A Property of A092506

Claim: For a prime p define $\mathbb{G}_p$ to be the set of positive integers congruent to a primitive root (mod p)
The set of p satisfying $\forall n \in \mathbb{G}_p \; \exists$ prime $q \in \mathbb{G}_p$ such that $q \mid n$ is exactly A092506

Proof: Consider a prime p with the given property.

Suppose $p-1$ is the product of at least two distinct primes. We can write $p-1 = ab$ with $\gcd(a,b) = 1$ and $a, b > 1$
It is a well known theorem that there exists a primitive root (mod p). Call it g. Then $\text{ord}_p(g^a) = b < ab$ and $\text{ord}_p(g^b) = a < ab$, and $\gcd(a+b, ab) = 1 \implies \text{ord}_p(g^{a+b}) = p-1$. Furthermore, by Dirichlet's Theorem, we can find primes s, t such that $s \equiv a$ and $t \equiv b \pmod p$. Then $st \in \mathbb{G}_p$ but st has no prime divisors in $\mathbb{G}_p$, contradiction.

So $p-1$ must be a power of a single prime; $p = r^x + 1$ for some prime r and $x \in \mathbb{Z}_{\geq 0}$. Unless $x = 0 \implies p = 2$, we have $p = r^x + 1 > 2 \implies 2 \nmid r^x + 1 \implies r = 2$. So $p = 2^x + 1$, and since $2 \nmid y,\; y \mid x \implies 2^y + 1 \mid 2^x + 1$, we must have $x = 2^z$ for some $z \in \mathbb{Z}_{\geq 0}$. So in this case $p = 2^{2^z} + 1$; overall p is either 2 or a Fermat Prime.

Now we will show that all the aforementioned primes satisfy the condition. The case where $p = 2$ is trivial, so we need only concern ourselves with the latter. Note that for any $0 \not\equiv i, j \pmod F$ where F is a Fermat Prime, $\text{ord}_F(ij) \mid \text{lcm}(\text{ord}_F(i), \text{ord}_F(j))$. But also $\text{ord}_F(a) \mid F - 1 = 2^{2^z}$. Hence $\text{ord}_F(i) = 2^{e_i}, \text{ord}_F(j) = 2^{e_j}$ with $i, j \in \mathbb{Z}_{\geq 0}$ and then $\text{ord}_F(ij) \mid \text{lcm}(\text{ord}_F(i), \text{ord}_F(j)) = \max(\text{ord}_F(i), \text{ord}_F(j))$. The conclusion follows by writing n as a product of its prime divisors (mod F).

So the set of p with this property is $\{2\} \cup$ A019434 = A092506 $\square$