Sridar Kuttan Pootheri
Counting classes of labeled 2–connected graphs
(Under the direction of Robert W. Robinson)

Applying the Tutte decomposition of 2–connected graphs into 3–block trees, unique structural characterizations of several classes of 2–connected graphs were provided in the PhD dissertation of the author under the title "Characterizing and counting classes of unlabeled 2–connected graphs".

In this thesis counting equations are derived for labeled minimally 2–connected graphs, labeled 2–connected minimally 2–edge–connected graphs and labeled 2–connected 3–edge–connected graphs based on these structural relations.

The appendices contain tables of the numbers of these three classes of 2–connected graphs, by number of nodes and number of edges, obtained by computer implementation of the counting equations. These are listed for node orders up to 34, 21 and 34 respectively.

Also tables of the numbers of labeled 3–edge–connected and minimally 2–edge–connected graphs (not necessarily 2–connected) are included for node orders up to 21 and 34.

Index words: Labeled graphs, Graph counting, Graph enumeration,
Minimally 2–connected graphs,
Minimally 2–edge–connected graphs, 3–edge–connected graphs,
Minimally 2–edge–connected blocks, 3–edge–connected blocks.

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by

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Dedication

To the Divine Self who nourishes every living creature.

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CHAPTER 1

Introduction

§1.1 Overview

The exact enumeration of graphs is one of the classical problems in combinatorics. In this thesis exact counting equations are derived for three classes of labeled 2–connected graphs : minimally 2–connected graphs, 3–edge–connected blocks and minimally 2–edge–connected blocks. Standard methods are then applied to count labeled 3–edge–connected graphs and minimally 2–edge–connected graphs (not necessarily 2–connected).

The equations are based on relations satisfied by exponential generating functions. These in turn are derived from the following characterization theorems, which are based on the minimal Tutte decomposition $T(G)$ of a 2–connected graph G into components which are cycles, bonds, or 3–connected graphs.

A 2–connected graph G is :

1. *minimally 2–connected if and only if each free edge in $T(G)$ belongs to a cyclic component;*
2. *minimally 2–edge–connected if and only if each free edge in $T(G)$ belongs to a cyclic component that contains at least one other free edge;*
3. *3–edge–connected if and only if each cyclic component has at most one free edge.*

These facts are discussed in Chapter 2. Detailed proofs appear in [15, Chapter 2]. The generating function relations are transformed algebraically to improve computational efficiency in sections $5, 6$ and 7 of Chapter 3. The resulting recurrences were implemented in Maple to calculate the numbers, which are tabulated in the appendices.

An important motivation for studying graphs is that they model interconnection networks for communication. These range in scale from the connection of processors in a multi–processor computer on up to a WAN connecting a global network. At any scale completeness of connectivity and efficiency are fundamental design considerations. The failure of a connection or a node can change the nature of the connectivity of the network. Minimality with respect to edge connectivity and node connectivity give ways of quantifying the fault tolerance of a network. Knowing the maximum number of configurations of networks satisfying a particular minimality condition helps in predicting the feasibility of synthesis or design approaches which require checking all possible network configurations with the specified properties and given parameters.

Of theoretical interest are the harmonic mean numbers of automorphisms of the graphs in our classes. These are obtained by comparing the numbers of labeled graphs with the corresponding numbers of unlabeled graphs which were calculated in [15]. The harmonic mean numbers of automorphisms are reported in the appendices.

§1.2 Background on counting

When we seek to count configurations too numerous to be feasibly listed, we employ a formal power series of the form $\sum_{n=0}^{\infty} a_n(y)\dfrac{x^n}{n!}$. These series, called *exponential generating functions*, are not required to be convergent. They are formal expressions which lie in the ring $\mathbb{Q}[y][[x]]$. By performing appropriate manipulations on these formal expressions, one arrives at the *counting series* of the desired configurations. As explained by Herbert Wilf in his book “Generatingfunctionology” [22], a counting series for a configuration is like a clothesline of numbers where each number a_n denotes the number of configurations of size n. In this thesis we will let egf abbreviate exponential generating function.

A *labeled graph* $\boldsymbol{G}$ is a pair $(V(\boldsymbol{G}), E(\boldsymbol{G}))$, where $V(\boldsymbol{G}) = \{1, 2, 3, \cdots, n\}$ for $n \geq 1$ and where $E(\boldsymbol{G})$ is a set of 2–element subsets of $V(\boldsymbol{G})$. Thus loops or multiple edges are not allowed in a graph. Two labeled graphs $\boldsymbol{G}$ and $\boldsymbol{H}$, both of order n, are *isomorphic* if there exists a permutation $\sigma \in S_n$, such that

$$\sigma(E(\boldsymbol{H})) = \Big\{(\sigma(i), \sigma(j)) : (i, j) \in E(\boldsymbol{H})\Big\} = E(\boldsymbol{G}).$$

We say that σ is an isomorphism from $\boldsymbol{H}$ to $\boldsymbol{G}$. In words, two labeled graphs are isomorphic if and only if there is a $1-1$ map between their node sets which preserves adjacency. An *unlabeled graph of order* n is an isomorphism class of labeled graphs of order n.

Let $\mathcal{A}$ denote the set of all the isomorphisms of a graph $\boldsymbol{G}$ onto itself. These form a group and there is a $1-1$ map between the left cosets of $\mathcal{A}$ in S_n and labeled graphs isomorphic to $\boldsymbol{G}$. Thus there are $n!/|\mathcal{A}|$ different ways of labeling $\boldsymbol{G}$. $\mathcal{A}$ is called the *automorphism group* of the graph $\boldsymbol{G}$ and $|\mathcal{A}|$ is the *symmetry number* of $\boldsymbol{G}$. Let $\boldsymbol{G}_1, \boldsymbol{G}_2, \cdots, \boldsymbol{G}_u$ are the u unlabeled graphs on n nodes. Let $s_1, s_1, \cdots, s_u$ be their symmetry numbers and l is their total number of the labeled versions. Then $l = n! \sum_{1 \leq i \leq u} 1/s_i$, so the *harmonic mean* of the symmetry numbers is given by $\dfrac{n!u}{l} = \dfrac{u}{\sum_{1 \leq i \leq u} 1/s_i}$. This gives a measure of symmetries in the graphs. For all general graphs this tends to 1 very quickly, as most of them have symmetry number 1 and hence are called *identity graphs*. In fact, suppose u_n denotes the number of all unlabeled graphs on n nodes and $l_n = 2^{\binom{n}{2}}$, then $u_n \sim \dfrac{l_n}{n!}$. This can also be seen from equation (9.1.1) in [9, pp. 196].

An obvious method of counting labeled graphs is by enumerating them one by one, keeping count as they are produced. But for our classes of graphs the numbers grow so rapidly that the maximum order for which this is feasible is quite small. This could be increased somewhat by enumerating only one graph of each isomorphism type, but still the numbers of unlabeled graphs grow quickly which again limit the maximum feasible order.

To overcome these limitations we turn to egfs, which have long been employed as an algebraic tool for labeled graph counting. The earliest known example of enumerating a general class of labeled graphs (as opposed to a very special class, such as trees) is the counting of connected graphs due to R. J. Riddell [12]. As noted at the beginning of Chapter 8 of [9], a famous theoretical physicist, George E. Uhlenbeck, in his Gibbs Lecture entitled "Unsolved problems in statistical mechanics," given at a meeting of the American Mathematical Society in 1950, cited the enumeration of blocks as one of these problems. Subsequently Riddell, and Ford and Uhlenbeck counted labeled blocks, but it was Robinson [16] who succeeded in solving the unlabeled problem.

§1.3 Outline of later chapters

Our method for counting graphs with prescribed properties relies on decomposing a graph into a core and components. In Chapter 2, we present decomposition characterizations of the classes of graphs counted in Chapter 3. Our characterizations are unique, as they all build on a version of the Tutte decomposition which is unique.

Temperley's partial differential equation (3.7) provides a recursive method for counting labeled blocks. Using this, in Chapter 3 we derive an explicit expression for computing the nth term of the counting series for labeled blocks.

Tutte's theorem decomposes blocks into 3–connected graphs, bonds with at least 3 edges and cycles with at least 3 edges. This can be translated into an equation relating the egfs of the cores and the components. In Chapter 3 we derive the equation corresponding to the decomposition of blocks. This equation was previously derived and used by Walsh [21] to count labeled 3–connected graphs. We then proceed to develop equations for the three special classes of 2–connected graphs in sections 5, 6 and 7, and to develop a general equation for counting connected graphs with given blocks in section 8. For each of the classes that we have counted in this thesis our decomposition theorem given in the Chapter 2 is translated into an equation involving egfs. Each of these equations and the one for blocks is inverted by composing over an appropriate egf. The resulting pair of equations have a set of terms that are common. By eliminating the common terms we obtain counting equations which do not require calculations of the numbers of labeled 3–connected graphs. In the counting schemes for the three classes of graphs there are several parts which they share. We collect these routines in section §**3.2** for later reference.

In the last Chapter, numerical results and related problems are discussed.

CHAPTER 2

Graph decompositions

Every graph in this thesis will be a loopless multigraph. A multigraph is said to be *simple* if between each pair of adjacent nodes there is exactly one edge. The *trivial graph* (a graph with just one node) and an isolated edge (a graph with two nodes and one edge) are both considered to be 1–connected and 1–edge–connected but not 2–connected or 2–edge–connected. For a graph G on $p \geq 3$ nodes and $n \geq 1$, we say that G is n–connected if and only if $p \geq n+1$ and G cannot be disconnected by removing fewer than n nodes and their incident edges. A simple non–trivial graph is defined to be *n–edge–connected* if it is connected and cannot be disconnected by removing fewer than n edges. A *block* is a non–trivial connected graph with no cut node. In other words, a graph is a block if and only if it is a 2–connected graph or else an isolated edge. Blocks are also called *non–separable graphs.* A graph is said to be *minimal* (respectively, *critical*) with respect to a property P if it has P but loses property P when any one of its edges is removed (respectively, when any one of its nodes along with all the edges incident to it are removed). Bonds (are graphs with two nodes joined by at least two edges) are the only kind of non–simple multigraphs considered in this thesis. All graphs not otherwise specified are assumed to be finite, loopless, simple, undirected 2–connected graphs. We refer the reader to [8] for any graph theoretic terminology not defined in this thesis.

