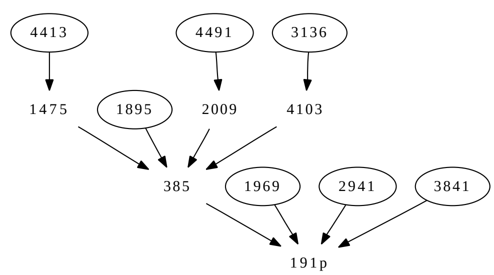


Figure 40. Prime family 191

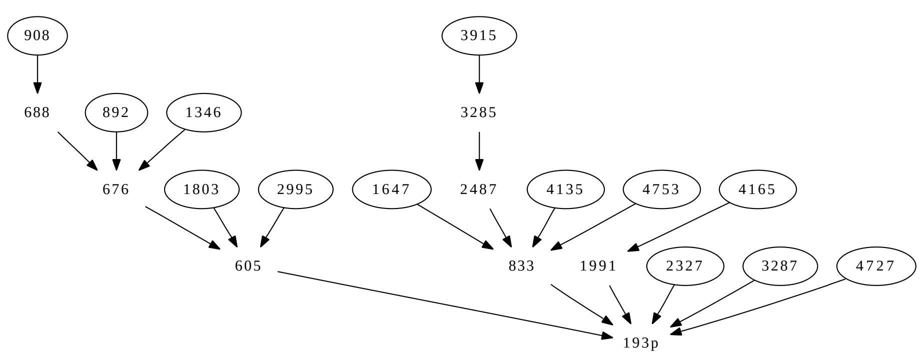


Figure 41. Prime family 193

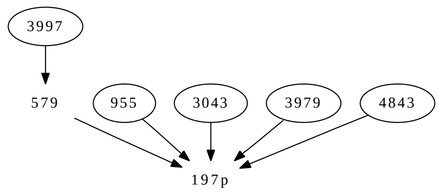


Figure 42. Prime family 197

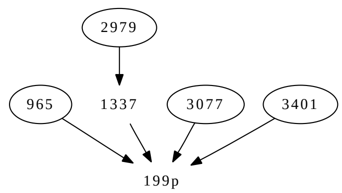


Figure 43. Prime family 199

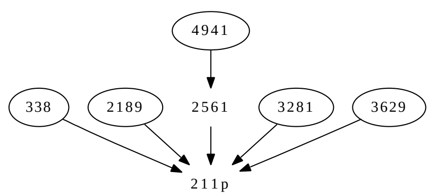


Figure 44. Prime family 211

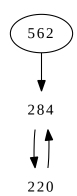


Figure 45. Aliquot cycle 220

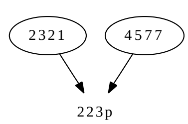


Figure 46. Prime family 223

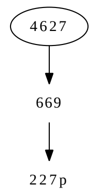


Figure 47. Prime family 227

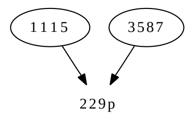


Figure 48. Prime family 229

