

Recurrences for the number of homeomorphically irreducible general graphs on m labeled nodes and with n edges, $m=1..10$

Recurrence for the number of homeomorphically irreducible general graphs on 1 node and n edges:

$$a(0) = 1, a(1) = 0, a(2) = 1$$
$$a(n) = a(n - 1)$$

Recurrence for the number of homeomorphically irreducible general graphs on 2 nodes and n edges:

$$a(0) = 1, a(1) = 1, a(2) = 4, a(3) = 6, a(4) = 11, a(5) = 17$$
$$a(n) = 3 a(n - 1) - 3 a(n - 2) + a(n - 3)$$

Recurrence for the number of homeomorphically irreducible general graphs on 3 nodes and n edges:

$$a(0) = 1, a(1) = 3, a(2) = 9, a(3) = 21, a(4) = 60, a(5) = 135, a(6) = 282, a(7) = 537,$$
$$a(8) = 945, a(9) = 1561$$
$$a(n) = 6 a(n - 1) - 15 a(n - 2) + 20 a(n - 3) - 15 a(n - 4) + 6 a(n - 5) - a(n - 6)$$

Recurrence for the number of homeomorphically irreducible general graphs on 4 nodes and n edges:

$$a(0) = 1, a(1) = 6, a(2) = 19, a(3) = 68, a(4) = 242, a(5) = 704, a(6) = 1981,$$
$$a(7) = 5140, a(8) = 12364, a(9) = 27614, a(10) = 57598, a(11) = 113108,$$
$$a(12) = 210812, a(13) = 375606, a(14) = 643646, a(15) = 1066196$$
$$a(n) = 10 a(n - 1) - 45 a(n - 2) + 120 a(n - 3) - 210 a(n - 4) + 252 a(n - 5)$$
$$- 210 a(n - 6) + 120 a(n - 7) - 45 a(n - 8) + 10 a(n - 9) - a(n - 10)$$

Recurrence for the number of homeomorphically irreducible general graphs on 5 nodes
and n edges:

$$\begin{aligned} a(0) &= 1, a(1) = 10, a(2) = 40, a(3) = 185, a(4) = 765, a(5) = 2845, a(6) = 10220, \\ a(7) &= 33885, a(8) = 105185, a(9) = 305465, a(10) = 830811, a(11) = 2119875, \\ a(12) &= 5091525, a(13) = 11565505, a(14) = 24977315, a(15) = 51552005, \\ a(16) &= 102175360, a(17) = 195301015, a(18) = 361365695, a(19) = 649360880, \\ a(20) &= 1136438375, a(21) = 1941722170, a(22) = 3245874555 \end{aligned}$$

$$\begin{aligned} a(n) &= 15 a(n-1) - 105 a(n-2) + 455 a(n-3) - 1365 a(n-4) + 3003 a(n-5) \\ &\quad - 5005 a(n-6) + 6435 a(n-7) - 6435 a(n-8) + 5005 a(n-9) - 3003 a(n-10) \\ &\quad + 1365 a(n-11) - 455 a(n-12) + 105 a(n-13) - 15 a(n-14) + a(n-15) \end{aligned}$$

Recurrence for the number of homeomorphically irreducible general graphs on 6 nodes
and n edges:

$$\begin{aligned} a(0) &= 1, a(1) = 15, a(2) = 81, a(3) = 441, a(4) = 2151, a(5) = 9957, a(6) = 43122, \\ a(7) &= 174162, a(8) = 666267, a(9) = 2403987, a(10) = 8183601, a(11) = 26281065, \\ a(12) &= 79660856, a(13) = 228180456, a(14) = 618992466, a(15) = 1595081266, \\ a(16) &= 3918506466, a(17) = 9211519476, a(18) = 20797923546, \\ a(19) &= 45258309066, a(20) = 95225448306, a(21) = 194283668576, \\ a(22) &= 385361919996, a(23) = 744815846526, a(24) = 1405609133956, \\ a(25) &= 2594810623242, a(26) = 4693273343496, a(27) = 8329200764212, \\ a(28) &= 14522804269452, a(29) = 24907046815512, a(30) = 42060395126646 \end{aligned}$$