Following Whitney's pioneering decomposition of connected graphs into non–separable graphs in [28], Mac Lane [11] was the first to introduce a decomposition of 2–connected graphs. He used this decomposition to extend Whitney's results on planar graphs. Later, in his seminal work [20], Tutte defined and established many properties of a decomposition of 2–connected graphs into graphs each of which is either a cycle, a bond or a 3–connected graph. Tutte's decomposition is equivalent to that of Mac Lane's when applied to planar graphs. Hopcroft and Tarjan independently discovered the same decomposition [13] and exploited the uniqueness of its minimal version for algorithmic purposes. Trakhtenbrot [19] also proved the existence and uniqueness of essentially the same decomposition, in the guise of a canonical decomposition of 2–pole networks. Cunningham and Edmonds [4] proved the existence and uniqueness of the minimal decomposition in a rather general framework. We refer the reader to [4, pp. 744–755] for a brief comparison of the various approaches to decomposing 2–connected graphs. We attribute the basic decomposition to Tutte, since he was the first to present it explicitly.

For the purposes of this thesis, a *bond* is defined to be a connected multigraph with exactly two nodes and at least two edges. A *cycle* is a connected simple graph with all nodes of degree two. Containing at least three edges and three nodes. A recent introduction to the Tutte decomposition can be found in [6], where it is used to give a structural characterization of locally finite 2–connected graphs. Below we follow the approach of Hopcroft and Tarjan [13].

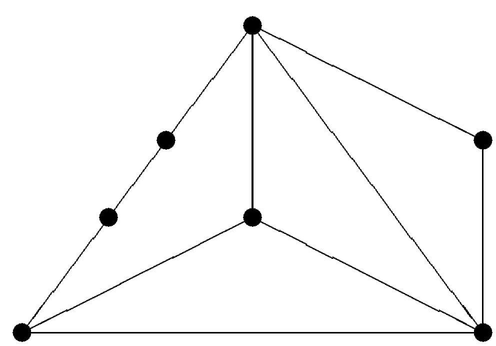

Figure 2.1: An example of a 2–connected graph G

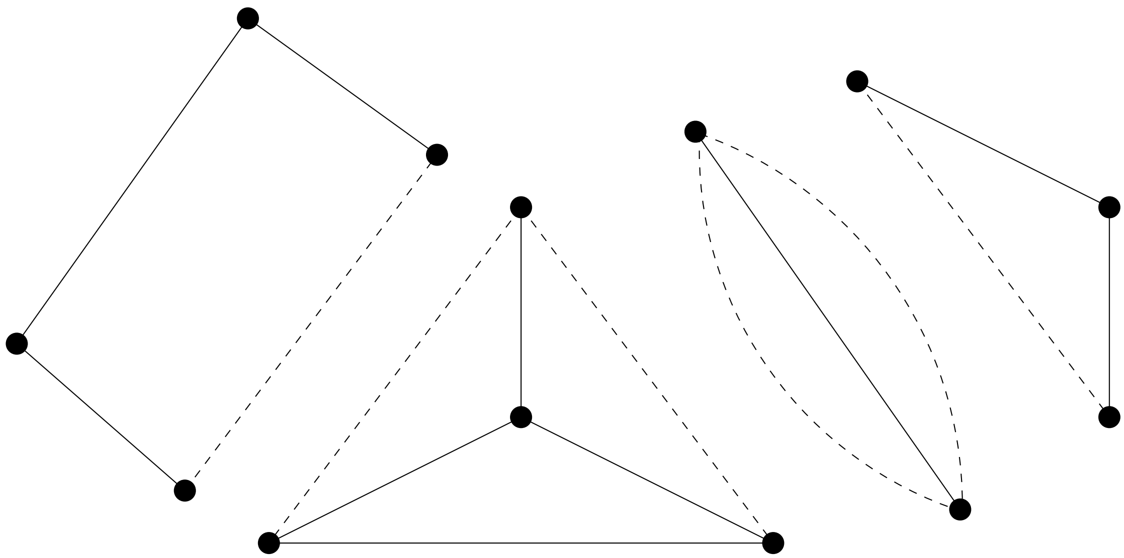

Figure 2.2: Tutte Decomposition $T(G)$ of the graph G

For a 2–connected graph G, a decomposition into smaller 2–connected graphs is performed by the following process. Choose two nodes u and v in G, and let E_1, E_2,..., E_n be the equivalence classes in G such that two edges are in the same class if and only if they lie on a common path not containing u or v except as an endpoint. Let $F_1 = \bigcup_{i=1}^{k} E_i$ and $F_2 = \bigcup_{i=k+1}^{n} E_i$ be such that each F_i has at least 2 edges. Form $G_i = (V(F_i), F_i \cup (u,v))$, for $i = 1, 2$. The new edge (u,v) in each G_i is called a *virtual edge*. If (u,v) is already an edge in G then a bond with three edges is included as a component, the middle edge being the original (u,v) and the other two being virtual edges for G_1 and G_2. In this case, since $\Big\{(u,v)\Big\}$ is a singleton equivalence class, (u,v) cannot be used by itself as one of the G_i's. The reason for including the virtual edge in G_i is to maintain 2–connectivity throughout the decomposition. G_1 and G_2 are then further decomposed until every graph in the decomposition is indecomposable. We say that G is *indecomposable* if for every pair of nodes in G, (i) there is only one class, or (ii) there are three classes and each class consists of a single edge, or (iii) there are exactly two classes and one class consists of a single

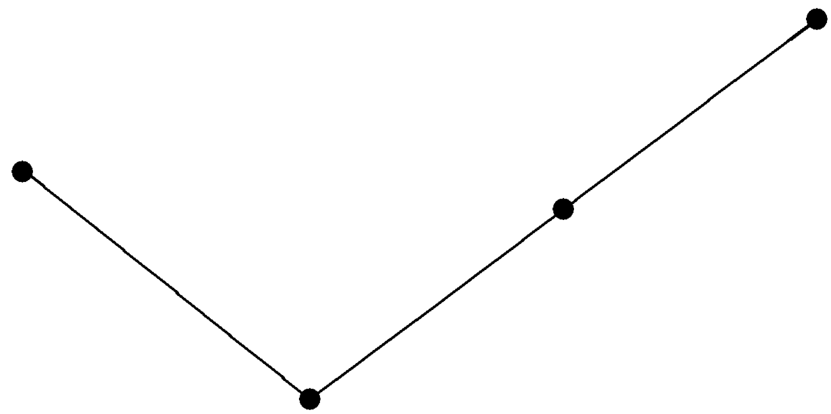

Figure 2.3: Underlying graph $\Gamma(G)$ of $T(G)$

edge. It turns out that the indecomposable elements are 3–connected graphs, bonds with exactly 3 edges and cycles with exactly 3 edges. In the essentially equivalent decomposition defined by Tutte, any cycle or bond with at least 3 edges is considered indecomposable.

Theorem 1 (Tutte, 1966). *For any 2–connected graph G, the indecomposable elements of the Tutte decomposition are 3–connected graphs, bonds with at least 3 edges and cycles with at least 3 edges. The underlying graph is a tree.*

This is Theorem **11.63** in [20]. The indecomposable components were termed 3–*blocks* by Tutte, but here we refer to them as *basic components* or just *components.* The reason for our choice of indecomposable components is the uniqueness criterion described below. Given a graph G, by $T(G)$ we mean the Tutte decomposition of G, which consists of the indecomposable elements of G and an associated underlying graph denoted by $\Gamma(G)$. If a 2–connected graph G is decomposed into G_1 and G_2 with a virtual edge (u, v), an edge is included in the underlying graph between the subgraphs corresponding to G_1 and G_2. If G_1 or G_2 is indecomposable, then it is represented by a node g_1 or g_2 in the underlying graph. If G_1 or G_2 can be further decomposed, then it corresponds to a subtree representing its decomposition. Thus, in $\Gamma(G)$ each node is a basic component and each edge corresponds to a pair of basic components (say, A and B) that share a single virtual edge (say, (u,v)). In this case, we say the nodes u and v are used as an *attachment pair* by the components A and B. When the two components are attached (and the virtual edge joining them is removed) for every pair adjacent in $\Gamma(G)$, the original graph G is recovered. Note that the order of attachment makes no difference to the recovery of G from $T(G)$.

For any 2–connected graph G, if no two bonds and no two cycles are adjacent in $T(G)$, then the corresponding decomposition is unique. This very useful uniqueness condition has been proved only in [3] and [14], as observed in [21, pp. 14] and in [4, pp. 745]. It may be observed that cycles (bonds) with more than 3 edges can be decomposed into cycles (bonds) of smaller size in a non–unique way. In [13], after decomposing a 2–connected graph using the procedure outlined above, it is suggested that adjacent cycles (bonds) should be merged into larger cycles (bonds) so as not to have any cycle (bond) adjacent to another cycle (bond). The resulting components are taken to be the triconnected components of the given graph. This decomposition is minimal in the sense that the number of components cannot be reduced. It is the unique decomposition of Cunningham and Edmond, and of Hopcroft and Tarjan. This uniqueness was not explicitly noted by Tutte. From now on we restrict our attention to the minimal decomposition and denote it by $T(G)$.

We remark that since the minimal decomposition for any 2–connected graph is unique and there is a $1-1$ correspondence between the set of all 2–connected graphs and the set of all minimal decompositions, any characterization based on minimal decompositions leads to a unique way of constructing the graphs under consideration, thereby avoiding duplicate representations and reducing considerably the need for isomorphism testing.