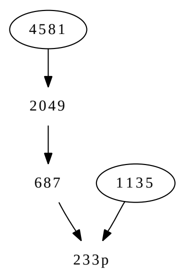


Figure 49. Prime family 233

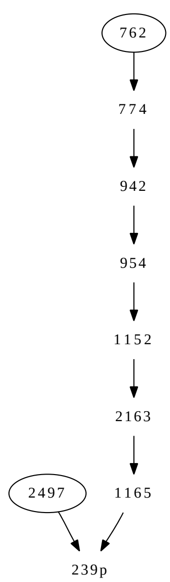


Figure 50. Prime family 239

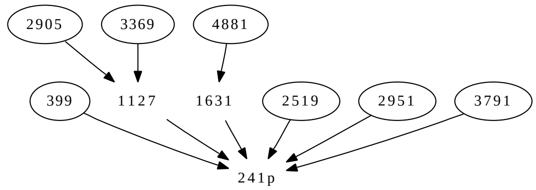


Figure 51. Prime family 241

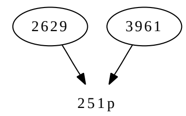


Figure 52. Prime family 251

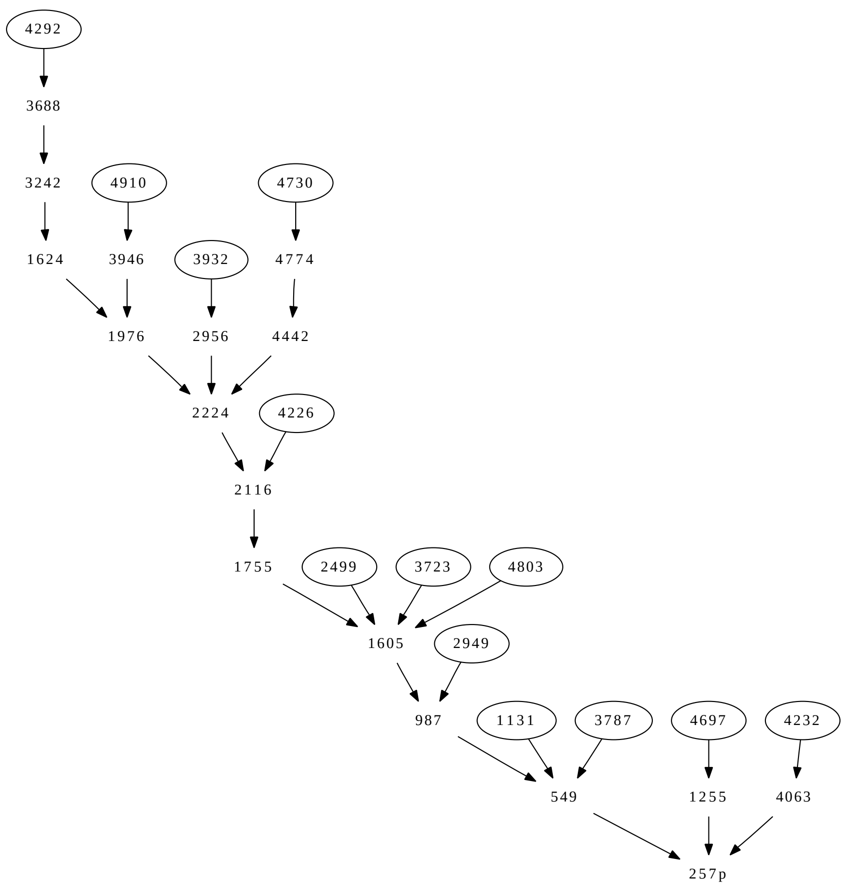


Figure 53. Prime family 257

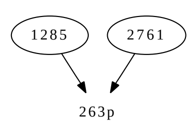


Figure 54. Prime family 263

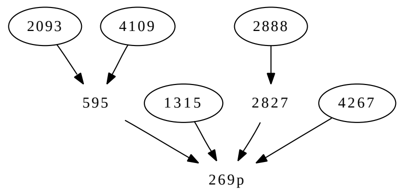


Figure 55. Prime family 269

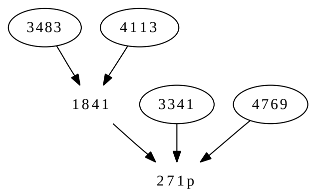


Figure 56. Prime family 271

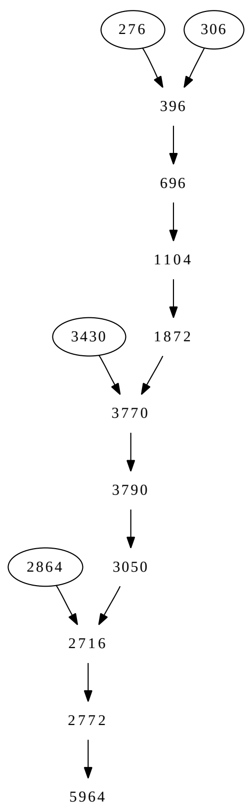


Figure 57. Sequence of 276 with unknown termination.