$$\begin{aligned} a(n) &= 21 a(n-1) - 210 a(n-2) + 1330 a(n-3) - 5985 a(n-4) + 20349 a(n-5) \\ &\quad - 54264 a(n-6) + 116280 a(n-7) - 203490 a(n-8) + 293930 a(n-9) \\ &\quad - 352716 a(n-10) + 352716 a(n-11) - 293930 a(n-12) + 203490 a(n-13) \\ &\quad - 116280 a(n-14) + 54264 a(n-15) - 20349 a(n-16) + 5985 a(n-17) \\ &\quad - 1330 a(n-18) + 210 a(n-19) - 21 a(n-20) + a(n-21) \end{aligned}$$

Recurrence for the number of homeomorphically irreducible general graphs on 7 nodes
and n edges:

$a(0) = 1, a(1) = 21, a(2) = 154, a(3) = 966, a(4) = 5586, a(5) = 30744, a(6) = 155918,$
 $a(7) = 749210, a(8) = 3423707, a(9) = 14823837, a(10) = 60947922,$
 $a(11) = 237925779, a(12) = 881718873, a(13) = 3101594041, a(14) = 10359403806,$
 $a(15) = 32880974266, a(16) = 99324356061, a(17) = 286133201956,$
 $a(18) = 788088924616, a(19) = 2081032222276, a(20) = 5283419212141,$
 $a(21) = 12932973298146, a(22) = 30604530293806, a(23) = 70187066891361,$
 $a(24) = 156353370015661, a(25) = 339035062258137, a(26) = 716959448978122,$
 $a(27) = 1481170591674410, a(28) = 2994008530022167,$
 $a(29) = 5929952998097492, a(30) = 11522718722910598,$
 $a(31) = 21992163680650912, a(32) = 41271306553739458,$
 $a(33) = 76227717606673451, a(34) = 138689289908437716,$
 $a(35) = 248763561963504392, a(36) = 440214270436656884,$
 $a(37) = 769075319036613659, a(38) = 1327308470019754586,$
 $a(39) = 2264243989259529991$

$a(n) = 28 a(n - 1) - 378 a(n - 2) - 20475 a(n - 4) + 3276 a(n - 3) - 376740 a(n - 6)$
 $+ 98280 a(n - 5) + 6906900 a(n - 9) - 3108105 a(n - 8) + 1184040 a(n - 7)$
 $- 30421755 a(n - 12) + 21474180 a(n - 11) - 13123110 a(n - 10)$
 $+ 21474180 a(n - 17) - 30421755 a(n - 16) + 37442160 a(n - 15)$
 $- 40116600 a(n - 14) + 37442160 a(n - 13) + 1184040 a(n - 21)$
 $- 3108105 a(n - 20) + 6906900 a(n - 19) - 13123110 a(n - 18) + 3276 a(n - 25)$
 $- 20475 a(n - 24) - 376740 a(n - 22) - a(n - 28) + 28 a(n - 27) - 378 a(n - 26)$
 $+ 98280 a(n - 23)$

Recurrence for the number of homeomorphically irreducible general graphs on 8 nodes
and n edges:

$a(0) = 1, a(1) = 28, a(2) = 274, a(3) = 1996, a(4) = 13525, a(5) = 85968, a(6) = 505394,$
 $a(7) = 2831180, a(8) = 15061258, a(9) = 76258884, a(10) = 368604140,$
 $a(11) = 1700575620, a(12) = 7488324419, a(13) = 31463799180,$
 $a(14) = 126112370512, a(15) = 482139677172, a(16) = 1758480501787,$
 $a(17) = 6122216564452, a(18) = 20367619370002, a(19) = 64844805959012,$
 $a(20) = 197929578509903, a(21) = 580430007986628, a(22) = 1638898915818538,$
 $a(23) = 4465720602506368, a(24) = 11768600734054453,$
 $a(25) = 30058722923763256, a(26) = 74558313889964788,$
 $a(27) = 179933254133360324, a(28) = 423220815208640661,$
 $a(29) = 971750057319600120, a(30) = 2181271567537385584,$
 $a(31) = 4793090000359514856, a(32) = 10323016053603779990,$
 $a(33) = 21815882883083143024, a(34) = 45285967905250924812,$
 $a(35) = 92425533065686897568, a(36) = 185625350985122781700,$
 $a(37) = 367154781453207935560, a(38) = 715731508822392823798,$
 $a(39) = 1376062424754386743404, a(40) = 2610893020554769052832,$
 $a(41) = 4891689107243079626844, a(42) = 9054932445218910658634,$
 $a(43) = 16568741053860934039152, a(44) = 29983210613040877819818,$
 $a(45) = 53683880386205067630508, a(46) = 95140893888558613499560,$
 $a(47) = 166961842109519116846300, a(48) = 290236922660003955867758,$
 $a(49) = 499945657302899186263104$

$$\begin{aligned} a(n) = & -58905 a(n-4) + 7140 a(n-3) - 630 a(n-2) + 36 a(n-1) \\ & - 30260340 a(n-8) + 8347680 a(n-7) - 1947792 a(n-6) + 376992 a(n-5) \\ & - 3796297200 a(n-14) + 2310789600 a(n-13) - 1251677700 a(n-12) \\ & + 600805296 a(n-11) - 254186856 a(n-10) + 94143280 a(n-9) \\ & + 8597496600 a(n-19) - 9075135300 a(n-18) + 600805296 a(n-25) \\ & - 254186856 a(n-26) + 94143280 a(n-27) - 30260340 a(n-28) \\ & + 8347680 a(n-29) - 7307872110 a(n-16) + 5567902560 a(n-15) \\ & + 8597496600 a(n-17) + 7140 a(n-33) - 58905 a(n-32) + 376992 a(n-31) \\ & - 1947792 a(n-30) - 630 a(n-34) + 36 a(n-35) - a(n-36) \\ & - 3796297200 a(n-22) + 2310789600 a(n-23) - 1251677700 a(n-24) \\ & - 7307872110 a(n-20) + 5567902560 a(n-21) \end{aligned}$$

Recurrence for the number of homeomorphically irreducible general graphs on 9 nodes
and n edges:

$a(0) = 1, a(1) = 36, a(2) = 459, a(3) = 3933, a(4) = 30762, a(5) = 222813,$
 $a(6) = 1503414, a(7) = 9633420, a(8) = 58624947, a(9) = 341006109,$
 $a(10) = 1899442287, a(11) = 10135772019, a(12) = 51845135796,$
 $a(13) = 254195050941, a(14) = 1194332805126, a(15) = 5375748277794,$
 $a(16) = 23173127544852, a(17) = 95654786286177, a(18) = 378142880620602,$
 $a(19) = 1432244962589832, a(20) = 5201428977802665,$
 $a(21) = 18132038605909458, a(22) = 60755711015992509,$
 $a(23) = 195990941057718699, a(24) = 609742765504037829,$
 $a(25) = 1832750801394452316, a(26) = 5332074935118772707,$
 $a(27) = 15041856297337739267, a(28) = 41216217214686250491,$
 $a(29) = 109877727886946718390, a(30) = 285429429962320091055,$
 $a(31) = 723544657178799300543, a(32) = 1792241565296684015961,$
 $a(33) = 4343479033272394336635, a(34) = 10310849546537644499709,$
 $a(35) = 24001303309302925402677, a(36) = 54839327573516836885261,$
 $a(37) = 123102644152592643235101, a(38) = 271726510203140516220465,$
 $a(39) = 590242178570805814714947, a(40) = 1262645519046986338978797,$
 $a(41) = 2661842002786938210737637, a(42) = 5533611431107012473513999,$
 $a(43) = 11350567924229561080867533, a(44) = 22985225791639010699913063,$
 $a(45) = 45975463342679550522893217, a(46) = 90878065860959159488335027,$
 $a(47) = 177600381772702519716182271, a(48) = 343292798615804160671281005,$
 $a(49) = 656590310520698938077431955,$
 $a(50) = 1243071203724589852825223949,$
 $a(51) = 2330358212177169584725976097,$
 $a(52) = 4327322102075602318296886101,$
 $a(53) = 7962003239532323870846811969,$
 $a(54) = 14519825833956479677200309633,$
 $a(55) = 26251788805859757870925920783,$
 $a(56) = 47068570447151509749035718261,$
 $a(57) = 83712058763907713538538611849,$
 $a(58) = 147718462591317000487999395267,$
 $a(59) = 258684868637343727537492621881,$
 $a(60) = 449667785193102305240123314815$