It is well understood that graphs with given connected components, and connected graphs with given blocks, have unique decompositions and constructions, given their connected components or their blocks. For this reason we restrict our attention to 2–connected graphs. Following Chaty and Chien [2], we note that it is straightforward to construct all separable minimally 2–edge–connected graphs by identifying two or more non–separable minimally 2–edge–connected graphs at nodes which become cut nodes after the identifications. Similarly, it is straightforward to construct all 3–edge–connected graphs by identifying two or more non–separable 3–edge–connected graphs at nodes which become cut nodes after the identifications.

Given the separation property of the basic components of a graph G, it follows that any two basic components of G can have at most two nodes in common. In $T(G)$, any edge which is not virtual is called *free*. For a component H, we denote by $E'(H)$ the set of free edges of H. There is a $1-1$ correspondence between the free edges in $T(G)$ and the edges of G. In our discussion, we often make no notational distinction between an edge in G and its corresponding free edge in $T(G)$. Although an edge e of G belongs to a unique component of $T(G)$, a node of G may belong to more than one component. We denote by H_e the component containing e as a free edge. For a node v, let H_v denote some component of $T(G)$ which contains v. By $\pi(H)$, we denote the node in $\Gamma(G)$ corresponding to the component H. A *terminal component* H is a component such that $\pi(H)$ is a terminal node in $\Gamma(G)$. An *internal component* H is a component such that $\pi(H)$ is an internal node in $\Gamma(G)$. It should be noted that removing an edge from a component does not affect the edge or node connectivity of any other component, while removing a node affects the edge or node connectivity of no component other than the ones to which it belongs. Also, removing an edge e of G from some component other than H leaves every free edge in H unaffected.

In [15], using Tutte decomposition we provided unique decomposition characterizations of several classes of 2–connected graphs. Three classes among them are particularly nice. We give the Theorems characterizing those three classes and refer the reader to [15] for proofs.

Theorem 2. *A 2–connected graph G is minimally 2–connected if and only if each free edge in $T(G)$ belongs to a cyclic component.*

Theorem 3. *A 2–connected graph G is minimally 2–edge–connected if and only if each free edge in $T(G)$ belongs to a cyclic component that contains at least one other free edge.*

Theorem 4. *A 2–connected graph G is 3–edge–connected if and only if each cyclic component has at most one free edge.*

Using these Theorems, applying the inversion of cycle index sum relations, we were able to count the unlabeled versions of these classes in [15]. In this thesis, we count the labeled versions of these three classes.

CHAPTER 3

Deriving counting equations

§3.1 Preliminaries

For each $k = 1, 2, 3, \ldots$, let a_k be the number of ways of labeling all graphs of order k which have some property $P(a)$. Then the formal power series $A(x) = \sum_{k\geq 1} a_k \dfrac{x^k}{k!}$ is the *egf* for the the class of graphs at hand. Suppose also that $B(x) = \sum_{k\geq 1} b_k \dfrac{x^k}{k!}$ is another *egf* for a class of graphs with property $P(b)$. Then

$$\begin{aligned}
A(x)B(x) &= \left\{\sum_{r\geq 1} \frac{a_r x^r}{r!}\right\}\left\{\sum_{s\geq 1} \frac{b_s x^s}{s!}\right\} \\
&= \sum_{r,s\geq 1} \frac{a_r b_s}{r!s!} x^{r+s} \\
&= \sum_{k\geq 1} x^k \left\{\sum_{r+s=k} \frac{a_r b_s}{r!s!}\right\}.
\end{aligned}$$

The coefficient of $\dfrac{x^k}{k!}$ is evidently

$$\begin{aligned}
\left[\frac{x^k}{k!}\right]\left\{A(x)B(x)\right\} &= \sum_{r+s=k} \frac{k! a_r b_s}{r!s!} \\
&= \sum_{1\leq r\leq k} \binom{k}{r} a_r b_{k-r}.
\end{aligned}$$

Then the following lemma provides an useful interpretation of the coefficients of the product $A(x)B(x)$ of these two generating functions.

Lemma 1 (Labeled Counting Lemma). *The coefficient of* $\dfrac{x^k}{k!}$ *in* $A(x)B(x)$ *is the number of ordered pairs* (G_1, G_2) *of two disjoint graphs, where* G_1 *has property* $P(a)$*,* G_2 *has property* $P(b)$*,* k *is the number of nodes in* $G_1 \cup G_2$ *and the labels* 1 *through* k *have been distributed over* $G_1 \cup G_2$*.*

To illustrate, let $C(x)$ be the egf for labeled, connected graphs,

$$C(x) = \sum_{k\geq 1} C_k \frac{x^k}{k!}.$$

Then $C(x)C(x)$ is the generating function for ordered pairs of labeled, connected graphs. On dividing this series by 2, we have the generating function for labeled graphs which have exactly two components. Similarly $\frac{C^n(x)}{n!}$ has the coefficient of $\frac{x^k}{k!}$, the number of labeled graphs of order k with exactly n components. If we let $G(x)$ to be the egf for labeled graphs, we then have

$$G(x) = \sum_{n\geq 1} \frac{C^n(x)}{n!}.$$

Thus we have the following exponential relationship for $G(x)$ and $C(x)$ found by Riddell [12].

Theorem 5. *The $egfs$ $G(x)$ and $C(x)$ for labeled graphs and labeled connected graphs come to terms in the following relation*

$$1 + G(x) = \exp\Big\{C(x)\Big\}. \tag{3.1}$$

Till now all our egfs have x as the only indeterminate. x denote nodes in the graphs. But we have to also keep track of the edges in the graphs. We let ys denote the edges. So, all coefficients in our egfs will be polynomials in y and the egfs are in $\mathbb{Q}[y][[x]]$. Thus (3.1) with edges look like

$$1 + G(x,y) = \exp\Big\{C(x,y)\Big\}.$$

Counting graphs with prescribed properties usually involves decomposing a graph into cores and components. In the case of connected graphs, the core is a set, in which each element is replaced by a component, which is a connected graph, to obtain an arbitrary graph.

Recall from last Chapter that bonds are graphs with two nodes and at least two parallel edges. Several times in this thesis bonds are taken as cores and edges of bonds are replaced by components, say, whose egf is $\beta(x, y)$. Since the edges of every bond is an unordered set, by the above argument, $\exp\Big\{\beta(x,y)\Big\}-\Big\{1+\beta(x,y)\Big\}$ is the egf for the resulting set of graphs. The $\Big\{1+\beta(x,y)\Big\}$ subtracted here corresponds to an empty bond (i.e., just a pair of nodes) and an edge (i.e., a pair of nodes with an edge between them). We subtract an empty bond and an edge because the bonds in the Tutte decomposition have at least three edges and an edge–rooted bond has at least two edges.

Before a component is attached at an edge (u, v) of the core, we root an edge of the core of the component. Edge rooting can be thought of as distinguishing, orienting and then deleting an edge of the graph. By this, the nodes incident to the edge becomes distinguished and labeled with a positive or negative sign (or a 0 or ∞). These distinguished nodes in the graph must be preserved in isomorphisms and automorphisms. In the literature these nodes are called *poles*. Then attaching the component is by identifying u with one of the poles and v with the other. 2–connected graphs that are rooted at an edge of their core are termed 2–*pole networks* by Walsh [21]. Note that for a 2–connected graph, core can be any one of a cycle, bond, and a 3–connected graph.

A 2–connected graph that is rooted at one of the edges of the cycle (respectively, bond, 3–connected graph) that is its core is called as a s–*network* (respectively, p–*network, h–network*). Since in the Tutte decomposition, a cycle component cannot be next to a cycle component (respectively, a bond component cannot be next to a bond component), to form an s–network (respectively, p–network), one has to take an edge–rooted cycle (bond) and replace each of its edges with a 2–pole network that is not a s–network (p–network).

On the other hand, any edge–rooted 2–connected graph with at least two edges belongs to exactly one of the three classes : s–networks, p–networks and h–networks. An h–network has a unique decomposition and a p–network (respectively, an s–network) can be uniquely decomposed into components which are not themselves p–networks (s–networks), where uniqueness is up to orientation of the edges of the core, and also up to their order if the core is a bond.

The above paragraph is Walsh's restatement of Trakhtenbrot's canonical network decomposition theorem. He states this as Proposition 1.1 in [21].

Also note that since we count classes of 2–connected graphs, each class has its own set of s–, p–, h– and 2–pole networks. Further, since counting labeled graphs is counting by their free edges, we count the 2–pole networks for each class and then put back the root edge and its end nodes to get all graphs in the class. For $m > 0$, for all (n, m)–graph, there is exactly $2m$ 2–pole–networks corresponding to it. Therefore there is no need to apply Otter's dissimilarity characteristic approach to labeled counting, as was done for unlabeled counting in [15].

§3.2 Some routines

§3.2.1 $L(R(x, y), \alpha(x, y))$

In the following we will need to compute the natural logarithm of a given egf, say $\alpha(x, y)$ with $\alpha(0, y) = 1$.

Let $R(x, y) = \log\Big\{\alpha(x, y)\Big\}$.
Taking $x\dfrac{\partial}{\partial x}$ on both sides, $xR_x(x, y) = \dfrac{1}{\alpha(x, y)}x\alpha_x(x, y)$.
Rearranging, $xR_x(x, y)\alpha(x, y) = x\alpha_x(x, y)$. Extracting the coefficient of $\dfrac{x^n}{n!}$ gives

$$n\alpha_n(y) = \sum_{0<k\leq n} k\binom{n}{k}R_k(y)\alpha_{n-k}(y)$$

$$= \sum_{0<k<n} k\binom{n}{k} R_k(y)\alpha_{n-k}(y) + nR_n(y)\alpha_0(y).$$

Rearranging, $nR_n(y)\alpha_0(y) = n\alpha_n(y) - \sum_{0<k<n} k\binom{n}{k} R_k(y)\alpha_{n-k}(y).$

Using the fact that $\alpha_0(y) = 1$, we have

$$R_n(y) = \alpha_n(y) - \frac{1}{n}\sum_{0<k<n} k\binom{n}{k} R_k(y)\alpha_{n-k}(y), \text{ where } R_0(y) = 0 \text{ and } R_1(y) = \alpha_1(y). \tag{3.2}$$

§3.2.2 Compose$(d(x,y), h(x,y))$

In the following we will need to compute the terms up to order n of composition of a given egf, say $h(x,y)$ into another egf $d(x,y)$ whose terms up to order n are known. Let

$$d(x,y) = \sum_{0\le k\le n} d_k(y)\frac{x^k}{k!}, \text{ where } d_k(y) = \sum_{0\le j\le m} a_j y^j.$$

Then $d_k(h) = \sum_{0\le j\le m} a_j \left[h^j\right]_{n-k}$, where $\left[h^j\right]_{n-k}$ denotes h^j with terms up to order $n-k$. Let $d_k(h) = \sum_{0\le i\le n-k} \mathfrak{h}_{i,k}(y)\frac{x^i}{i!}$.