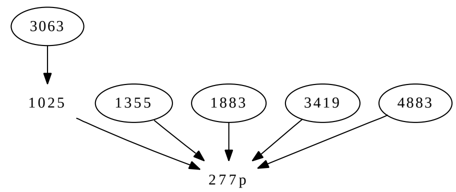


Figure 58. Prime family 277

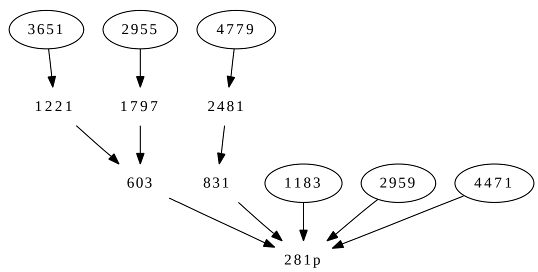


Figure 59. Prime family 281

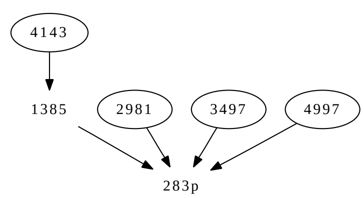


Figure 60. Prime family 283

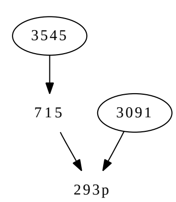


Figure 61. Prime family 293

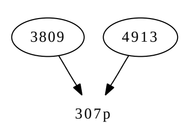


Figure 62. Prime family 307

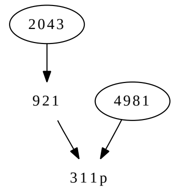


Figure 63. Prime family 311

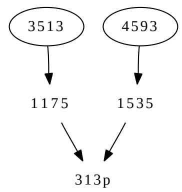


Figure 64. Prime family 313

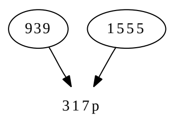


Figure 65. Prime family 317

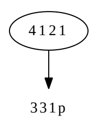


Figure 66. Prime family 331

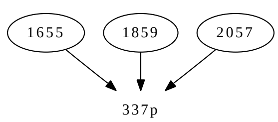


Figure 67. Prime family 337

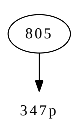


Figure 68. Prime family 347

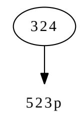


Figure 69. Prime family 523

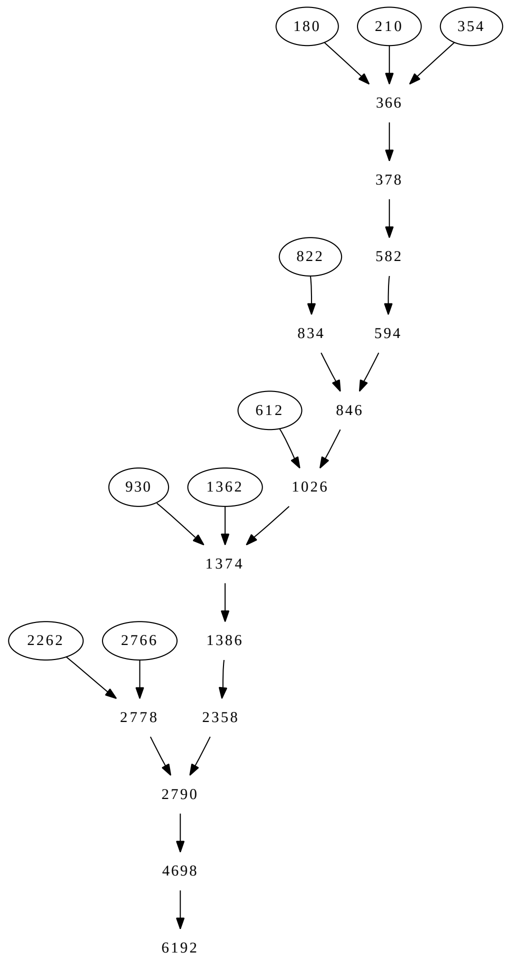


Figure 70. Prime family 601

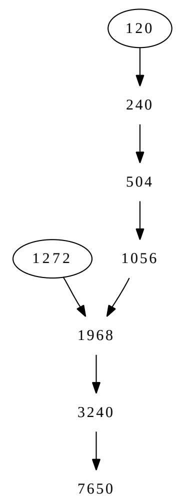


FIGURE 71. Prime family 12161

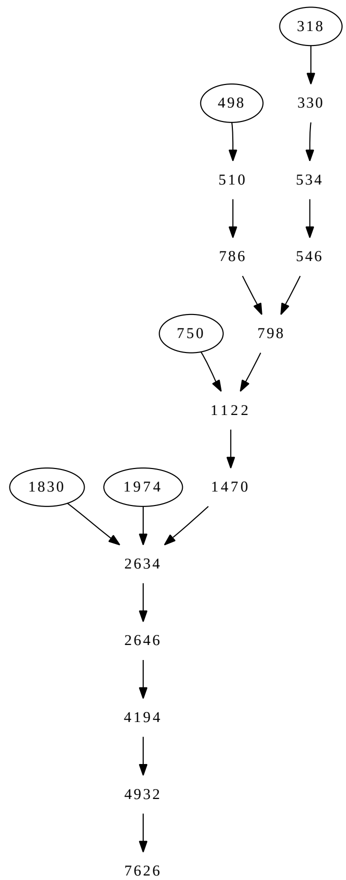


FIGURE 72. Prime family 321329

3. Unitary Sigma Illustrations

Mapping n to the sum of the unitary divisors of n [2, 3, 4] as tabulated in A034448 [9] defines another set of merger families, illustrated in Figures 73–77.