$$\begin{aligned}
a(n) = & 166871334960 a(n-31) - 344867425584 a(n-30) + 646626422970 a(n-29) \\
& - 1103068603890 a(n-28) + 3190187286 a(n-35) - 10150595910 a(n-34) \\
& + 28760021745 a(n-33) - 73006209045 a(n-32) + 215553195 a(n-37) \\
& - 886163135 a(n-36) - 45379620 a(n-38) + a(n-45) - 45 a(n-44) \\
& + 990 a(n-43) - 14190 a(n-42) + 148995 a(n-41) - 1221759 a(n-40) \\
& - 990 a(n-2) + 45 a(n-1) + 14190 a(n-3) - 148995 a(n-4) \\
& - 8145060 a(n-6) + 1221759 a(n-5) + 45379620 a(n-7) + 886163135 a(n-9) \\
& - 215553195 a(n-8) - 3190187286 a(n-10) + 10150595910 a(n-11) \\
& + 8145060 a(n-39) - 28760021745 a(n-12) + 73006209045 a(n-13) \\
& - 166871334960 a(n-14) + 344867425584 a(n-15) - 646626422970 a(n-16) \\
& + 1103068603890 a(n-17) - 1715884494940 a(n-18) \\
& + 2438362177020 a(n-19) - 3169870830126 a(n-20) \\
& + 3773655750150 a(n-21) - 4116715363800 a(n-22) \\
& + 4116715363800 a(n-23) - 3773655750150 a(n-24) \\
& + 3169870830126 a(n-25) - 2438362177020 a(n-26) \\
& + 1715884494940 a(n-27)
\end{aligned}$$

**Recurrence for the number of homeomorphically irreducible general graphs on 10 nodes
and n edges:**

$$\begin{aligned}
a(0) = 1, & a(1) = 45, a(2) = 730, a(3) = 7420, a(4) = 66280, a(5) = 543370, \\
a(6) = 4160200, & a(7) = 30091090, a(8) = 207053950, a(9) = 1364245160, \\
a(10) = 8620401212, & a(11) = 52330072460, a(12) = 305510516285, \\
a(13) = 1715790008165, & a(14) = 9269583306080, a(15) = 48163386772910, \\
a(16) = 240597999521720, & a(17) = 1155147712521320, a(18) = 5328887173244440, \\
a(19) = 23617478578000630, & a(20) = 100568027220420166, \\
a(21) = 411577589004753520, & a(22) = 1619786490094921690, \\
a(23) = 6135328335120981760, & a(24) = 22389830688304331230, \\
a(25) = 78820026063765182218, & a(26) = 268036513817855615050, \\
a(27) = 881777583685763948520, & a(28) = 2810512692998158552690, \\
a(29) = 8692176642956394358410, & a(30) = 26123539450390551015162, \\
a(31) = 76404658719034033143120, & a(32) = 217765814038982756703780, \\
a(33) = 605632915830209791001640, & a(34) = 1645566085602011936107200, \\
a(35) = 4373320050521528768142900, & a(36) = 11380712935321428321197340, \\
a(37) = 29028954696097105871618960, & a(38) = 72645575686873690572238770,
\end{aligned}$$