Then $d(x,h) = \sum_{0\le k\le n} d_k(h)\frac{x^k}{k!}$

$$= \sum_{0\le k\le n}\sum_{0\le i\le n-k} \mathfrak{h}_{i,k}(y)\binom{i+k}{i}\frac{x^{(i+k)}}{(i+k)!}.$$

Now $\left[h^j\right]_{n-k}$ can be computed using the following recursive scalar exponentiation algorithm.

Let $W(x) = V(x)^\alpha = \left\{\sum_{0\le k\le n} V_k x^k\right\}^\alpha$, where α can be any fixed real number.

Differentiating both sides with respect to x gives

$$W'(x) = \alpha V(x)^{\alpha-1}V'(x) = \alpha\frac{V(x)^\alpha}{V(x)}V\prime(x) = \alpha\frac{W(x)}{V(x)}V'(x).$$

Thus $V(x)W'(x) = \alpha W(x)V'(x)$. Collecting the coefficient of x^n on both sides,

$$\sum_{0\le k\le n} (n-k)V_k W_{n-k} = \alpha \sum_{1\le k\le n} kV_k W_{n-k}.$$

Rearranging,

$$\begin{aligned} nW_nV_0 &= \sum_{1\le k\le n} \alpha k V_k W_{n-k} - \sum_{1\le k\le n} (n-k)V_k W_{n-k} \\ &= \sum_{1\le k\le n} (\alpha k - n + k)V_k W_{n-k} = \sum_{1\le k\le n} ((\alpha+1)k - n)V - kW_{n-k}. \end{aligned}$$

Or, $W_nV_0 = \sum_{1\le k\le n} \left\{\frac{\alpha+1}{n}k - 1\right\} V_k W_{n-k}$ and $W_0 = V_0^{\alpha}$.

This is due to Leonhard Euler and is listed in [10, pp. 526]. Let the maximum degree of y be $O(n^{\alpha})$ in every coefficient of $\frac{x^n}{n!}$ of all power series that we use in a single computation. Then time taken to compute the product of two such coefficients is in order $O(n^{2\alpha})$. Since the above scalar exponentiation algorithm is linear, time taken to compute the the power series exponentiation using that is of order $O(n)O(n^{2\alpha}) = O(n^{1+2\alpha})$. Therefore the power series composition into y for a single $d_k(y)$ takes $O(n^{\alpha})O(n^{1+2\alpha}) = O(n^{1+3\alpha})$. But then every time when $h(x,y)$ is composed into $d(x,y)$, since all the coefficients $d_k(y)$, for all $1 \le k \le n$ has to be composed, the over all time taken for composition for the nth iteration is $O(n)O(n^{1+3\alpha}) = O(n^{2+3\alpha})$.

It must be noted that this a pessimistic estimate of the running time. It only tells that the time is at most a polynomial in n. For a given n, the number of terms needed is assumed to be n. But only terms up to order $n-k$ are computed. Further, all polynomials are assumed to be of size $O(n^{\alpha})$. This is true only for the coefficient of $\frac{x^n}{n!}$. For the coefficient of $\frac{x^k}{k!}$ for $k < n$, is of size $O(k^{\alpha})$, as it depends on the number of possible edges in k node graphs of the type that we work with.

Now since the number of edges in a 3–edge–connected block on n nodes is $O(n^2)$, $\alpha = 2$. So the composition for this case takes $O(n^8)$. As noted by Dirac [5, pp. 214], there are at most $2n-4$ edges in any minimally 2–connected graph on n nodes. These contain the minimally 2–edge–connected blocks, so these also have at most $2n-4$ edges. Thus in both the cases, $\alpha = 1$. So the composition for these cases take $O(n^5)$.

§3.2.3 $MP(R(x,y), \beta(x,y))$

In the following we will need to root an edge of a cycle on m nodes and replace each of the remaining edges with graphs from a set of graphs whose egf is denoted by $\beta(x,y)$. Let $R(x,y)$ denote the egf of the resulting set of graphs. An edge–rooted m–cycle has $m-2$ nodes and $m-1$ edges. Let $n = m-2$. Since all cycles in the Tutte decomposition contain at least 3 nodes, n here starts at 1. Hence the egf of the resulting set of graphs is given by

$$\text{Let } R(x,y) = \sum_{n\geq 1} x^n \beta(x,y)^{n+1} = x\beta(x,y)^2 \sum_{n\geq 0} x^n \beta(x,y)^n$$
$$= \frac{x\beta(x,y)^2}{1-x\beta(x,y)} = xR(x,y)\beta(x,y) + x\beta(x,y)^2.$$

Extracting the coefficient of $\dfrac{x^n}{n!}$ of $R(x,y)$, for $n > 1$, yields,

$$R_n(y) = \sum_{1\leq k\leq n} k\binom{n}{k}\Big\{R_{k-1}(y)\beta_{n-k}(y) + \beta_{k-1}(y)\beta_{n-k}(y)\Big\}$$

$$\text{with } R_0(y) = 0 \text{ and } R_1(y) = \beta_0(y)\beta_0(y). \tag{3.3}$$

We call this routine $MP(R(x,y), \beta(x,y))$.

§3.2.4 $M0P(R(x,y), \beta(x,y))$

In the following we will need to count configurations which have exactly one free edge in a rooted cycle. To accomplish this we root an edge of a cycle on m nodes and replace all but one of the remaining edges with members of a set of graphs whose egf is denoted by $\beta(x,y)$. Let $R(x,y)$ denote the egf of the resulting set of graphs. An edge–rooted m–cycle has $m-2$ nodes and $m-1$ edges. There are $m-1$ ways of choosing an edge to maintain as free. We replace only the remaining $m-2$ edges with graphs counted by $\beta(x,y)$. Let $n=m-2$. Since all cycles in the Tutte decomposition contain at least 3 nodes, n here starts at 1. Hence the *egf* of the resulting set of graphs is given by

$$\text{Let } R(x,y) = y\sum_{n\geq 1}(n+1)x^n\beta(x,y)^n = y\Big\{\frac{1}{(1-x\beta(x,y)^2}-1\Big\}$$

$$= \frac{y-y(1-x\beta(x,y))^2}{(1-x\beta(x,y))^2} = \frac{y\Big\{2x\beta(x,y)-x^2\beta(x,y)^2\Big\}}{(1-x\beta(x,y))^2}$$

$$= R(x,y)\Big\{2x\beta(x,y)-x^2\beta(x,y)^2\Big\} + y\Big\{2x\beta(x,y)-x^2\beta(x,y)^2\Big\}$$

$$= 2xy\beta(x,y) + 2xR(x,y)\beta(x,y) - x^2\Big\{y\beta(x,y)^2 + R(x,y)\beta(x,y)^2\Big\}.$$

Taking the coefficient of $\frac{x^n}{n!}$ on both sides, yields,

$$R_n(y) = 2ny\beta_{n-1}(y) + 2\sum_{1\leq k\leq n} k\binom{n}{k}R_{n-k}(y)\beta_{k-1}(y)$$

$$-\sum_{2\leq k\leq n} k(k-1)\binom{n}{k}\Bigg\{y\beta_{n-k}(y)\beta_{k-2}(y)$$

$$+ R_{n-k}(y)\sum_{0\leq r\leq k-2}\binom{k-2}{r}\beta_r(y)\beta_{k-2-r}(y)\Bigg\},$$

for $n>1$ with $R_0(y)=0$ and $R_1(y) = 2\beta_0(y)(R_0(y)+y) = 2\beta_0(y)y$. (3.4)

We call this routine $M0P(R(x,y),\beta(x,y))$.

§3.2.5 $M1P(R(x,y),\beta(x,y))$

Let $\mathcal{S}$ denote a set of graphs whose *egf* is denoted by $\beta(x,y)$. In the following we will need to count configurations which have at most one free edge of a rooted cycle. To accomplish this we root an edge of a cycle on m nodes and replace each of the remaining edges with graphs from $\mathcal{S}$ or replace all but one of the remaining edges with graphs from $\mathcal{S}$. Let $R(x,y)$ denote the *egf* of the resulting set of graphs. An edge–rooted m–cycle has $m-2$ nodes and $m-1$ edges. There are $m-1$ ways of choosing an edge to maintain as free. We replace all $m-1$ edges or only the remaining $m-2$ edges with graphs in $\mathcal{S}$. Let $n = m-2$. Since all cycles in Tutte the decomposition contain at least 3 nodes, n here starts at 1. Hence the egf of the resulting set of graphs is given by

$$\begin{aligned}
\text{Let } R(x,y) &= \sum_{n\geq 1}\Big\{x^n\beta(x,y)^{n+1} + y(n+1)x^n\beta(x,y)^n\Big\}\\
&= \frac{x\beta(x,y)^2}{1-x\beta(x,y)} + \frac{y - y(1-x\beta(x,y))^2}{(1-x\beta(x,y))^2}\\
&= \left\{\frac{x\beta(x,y)^2 - x^2\beta(x,y)^3 + 2xy\beta(x,y) - x^2y\beta(x,y)^2}{(1-x\beta(x,y))^2}\right\}\\
&= R(x,y)\Big\{2x\beta(x,y) - x^2\beta(x,y)^2\Big\}\\
&+ x\beta(x,y)^2 - x^2\beta(x,y)^3 + 2xy\beta(x,y) - x^2y\beta(x,y)^2\\
&= 2xy\beta(x,y) + x\beta(x,y)\Big\{2R(x,y) + \beta(x,y)\Big\}\\
&- x^2\Big\{y\beta(x,y)^2 + \Big\{R(x,y) + \beta(x,y)\Big\}\beta(x,y)^2\Big\}.
\end{aligned}$$

Taking the coefficient of $\frac{x^n}{n!}$ on both sides gives

$$\begin{aligned}
R_n(y) &= 2ny\beta_{n-1}(y) + \sum_{1\leq k\leq n} k\binom{n}{k}\beta_{k-1}(y)\Big\{2R_{n-k}(y) + \beta_{n-k}(y)\Big\}\\
&- \sum_{2\leq k\leq n} k(k-1)\binom{n}{k}\Big\{y\beta_{n-k}(y)\beta_{k-2}(y)\\
&+ \Big\{R_{n-k}(y) + \beta_{n-k}(y)\Big\}\Big\{\sum_{0\leq r\leq k-2}\binom{k-2}{r}\beta_r(y)\beta_{k-2-r}(y)\Big\}\Big\},
\end{aligned}$$

for $n > 1$ with $R_1(y) = 2y\beta_0(y) + \beta_0(y)(2R_0(y) + \beta_0(y)) = 2y\beta_0(y) + \beta_0(y)^2$. (3.5)

We call this routine $M1P(R(x,y), \beta(x,y))$.

