Specific Assertions for Quasi-Period 9 of Sequence A105730.

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Claim

Sequence A105730 has quasi-period 6 - x - 8 - 6 - y - 8 - 6 - 9 - z starting at n=0, where x, y, z > 10.

If we let |F(k)| be the size of the fan, i.e., the number of different initial values leading to maximum k in their Collatz trajectory then A105730(n) = A087256(6n+4) = $\left| F\left(2^{6\times n+4}\right) \right|$.

For each of the sizes of the fans the respective sets of the modular identities as stated in the link of A033496 must be verified. All identities are proved by induction in a fashion similar to the proof in the link of A087256.

The specific assertions are:

1. A105730(3n) = 6 = $\left| F\left(2^{18\times n+4}\right) \right|$
2. A105730(9n+2) = 8 = $\left| F\left(2^{54\times n+16}\right) \right|$
3. A105730(9n+5) = 8 = $\left| F\left(2^{54\times n+34}\right) \right|$
4. A105730(9n+7) = 9 = $\left| F\left(2^{54\times n+46}\right) \right|$
5. A105730(27n+8) = 11 = $\left| F\left(2^{162\times n+52}\right) \right|$
6. A105730(27n+1) = 12 = $\left| F\left(2^{162\times n+10}\right) \right|$
7. A105730(27n+4) = 13 = $\left| F\left(2^{162\times n+28}\right) \right|$
8. A105730(9n+1), A105730(9n+4), A105730(9n+8) > 10