Figure 73. Unitary sigma family 2

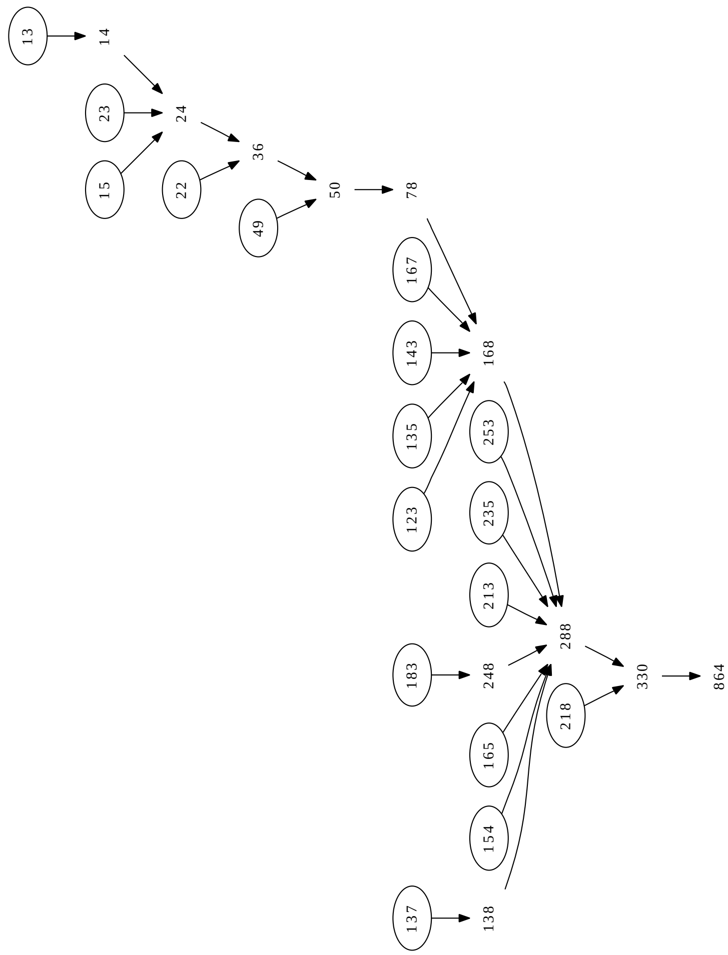


FIGURE 74. Unitary sigma family 13

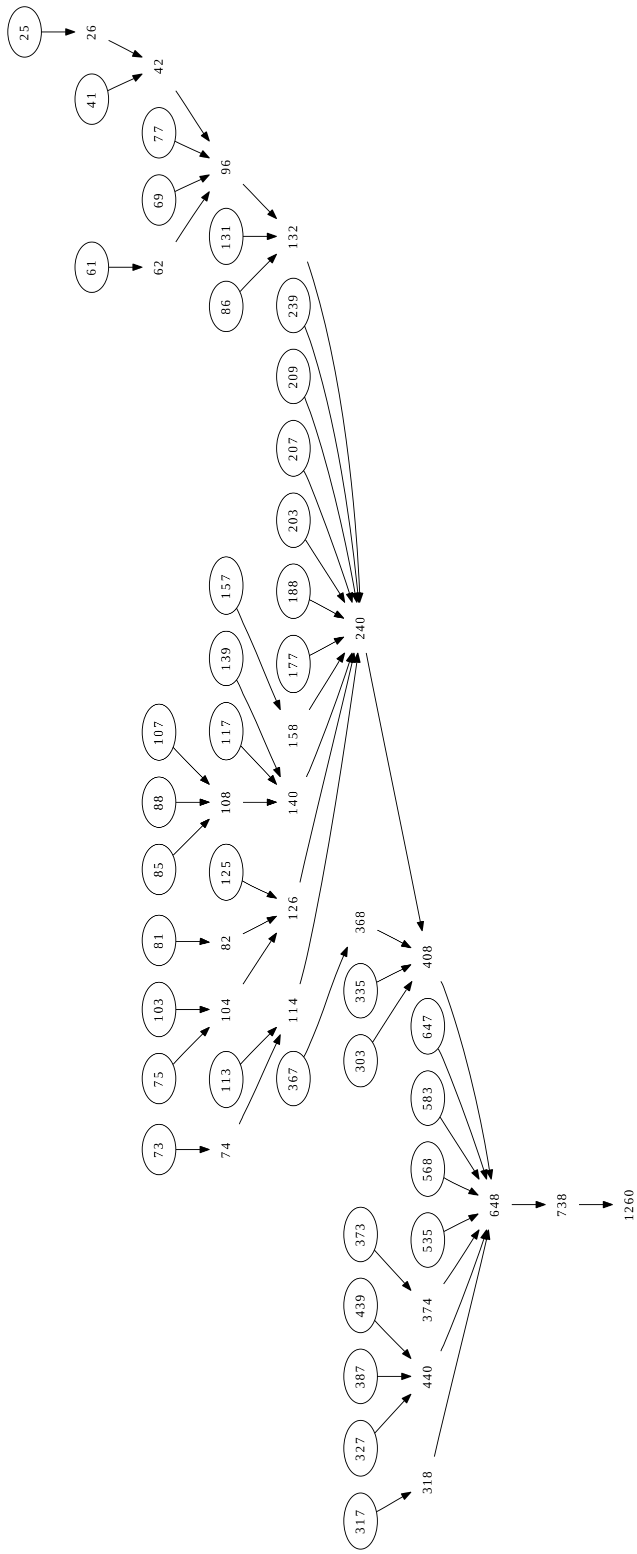


Figure 75. Unitary sigma family 25

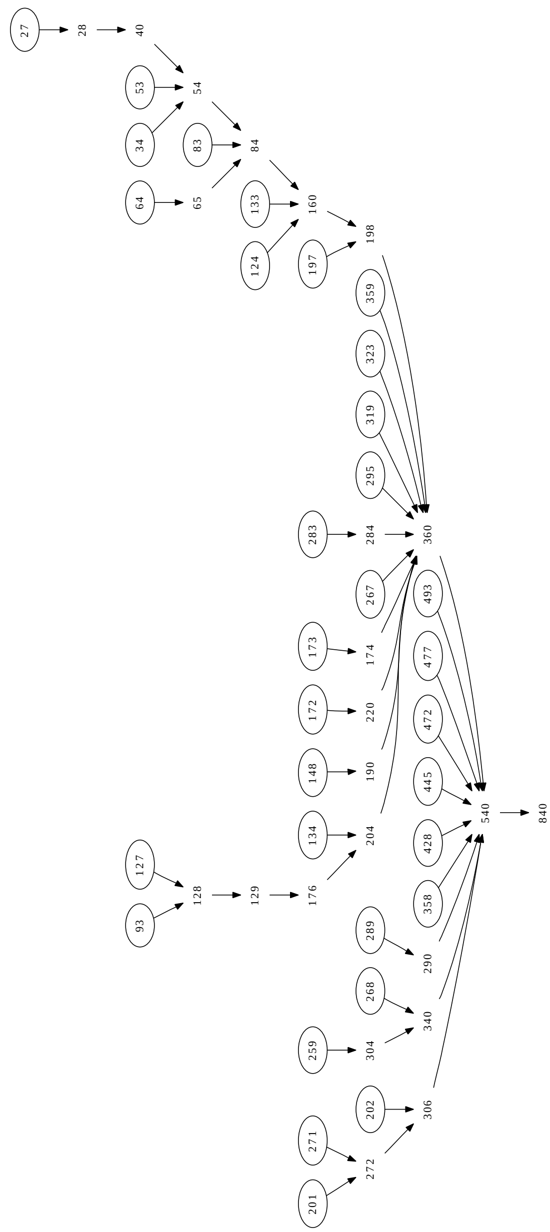


FIGURE 76. Unitary sigma family 27

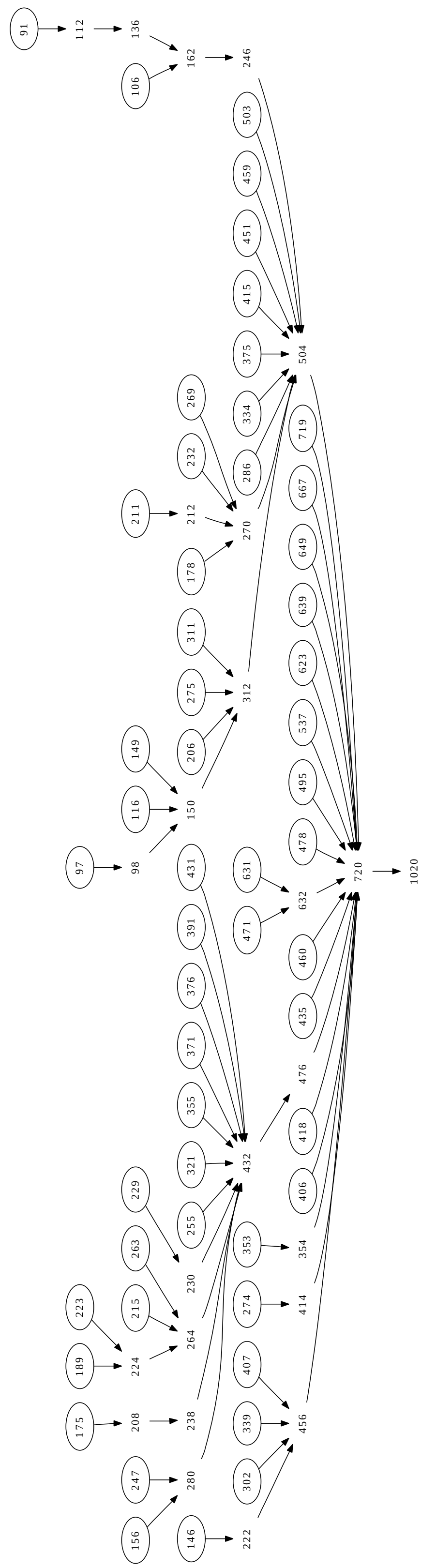


Figure 77. Unitary sigma family 91

References

1. Manuel Benito and Juan L. Varona, *Advances in aliquot cycles*, Math. Comp. **68** (1999), no. 225, 389–393. MR 1489967 (99c:11162)