§3.3 Counting labeled blocks

Suppose $R(x,y), N(x,y)$ and $D(x,y)$ are three *egf*s related by $R(x,y) = \dfrac{N(x,y)}{D(x,y)}$, and $D(0,0) \neq 0$. Then multiplying by $D(x,y)$ and collecting the coefficients of $\dfrac{x^n}{n!}$ on both sides, we have

$$\begin{aligned} N_n(y) &= \sum_{0\le k\le n} \binom{n}{k} D_{n-k}(y) R_k(y) \\ &= \sum_{0\le k\le n-1} \binom{n}{k} D_{n-k}(y) R_k(y) + D_0(y) R_n(y). \end{aligned}$$

Rearranging, $$R_n(y) = \frac{1}{D_0(y)} \left\{ N_n(y) - \sum_{0\le k\le n-1} \binom{n}{k} D_{n-k}(y) R_k(y) \right\}. \tag{3.6}$$

Let $\mathfrak{b}(x,y)$ and $\mathfrak{f}(x,y)$ denote the *egf*s of the labeled blocks and 3–connected graphs respectively. In the following let $\mathfrak{b}_y$ denote $\dfrac{\partial \mathfrak{b}}{\partial y}$ and let $\mathcal{N}$ denote $\dbinom{n}{2}$. From $\log \dfrac{\partial C}{\partial x} = \dfrac{\partial \mathfrak{b}(z,y)}{\partial z}$, where $C(x,y)$ is the *egf* of connected graphs and $z = x\dfrac{\partial C}{\partial x}$, Temperley [18] used the calculus to deduce that

$$2(1+y)\mathfrak{b}_y = x^2 \left\{ 1 + \frac{\mathfrak{b}_{xx}}{1 - x\mathfrak{b}_{xx}} \right\}. \tag{3.7}$$

N. C. Wormald in [25], has given a direct combinatorial proof for this remarkable equation. It was employed by E. M. Wright in [26] to find the exact formulae for $u(n, n+k)$ for successive k and general n, where $u(n,q)$ is the number of labeled blocks with n nodes and q edges, and in [27] to derive an asymptotic approximation to $u(n, n+k)$ for fixed k and large n.

Here we use (3.7) to compute the terms of $\mathfrak{b}(x,y)$. Rearranging (3.7),

$$\begin{aligned} \mathfrak{b}_y &= \frac{x^2}{2(1+y)} \left\{ \frac{1 - x\mathfrak{b}_{xx} + \mathfrak{b}_{xx}}{1 - x\mathfrak{b}_{xx}} \right\} \\ &= \frac{x^2 - x^3\mathfrak{b}_{xx} + x^2\mathfrak{b}_{xx}}{2(1+y)(1 - x\mathfrak{b}_{xx})}. \end{aligned} \tag{3.8}$$

Let $\mathfrak{b}(x,y) = \sum_{n\ge 2} b_n(y) \dfrac{x^n}{n!}$ and let $\mathfrak{b}_y(x,y) = \sum_{n\ge 2} b'_n(y) \dfrac{x^n}{n!}$, so that $\mathfrak{b}_{xx}(x,y) = \sum_{n\geq 2} n(n-1)b_n(y)\frac{x^{n-2}}{n!}$. (3.9)

Then for the numerator of (3.8) we have $x^2 - x^3\mathfrak{b}_{xx} + x^2\mathfrak{b}_{xx}$

$$= x^2 - \sum_{n\geq 2}(n+1)n(n-1)b_n(y)\frac{x^{n+1}}{(n+1)!} + \sum_{n\geq 2} n(n-1)b_n(y)\frac{x^n}{n!}$$

$$= x^2 + \sum_{n\geq 3}\Big\{n(n-1)b_n(y) - n(n-1)(n-2)b_{n-1}(y)\Big\}\frac{x^n}{n!} + 2b_2(y)\frac{x^2}{2!}$$

$$= 2(1+b_2(y))\frac{x^2}{2!} + \sum_{n\geq 3}\Big\{n(n-1)b_n(y) - n(n-1)(n-2)b_{n-1}(y)\Big\}\frac{x^n}{n!}.$$

For the denominator of (3.8),

$$2(1+y)(1-x\mathfrak{b}_{xx}) = 2(1+y)\left\{1 - \sum_{n\geq 2}(n-1)b_n(y)\frac{x^{n-1}}{(n-1)!}\right\}$$

$$= 2(1+y) - \sum_{n\geq 1} 2n(1+y)b_{n+1}(y)\frac{x^n}{n!}.$$

Then applying (3.6) to (3.8) we find that, for $n \geq 3$,

$$b_n'(y) = \frac{1}{2(1+y)}\left\{n(n-1)b_n(y) - n(n-1)(n-2)b_{n-1}(y)\right.$$

$$\left. + \sum_{0\leq k\leq n-1}\binom{n}{k} b_k'(y)(n-k)2(1+y)b_{n-k+1}(y)\right\}.$$

Rearranging, $n(n-1)b_n(y) - 2(1+y)b_n'(y)$

$$= n(n-1)(n-2)b_{n-1}(y) - \sum_{0\leq k\leq n-1}\binom{n}{k} 2(n-k)(1+y)b_k'(y)b_{n-k+1}(y) \qquad (3.10)$$

$$= \sum_{n\leq k\leq \mathcal{N}} \alpha(n,k)y^k, \text{ say.}$$

Note that from our definitions, the numbers $u(m,j)$ for $m < n$ and all j can be used on the left side of (3.10) to determine $\alpha(n,k)$ for all k.

Now let $b_n(y) = \sum_{n\leq k\leq \mathcal{N}} u(n,k)y^k$. Then

$$b_n'(y) = \sum_{n\leq k\leq \mathcal{N}} ku(n,k)y^{k-1} = \sum_{n-1\leq k\leq \mathcal{N}-1} (k+1)u(n,k+1)y^k$$

and $yb'_n(y) = \sum_{n \le k \le \mathcal{N}} ku(n,k)y^k$.

Then $n(n-1)b_n(y) - 2yb'_n(y) - 2b'_n(y)$

$$= \sum_{n\le k\le \mathcal{N}} \Big\{ n(n-1)u(n,k) - 2ku(n,k) \Big\} y^k - \sum_{n-1 \le k \le \mathcal{N}-1} 2(k+1)u(n,k+1)y^k$$

$$= -2nu(n,n)y^{n-1} + \sum_{n \le k \le \mathcal{N}-1} \Big\{ n(n-1)u(n,k) - 2ku(n,k) - 2(k+1)u(n,k+1) \Big\} y^k$$

$$+ \Big\{ n(n-1)u(n,\mathcal{N}) - 2\mathcal{N}u(n,\mathcal{N}) \Big\} y^{\mathcal{N}}.$$

Then for $n \le k \le \mathcal{N}-1$,

$\alpha(n,k) = n(n-1)u(n,k) - 2ku(n,k) - 2(k+1)u(n,k+1)$.

Rearranging,

$u(n,k+1) = \dfrac{n(n-1)u(n,k) - 2ku(n,k) - \alpha(n,k)}{2(k+1)}$, for all $k, n \le k \le \mathcal{N}-1$.

Thus for any n, given $\alpha(n,n), \ldots, \alpha(n,\mathcal{N})$ and the fact that $u(n,n) = \dfrac{(n-1)!}{2}$

we can compute all $u(n,n+1), \ldots, u(n,\mathcal{N})$. (3.11)

Maple code for counting labeled blocks using the above derivation is :

$blocks$:= **proc**(n, By, B)

local k, S, R, N, T;

$S := n \times (n-1) \times (n-2) \times \text{coeff}(B,\ x,\ n-1)$;

for k **from** 0 **to** $n-1$ **do**

$S := S - 2 \times \text{binomial}(n,\ k) \times (n-k) \times (1+y)$

$\times\ \text{coeff}(By,\ x,\ k) \times \text{coeff}(B,\ x,\ n-k+1)$

od;

$S := \text{collect}(S,\ y,\ sort)$;

$N := \text{binomial}(n,\ 2)$;

$R := 1/2 \times (n-1)! \times y^n$;

for k **from** n **to** $N-1$ **do**

$T := (\, n \times (n-1) \times \text{coeff}(R,\, y,\, k)$

$\quad - 2 \times k \times \text{coeff}(R,\, y,\, k) - \text{coeff}(S,\, y,\, k)\,)/(2 \times k + 2)\,;$

$R := R + T \times y^{(k+1)}$

od;

collect(simplify(R), y, $sort$)

end

$B := y\,x^2 + y^3\,x^3;$

$By := x^2 + 3\,y^2\,x^3;$

for $iter$ **from** 4 **to** 15 **do**

$B := B + x^{iter} \times$ **blocks**$(iter, By, B);$

$By := By + x^{iter} \times \text{diff}(\text{coeff}(B, x, iter), y);$

od;

It can be seen that this method of computing the number of labeled blocks for any given number of nodes and all possible edges is simpler and more efficient than the computational method derived in [26]. Of course, it does not lead to the sort of asymptotic expressions determined in [27].