$a(39) = 178520147640484228415464230$, $a(40) = 431145781849274155587076602$,
 $a(41) = 1024128129423158517695890170$,
 $a(42) = 2394361566002926154439205890$,
 $a(43) = 5513429108453708190996952050$,
 $a(44) = 12511884327909292811075761500$,
 $a(45) = 27999297958733352219137090328$,
 $a(46) = 61820636470337214565366933110$,
 $a(47) = 134742681796212147202388909010$,
 $a(48) = 290049424847175506386207975320$,
 $a(49) = 616923036239683085103717969240$,
 $a(50) = 1297082182543337925974478437526$,
 $a(51) = 2696845863549737077477389489570$,
 $a(52) = 5547043037073041070683108146050$,
 $a(53) = 11291176282140734035088337362790$,
 $a(54) = 22752862769856694959156932691960$,
 $a(55) = 45403696220021046015148720443900$,
 $a(56) = 89750416629432804180054871157130$,
 $a(57) = 175791094056543576654657971218350$,
 $a(58) = 341264517529246792157308788606660$,
 $a(59) = 656798511232454417922111951857220$,
 $a(60) = 1253508773874649390502141134035258$,
 $a(61) = 2372898406491547834189661999798790$,
 $a(62) = 4456409259587740164637541526157200$,
 $a(63) = 8304974833760173814635851651741540$,
 $a(64) = 15361298373487854487349669271196425$,
 $a(65) = 28205838339014111898332390976088905$,
 $a(66) = 51422485843109442737834729988532350$,
 $a(67) = 93099671141437134748250740826377980$,
 $a(68) = 167416819666660361554591575674489550$,
 $a(69) = 299073335175192883101697338482458800$,
 $a(70) = 530826888807411318425409489793575816$,
 $a(71) = 936248211473924702167589202067237950$,
 $a(72) = 1641177921716392261128178347798012255$

$$\begin{aligned}
a(n) = & 29749251314475 a(n-39) + 29248649430 a(n-45) + 438729741450 a(n-43) \\
& - 1451182990950 a(n-42) - 119653565850 a(n-44) + 4353548972850 a(n-41) \\
& + 341055 a(n-51) + 55 a(n-1) - 3478761 a(n-50) + 28989675 a(n-49) \\
& + 1217566350 a(n-47) - 202927725 a(n-48) - 6358402050 a(n-46) \\
& + a(n-55) + 26235 a(n-3) - 1485 a(n-2) + 1485 a(n-53) - 55 a(n-54) \\
& - 26235 a(n-52) + 3478761 a(n-5) - 341055 a(n-4) + 202927725 a(n-7) \\
& - 28989675 a(n-6) + 6358402050 a(n-9) - 1217566350 a(n-8) \\
& + 119653565850 a(n-11) - 29248649430 a(n-10) + 1451182990950 a(n-13) \\
& - 438729741450 a(n-12) - 4353548972850 a(n-14) \\
& - 29749251314475 a(n-16) + 11899700525790 a(n-15) \\
& - 144079707346575 a(n-18) + 68248282427325 a(n-17) \\
& - 505037289962205 a(n-20) + 280576272201225 a(n-19) \\
& + 1866442158555975 a(n-23) - 1300853625660225 a(n-22) \\
& + 841728816603675 a(n-21) + 3085851035479212 a(n-25) \\
& - 2488589544741300 a(n-24) - 3560597348629860 a(n-26) \\
& - 3824345300380220 a(n-28) + 3824345300380220 a(n-27) \\
& - 3085851035479212 a(n-30) + 3560597348629860 a(n-29) \\
& - 11899700525790 a(n-40) + 2488589544741300 a(n-31) \\
& - 1866442158555975 a(n-32) + 1300853625660225 a(n-33) \\
& + 505037289962205 a(n-35) - 841728816603675 a(n-34) \\
& + 144079707346575 a(n-37) - 280576272201225 a(n-36) \\
& - 68248282427325 a(n-38)
\end{aligned}$$