2. Eckford Cohen, *Arithmetical functions associated with the unitary divisors of an integer*, Math. Zeitschr. **74** (1960), 66–80. MR 0112861 (22#3707)
3. ———, *The number of unitary divisors of an integer*, Am. Math. Monthly **67** (1960), no. 9, 879–880. MR 0122790 (23#A124)
4. Graeme L. Cohen, *On an integer's infinitary divisors*, Math. Comp. **54** (1990), no. 189, 395–411. MR 0993927 (90e:11011)
5. P. Erdős, *On asymptotic properties of aliquot sequences*, Math. Comp. **30** (1976), no. 135, 641–645. MR 0404115 (53#7919)
6. Richard K. Guy, *Unsolved problems in number theory*, 3 ed., Springer, Heidelberg, 2004. MR 2076335 (2005h:11003)
7. Richard K. Guy and J. L. Selfridge, *What drives an aliquot sequence?*, Math. Comp. **29** (1975), no. 129, 101–107. MR 0384669 (52#5542)
8. David Moews and Paul C. Moews, *A search for aliquot cycles and amicable pairs*, Math. Comp. **61** (1993), no. 204, 935–938. MR 1185249 (94a:11198)
9. Neil J. A. Sloane, *The On-Line Encyclopedia Of Integer Sequences*, Notices Am. Math. Soc. **50** (2003), no. 8, 912–915, http://oeis.org/. MR 1992789 (2004f:11151)

URL: http://www.mpia.de/~mathar

Hoeschstr. 7, 52372 Kreuzau, Germany