§3.4 Counting labeled 3–connected graphs

Let $\mathfrak{f}(x, y)$ denote the egf of all labeled 3–connected graphs. Labeled 3–connected graphs are independently counted in [1, 21, 23]. Our discussion here is identical to that of Walsh[21], for it is based on the same decomposition characterization. Let $d(x, y)$ denote the egf of all labeled 2–pole–networks corresponding to blocks, $s(x, y)$

the egf of all corresponding s–networks, and $p(x,y)$ the egf of all corresponding p–networks. Let $k(x,y) = \frac{2}{x^2}\mathfrak{b}_y(x,y)$. Then

$$d(x,y) = (1+y)k(x,y) - 1 = (1+y)\frac{2}{x^2}\mathfrak{b}_y(x,y) - 1. \tag{3.12}$$

Also, rearranging this gives $1 + y = \frac{x^2}{2}\frac{d(x,y)+1}{\mathfrak{b}_y(x,y)}$. (3.13)

Now, $d(x,y) - s(x,y)$ counts all 2–pole networks that are not s–networks. Since in Tutte decomposition cycles cannot be next to cycles, applying $MP(s(x,y), d(x,y) - s(x,y))$ from equation (3.3) allows one to compute terms of $s(x,y)$, from its own lower order terms and the lower order terms of $d(x,y) - s(x,y)$.

Now, $d(x,y) - p(x,y)$ counts all 2–pole networks with at least two edges that are not p–networks. All p–networks with non–adjacent poles are obtained by parallel unions of networks in $d(x,y) - p(x,y)$. Since the parallel unions are unordered k–tuples, $k \geq 2$, we can write,

$$d(x,y) - p(x,y) = \log\Big\{k(x,y)\Big\} = \log\Big\{\frac{2}{x^2}\mathfrak{b}_y(x,y)\Big\}.$$

Now, the root edge of a 2–pole network can belong to any one of the three types of the components. Writing this in terms of egfs,

$$d(x,y) = \frac{2}{x^2}\mathfrak{f}_y(x,d) + s(x,y) + p(x,y).$$

Rearranging,
$$\begin{aligned} \frac{2}{x^2}\mathfrak{f}_y(x,d) &= \Big\{d(x,y) - p(x,y)\Big\} - s(x,y) \\ &= \log\Big\{\frac{2}{x^2}\mathfrak{b}_y(x,y)\Big\} - s(x,y). \end{aligned} \tag{3.14}$$

Equations (3.12) and (3.14) are equations (9) and (8) respectively on page 2 of [21]. To count labeled 3–connected graphs, first compute $\mathfrak{b}(x,y)$ independently from (3.7). Then compute $d(x,y)$ from (3.12) and $s(x,y)$ from $MP(s(x,y), d(x,y) - s(x,y))$. These along with (3.14) give $\mathfrak{f}(x,y)$ as needed.

§3.5 Counting labeled minimally 2–connected graphs

Let $\hat{\mathfrak{b}}(x,y)$ denote the egf of all labeled minimally 2–connected graphs, $\hat{d}(x,y)$ the efg for the corresponding 2–pole–networks, $\hat{p}(x,y)$ the egf for the corresponding p–networks, and let $\hat{s}(x,y)$ the egf for the corresponding s–networks. Now, the root edge of a 2–pole network can belong to any one of the three types of the components. Writing this in terms of egfs,

$$\hat{d}(x,y) = \frac{2}{x^2}\mathfrak{f}_y(x,\hat{d}) + \hat{p}(x,y) + \hat{s}(x,y). \tag{3.15}$$

Let $h(x,y)$ be an egf satisfying $d(x,h(x,y)) = \hat{d}(x,y)$. Then composing $h(x,y)$ on both sides for the second parameter of (3.14) gives

$$\frac{2}{x^2}\mathfrak{f}_y(x,\hat{d}) = \log\left\{\frac{2}{x^2}\mathfrak{b}_y(x,h)\right\} - s(x,h).$$

This with (3.15) gives

$$\hat{d}(x,y) = \log\left\{\frac{2}{x^2}\mathfrak{b}_y(x,h)\right\} - s(x,h) + \hat{p}(x,y) + \hat{s}(x,y). \tag{3.16}$$

Now, $\hat{d}(x,y) - \hat{s}(x,y)$ counts all 2–pole networks that are not s–networks. Since in the Tutte decomposition cycles cannot be next to cycles and for minimally 2–connected graphs arbitrary numbers of free edges are allowed in cyclic components, each edge of an edge–rooted cycle has to be replaced either by an edge or a non–s–network. Whence applying $MP(\hat{s}(x,y), y + \hat{d}(x,y) - \hat{s}(x,y))$ from equation (3.3) one can compute $\hat{s}(x,y)$ from its own lower order terms and the lower order terms of $y + \hat{d}(x,y) - \hat{s}(x,y)$.

Now $\hat{d}(x,y) - \hat{p}(x,y)$ counts all 2–pole networks that are not p–networks. In the Tutte decomposition bonds contain at least three edges. So in an edge–rooted bond, there are at least two edges. Since a bond cannot be next to a bond, each edge of an edge–rooted bond has to replaced by a non–p–network. As noted before the edges of a bond form an unordered set; writing this in terms of egfs,

$$\hat{p}(x,y) = \exp\left\{\hat{d}(x,y) - \hat{p}(x,y)\right\} - \left\{1 + \hat{d}(x,y) - \hat{p}(x,y)\right\}. \tag{3.17}$$

Let $dp(x,y)$ denote $\hat{d}(x,y) - \hat{p}(x,y)$. Then

$\hat{p}(x,y) = \exp\Big\{dp(x,y)\Big\} - 1 - dp(x,y).$

Applying $x\dfrac{\partial}{\partial x}$ to both sides gives

$$\begin{aligned} x\hat{p}_x(x,y) &= \exp\Big\{dp(x,y)\Big\}xdp_x(x,y) - xdp_x(x,y) \\ &= \Big\{\hat{p}(x,y) + 1 + dp(x,y)\Big\}xdp_x(x,y) - xdp_x(x,y) \\ &= \Big\{\hat{p}(x,y) + dp(x,y)\Big\}xdp_x(x,y) + 0 \\ &= \hat{d}(x,y)x\Big\{\hat{d}_x(x,y) - \hat{p}_x(x,y)\Big\}. \end{aligned}$$

Collecting the coefficients of $\dfrac{x^n}{n!}$ on both sides, gives

$$n\hat{p}_n(y) = \sum_{0<k<n} \binom{n}{k} k\Big\{\hat{d}_k(y) - \hat{p}_k(y)\Big\}\hat{d}_{n-k}(y), \tag{3.18}$$

for $n > 0$, since $\hat{d}_0(y) = 0$. Also, $\hat{p}_0(y) = \hat{p}_1(y) = 0$.

Further, composing $h(x,y)$ on both sides of (3.13), $1 + h(x,y) = \dfrac{\hat{d}(x,y)+1}{\left(\frac{2}{x^2}\right)\mathfrak{b}_y(x,h)}$.

Terms of $h(x,y)$ can be computed using this relation. The division in this can be performed using (3.6).

Thus, $\hat{d}(x,y)$ and hence $\hat{s}(x,y)$ can be computed recursively. Since every edge in a minimally 2–connected graph is from a cyclic component, we have $\hat{\mathfrak{b}}_y(x,y) = \dfrac{x^2}{2}\hat{s}(x,y)$, so that the terms of $\hat{\mathfrak{b}}(x,y)$ can be immediately computed, since $\hat{\mathfrak{b}}(x,0) = 0$.

§3.6 Counting labeled 3–edge–connected blocks

An arbitrary 3–edge–connected graph could have a cut node. Since the Tutte decomposition deals only 2–connected graphs, here we only count 3–edge–connected graphs that don't have any cut nodes. These graphs are also called 3–edge–connected blocks.

Let $\tilde{\mathfrak{b}}(x,y)$ denote the *egf* of all labeled 3–edge–connected blocks, $\tilde{d}(x,y)$ the *egf* for the corresponding 2–pole–networks, $\tilde{p}(x,y)$ the *egf* for the corresponding p–networks, and let $\tilde{s}(x, y)$ the egf for the corresponding s–networks. Now, the root edge of a 2–pole network belongs to one of the three types of the components. Writing this in terms of of egfs,

$$\tilde{d}(x, y) = \frac{2}{x^2}\mathfrak{f}_y(x, y + \tilde{d}) + \tilde{p}(x, y) + \tilde{s}(x, y). \tag{3.19}$$

Let $h(x, y)$ be an egf satisfying $d(x, h(x, y)) = y + \tilde{d}(x, y)$. Then, as before, composing $h(x, y)$ on both sides for the second parameter of (3.14) gives

$$\frac{2}{x^2}\mathfrak{f}_y(x, y + \tilde{d}) = \log\left\{\frac{2}{x^2}\mathfrak{b}_y(x, h)\right\} - s(x, h).$$

This with (3.19) gives

$$\tilde{d}(x, y) = \log\left\{\frac{2}{x^2}\mathfrak{b}_y(x, h)\right\} - s(x, h) + \tilde{p}(x, y) + \tilde{s}(x, y). \tag{3.20}$$

Now, $\tilde{d}(x, y) - \tilde{s}(x, y)$ counts all 2–pole networks that are not s–networks. In the Tutte decomposition cycles cannot be next to cycles and from the decomposition theorem for 3–edge–connected graphs at most one free edge is allowed in cyclic components. Whence applying $M1P(\tilde{s}(x, y), \tilde{d}(x, y) - \tilde{s}(x, y))$ from equation (3.5) one can compute $\tilde{s}(x, y)$ from its own lower order terms and the lower order terms of $\tilde{d}(x, y) - \tilde{s}(x, y)$.

Now, $\tilde{d}(x, y) - \tilde{p}(x, y)$ counts all 2–pole networks that are not p–networks. Let $dp(x, y)$ denote $\tilde{d}(x, y) - \tilde{p}(x, y)$. In the Tutte decomposition bonds contain at least three edges. So, in an edge–rooted bond there are at least two edges. As before, since a bond cannot be next to a bond and the edges of a bond is an unordered set, each of the edges has to be replaced by a non–p–network, giving $\exp\left\{dp(x, y)\right\} - 1 - dp(x, y)$. But from the decomposition theorem for 3–edge–connected graphs at most one free edge is allowed in bond components. So, apart from replacing all the edges of edge–rooted bonds, we need to have configurations where there is exactly one free edge. To accomplish this we distinguish one of edges of edge–rooted bonds by moding out an edge, replace all other edges and then add it back after all the substitutions are over by multiplying by a y. After distinguishing one of the edges, an edge-rooted bond has only one or more edges available for the substitution. Thus for this case we have $y\exp\Big\{dp(x,y)\Big\}-y$. Adding both the cases,

$$\tilde{p}(x,y)=(1+y)\exp\Big\{dp(x,y)\Big\}-(1+y)-dp(x,y).$$

Taking $x\dfrac{\partial}{\partial x}$ on both sides give

$$\begin{aligned}x\tilde{p}_x(x,y)&=(1+y)\exp\Big\{dp(x,y)\Big\}xdp_x(x,y)-xdp_x(x,y)\\&=\Big\{\tilde{p}(x,y)+(1+y)+dp(x,y)\Big\}xdp_x(x,y)-xdp_x(x,y)\\&=\Big\{\tilde{p}(x,y)+dp(x,y)\Big\}xdp_x(x,y)+yxdp_x(x,y)\\&=\tilde{d}(x,y)x\Big\{\tilde{d}_x(x,y)-\tilde{p}_x(x,y)\Big\}+yx\Big\{\tilde{d}_x(x,y)-\tilde{p}_x(x,y)\Big\}.\end{aligned}$$

Collecting the coefficient of $\dfrac{x^n}{n!}$ on both sides, yields

$$n\tilde{p}_n(y)=\sum_{0<k<n}\binom{n}{k}k\Big\{\tilde{d}_k(y)-\tilde{p}_k(y)\Big\}\tilde{d}_{n-k}(y)+ny\Big\{\tilde{d}_n(y)-\tilde{p}_n(y)\Big\},$$

for $n>0$, since $\tilde{d}_0(y)=0$. Then

$$n(1+y)\tilde{p}_n(y)=\sum_{0<k<n}\binom{n}{k}k\Big\{\tilde{d}_k(y)-\tilde{p}_k(y)\Big\}\tilde{d}_{n-k}(y)+ny\tilde{d}_n(y).$$

Or, $$\tilde{p}_n(y)=\frac{1}{n(1+y)}\sum_{0<k<n}\binom{n}{k}k\Big\{\tilde{d}_k(y)-\tilde{p}_k(y)\Big\}\tilde{d}_{n-k}(y)+\frac{y}{1+y}\tilde{d}_n(y). \tag{3.21}$$

Let $LB(x,y)=\log\Big\{\dfrac{2}{x^2}\mathfrak{b}_y(x,h)\Big\}$ and $S(x,y)=s(x,h)$.

Then collecting the coefficient of $\dfrac{x^n}{n!}$ on both sides, yields

$$\begin{aligned}\tilde{d}_n(y)&=LB_n(y)-S_n(y)+\tilde{s}_n(y)+\tilde{p}_n(y) \qquad (3.22)\\&=LB_n(y)-S_n(y)+\tilde{s}_n(y)\\&+\frac{1}{n(1+y)}\sum_{0<k<n}\binom{n}{k}k\Big\{\tilde{d}_k(y)-\tilde{p}_k(y)\Big\}\tilde{d}_{n-k}(y)+\frac{y}{1+y}\tilde{d}_n(y).\end{aligned}$$

Now, $\tilde{d}_n(y)-\dfrac{y}{1+y}\tilde{d}_n(y)=\dfrac{(1+y)\tilde{d}_n(y)-y\tilde{d}_n(y)}{1+y}=\dfrac{\tilde{d}_n(y)}{1+y}.$

Therefore

$$\tilde{d}_n(y) = (1+y)\Big\{LB_n(y) - S_n(y) + \tilde{s}_n(y)\Big\} + \frac{1}{n}\sum_{0<k<n}\binom{n}{k}k\Big\{\tilde{d}_k(y) - \tilde{p}_k(y)\Big\}\tilde{d}_{n-k}(y), \tag{3.23}$$

for $n > 0$. Thus $\tilde{d}_n(y)$ can be computed from (3.23) from its lower terms and terms of other egfs that can be computed independent of $\tilde{d}(x,y)$. Yet, we also need to keep track of the terms of $\tilde{d}(x,y) - \tilde{p}(x,y)$. For this, we rearrange (3.22) to get

$$\tilde{d}_n(y) - \tilde{p}_n(y) = LB_n(y) - S_n(y) + \tilde{s}_n(y). \tag{3.24}$$

Further, composing $h(x,y)$ on both sides of (3.13), $1 + h(x,y) = \dfrac{\tilde{d}(x,y)+1}{\left(\frac{2}{x^2}\right)\mathfrak{b}_y(x,h)}$.

$$\tag{3.25}$$

Terms of $h(x,y)$ can be computed using this relation. The division in this can be performed using (3.6). Let $u(x,y)$ come from $M0P(u(x,y), \tilde{d}(x,y) - \tilde{s}(x,y))$ of equation (3.4). Then $u(x,y)$ counts s–networks in $\tilde{s}(x,y)$ that have exactly one free edge. Let $v(x,y) = y\exp\Big\{dp(x,y)\Big\} - y$. Then, as discussed before, $v(x,y)$ counts p–networks in $\tilde{p}(x,y)$ that have exactly one free edge.

Taking $x\dfrac{\partial}{\partial x}$ on both sides gives

$$\begin{aligned} xv_x(x,y) &= y\exp\Big\{\tilde{d}(x,y) - \tilde{p}(x,y)\Big\}x\Big\{\tilde{d}_x(x,y) - \tilde{p}_x(x,y)\Big\} \\ &= \Big\{v(x,y) + y\Big\}x\Big\{\tilde{d}_x(x,y) - \tilde{p}_x(x,y)\Big\} \\ &= yx\Big\{\tilde{d}_x(x,y) - \tilde{p}_x(x,y)\Big\} + v(x,y)x\Big\{\tilde{d}_x(x,y) - \tilde{p}_x(x,y)\Big\}. \end{aligned}$$

Collecting the coefficient of $\dfrac{x^n}{n!}$ on both sides, yields

$$nv_n(y) = yn\Big\{\tilde{d}_n(y) - \tilde{p}_n(y)\Big\} + \sum_{0<k<n} k\binom{n}{k}v_{n-k}(y)\Big\{\tilde{d}_k(y) - \tilde{p}_k(y)\Big\},$$

for $n > 0$, since $v_0(y) = 0$. Let $z(x,y) = \tilde{d}(x,y) - u(x,y) - v(x,y)$.
Then $\tilde{\mathfrak{b}}_y(x,y) = \dfrac{x^2}{2}z(x,y)$, so that the terms of $\tilde{\mathfrak{b}}(x,y)$ can be immediately computed, since $\tilde{\mathfrak{b}}(x,0) = 0$.

§3.7 Counting labeled minimally 2–edge–connected blocks

An arbitrary minimally 2–edge–connected graph could have a cut node. Since the Tutte decomposition deals only 2–connected graphs, here we only count minimally 2–edge–connected graphs that don't have any cut nodes. These graphs are also called minimally 2–edge–connected blocks. Let $\check{\mathfrak{b}}(x,y)$ denote the *egf* of all labeled minimally 2–edge–connected blocks, $\check{d}(x,y)$ for all corresponding 2–pole–networks, $\check{p}(x,y)$ the *egf* for all corresponding p–networks, and let $\check{s}(x,y)$ the *egf* for all corresponding s–networks. Now, the root edge of a 2–pole network can belong to any one of the three types of the components. Writing this in terms of *egf*s,

$$\check{d}(x,y) = \frac{2}{x^2}\mathfrak{f}_y(x,\check{d}) + \check{p}(x,y) + \check{s}(x,y). \tag{3.26}$$

Let $h(x,y)$ be such that $d(x,h(x,y)) = \check{d}(x,y)$. Then composing $h(x,y)$ on both sides to the second parameter of (3.14) gives

$$\frac{2}{x^2}\mathfrak{f}_y(x,\check{d}) = \log\left\{\frac{2}{x^2}\mathfrak{b}_y(x,h)\right\} - s(x,h).$$

This with (3.26) gives

$$\check{d}(x,y) = \log\left\{\frac{2}{x^2}\mathfrak{b}_y(x,h)\right\} - s(x,h) + \check{p}(x,y) + \check{s}(x,y). \tag{3.27}$$

Let $w(x,y)$ come from $MP(w(x,y), y + \check{d}(x,y) - \check{s}(x,y))$ of equation (3.3). Then $w(x,y)$ counts s–networks in $\check{s}(x,y)$ that have arbitrary number (possibly zero) of free edges. Let $u(x,y)$ come from $M0P(u(x,y), \check{d}(x,y) - \check{s}(x,y))$ of equation (3.4). Then $u(x,y)$ counts minimally 2–edge–connected s–networks that have exactly one free edge. By the decomposition theorem, we have $\check{s}(x,y) = w(x,y) - u(x,y)$. Thus $\check{s}(x,y)$ can be computed from its own lower order terms and the lower order terms of $\check{d}(x,y) - \check{s}(x,y)$.

Now, $\check{d}(x,y) - \check{p}(x,y)$ counts all 2–pole networks that are not p–networks. In the Tutte decomposition bonds contain at least three edges. So in an edge–rooted bond, there are at least two edges. Since a bond cannot be next to a bond, each edge of an edge–rooted bond has to be replaced by a non–p–network. As noted before, since the edges of a bond is an unordered set, writing this in terms of egfs,

$$\check{p}(x,y) = \exp\Big\{\check{d}(x,y) - \check{p}(x,y)\Big\} - \Big\{1 + \check{d}(x,y) - \check{p}(x,y)\Big\}.$$

Since this looks exactly like (3.17), from (3.18),

$$n\check{p}_n(y) = \sum_{0<k<n} \binom{n}{k} k\Big\{\check{d}_k(y) - \check{p}_k(y)\Big\}\check{d}_{n-k}(y), \text{ since } \check{d}_0(y) = 0. \tag{3.28}$$

Also, $\check{p}_0(y) = \check{p}_1(y) = 0$.

Further Composing $h(x,y)$ on both sides of (3.13), $1 + h(x,y) = \dfrac{\check{d}(x,y)+1}{\left(\frac{2}{x^2}\right)\mathfrak{b}_y(x,h)}$.

Terms of $h(x,y)$ can be computed using this relation. The division in this can be performed using (3.6).

Thus, $\check{d}(x,y)$ and hence $\check{s}(x,y)$ can be computed recursively. Let $v(x,y)$ come from $MP(v(x,y), \check{d}(x,y) - \check{s}(x,y))$ of equation (3.3). Then $v(x,y)$ counts s–networks in $\check{s}(x,y)$ that have no free edge. Let $z(x,y) = w(x,y) - v(x,y)$.

Then $\check{\mathfrak{b}}_y(x,y) = \dfrac{x^2}{2} z(x,y)$, so that the terms of $\check{\mathfrak{b}}(x,y)$ can be immediately computed, since $\check{\mathfrak{b}}(x,0) = 0$.

§3.8 Counting labeled connected graphs from blocks

In last two sections we have discussed counting methods for labeled 3–edge–connected blocks and labeled minimally 2–edge–connected blocks. To count all of 3–edge–connected graphs and minimally 2–edge–connected graphs, we have to see how connected graphs are related to blocks in general. Let $\mathfrak{c}(x,y)$ denote the egf of all labeled connected graphs. An obvious way to construct all connected graphs from all blocks is by attaching blocks at various nodes. A more precise definition of this construction involves a core, components, and node–rooting argument. Node–rooting can be thought of as distinguishing a node and removing it. In terms of efg

this is achieved by taking partial derivative with respect to x. Hence in the case of counting all blocks, the core is a set of node–rooted blocks joined at the root; every other node has a node rooted connected graph attached to it, to obtain another node–rooted connected graph. In terms of egfs this is

$$x\mathfrak{c}_x(x,y) = x\exp\Big\{\mathfrak{b}_x(x\mathfrak{c}_x(x,y),y)\Big\}. \tag{3.29}$$

Now, let $\mathfrak{c}(x,y) = \sum_{n\geq 0} c_n(y)\frac{x^n}{n!}$.

Then, $x\mathfrak{c}_x(x,y) = \sum_{n\geq 1} nc_n(y)\frac{x^n}{n!}$ and $x^2\mathfrak{c}_{xx}(x,y) = \sum_{n\geq 1} n(n-1)c_n(y)\frac{x^n}{n!}$.

Also, rewriting (3.9), $\mathfrak{b}_{xx}(x,y) = \sum_{n\geq 0} b_{n+2}(y)\frac{x^n}{n!}$.

Cancelling x and taking $\frac{\partial}{\partial x}$ on both sides of (3.29) gives

$$\mathfrak{c}_{xx}(x,y) = \mathfrak{c}_x(x,y)\mathfrak{b}_{xx}(x\mathfrak{c}_x(x,y),y)\Big\{\mathfrak{c}_x(x,y) + x\mathfrak{c}_{xx}(x,y)\Big\}.$$

Multiplying both sides by x^2 gives

$$x^2\mathfrak{c}_{xx}(x,y) = x\mathfrak{c}_x(x,y)\mathfrak{b}_{xx}(x\mathfrak{c}_x(x,y),y)\Big\{x\mathfrak{c}_x(x,y) + x^2\mathfrak{c}_{xx}(x,y)\Big\}. \tag{3.30}$$

Let $T(x,y)$ denote $\mathfrak{b}_{xx}(x\mathfrak{c}_x(x,y),y)$.

Then collecting the coefficient for $\frac{x^n}{n!}$ on both sides, yields

$$n(n-1)c_n(y) = \sum_{0\leq k\leq n}\binom{n}{k}T_{n-k}(y)$$
$$\Bigg\{\sum_{0\leq r\leq k}\binom{k}{r}(k-r)c_{k-r}(y)\Big\{rc_r(y) + r(r-1)c_r(y)\Big\}\Bigg\}$$

Rearranging,

$$c_n(y) = \frac{1}{n(n-1)}\sum_{0<k<n}\binom{n}{k}T_{n-k}(y)\Bigg\{\sum_{0<r<k}\binom{k}{r}(k-r)c_{k-r}(y)r^2c_r(y)\Bigg\}, \tag{3.31}$$

since $T_0(y) = 0$.

This equation is so general that it can be used to count the number of labeled versions of any class of graphs with cut nodes, given the corresponding numbers of its blocks. Supplying $\tilde{\mathfrak{b}}(x,y)$ (respectively, $\check{\mathfrak{b}}(x,y)$) in equation (3.30) instead of

$\mathfrak{b}(x, y)$ gives number of all labeled 3–edge–connected graphs (respectively, minimally 2–edge–connected graphs), except that for both cases, $\mathfrak{c}(x, y)$ has to start from x instead of x^4y^6 or x^3y^3. This is because the graph with a single node is considered to be a connected graph with 0 number of blocks.

An estimate of time taken for composition into x is $O(n)O(n^{1+2\alpha}) = O(n^{2+2\alpha})$, where α is the maximum degree of the polynomial coefficient of $\frac{x^n}{n!}$ in the indeterminate y which denotes the edges. Since the number of edges in a minimally 2–edge–connected graph need not be of $O(n)$, we contend that the running time in both the cases of minimally 2–edge–connected graphs and 3–edge–connected graphs is $O(n^6)$.

CHAPTER 4

Conclusions

§4.1 Numerical Results

All the sequences computed in this thesis are new, according to N. J. Sloane's online encyclopedia of Integer sequences ⟨*http://www.research.att.com/˜njas/ sequences/Seis.html* ⟩. In Sloane's online encyclopedia, the number of labeled minimally 2–connected graphs, labeled minimally 2–edge–connected blocks, labeled minimally 2–edge–connected graphs, labeled 3–edge–connected blocks, and labeled 3–edge–connected graphs, and have been assigned sequence numbers $A054849$, $A054850$, $A054851$, $A054852$, and $A054853$ respectively.

Programs for all of these sequences were written in Maple, a popular symbolic computation package. The computations in all took less than a day on a Sun Solaris workstation (330 MHz). In the appendices we list only those numbers which can be accommodated within the margins of the thesis; many more numbers were computed.

The computational cost for any of the three classes, namely minimally 2–connected graphs, 3–edge–connected blocks and minimally 2–edge–connected blocks is dominated by the compositions of the form $d(x,h) = \hat{d}$. In section §**3.2.2** it is argued that compositions can be done in $O(n^8)$ or $O(n^5)$, according to the maximum number of edges possible in the type of graphs counted.

But the analysis there assumes that multiplication of two arbitrary long integers is $O(1)$. This is not usually the situation. In the case of unlabeled counting [15], we did all the computations modulo 16 bit primes and then combined the results using

Chinese remainder Theorem. If such an approach is taken for the computations here, a complete analysis would need to consider the bit complexity and the number of primes needed as a function of n. Not to mention about the fact that we will have to to consider the availability of primes numbers, which will force the use of primes too long for a single word. If m is the number of bits used to store a prime number, then multiplications will take $O(m^2)$ time using the most straightforward algorithm. This can be replaced by $O(m \log m \log\log m)$ using Schönhage and Strassen's algorithm [17], but assume $O(m^2)$ is used.

The $\log_2$ of the product of the prime numbers expressible with m bits is approximately 2^m, so $m = O(\log n)$ will suffice for applying the Chinese remainder Theorem to calculate the unlabeled numbers for graphs on up to n nodes. For the latter are bounded above by $2^{n^2/2}$. Hence an additional factor of $O(\log^2 n)$ will cover the bit complexity of the arithmetic operations required by any of our counting algorithms.

Numerical results include the harmonic means of the symmetry numbers for minimally 2–connected graphs, 3–edge–connected blocks and minimally 2–edge–connected blocks up to the order of 32, 24 and 34 respectively. Harmonic means of the symmetry numbers of a class of graphs is defined in section §**1.2**. To compute these numbers for a class of graph one needs the numbers of both labeled and unlabeled versions of that class. From the tables that we computed in [15], we have the numbers of unlabeled versions of these three classes of graphs. Needless to say again that in this thesis we have computed the labeled versions of these three classes of graphs. So we were able to compute and list these numbers in the appendices without any difficulty. Apart from listing them by nodes, we also provide the edge break up for the highest possible node to show the reader how the means are distributed among the edges.

§4.2 Related results and problems

We wish to introduce the reader to [15], where the same author has counted the unlabeled versions of the three 2–connected classes of graphs counted in this thesis and has provided unique characterizations of several additional classes of 2–connected graphs. This is the source of the unlabeled numbers, which were used with labeled counts to calculate the harmonic means of the symmetry numbers.

Unsolved problems include asymptotic estimates for minimally 2–connected graphs and minimally 2–edge–connected blocks. The harmonic means data suggests that this will be challenging, since these means seem to be at least linear in n and perhaps more like $O(n^{3/2})$. For 3–edge–connected blocks it is well known that almost all graphs are 3–edge–connected and identity graphs. However for $m \sim 3n/2$, this argument does not apply, and the asymptotic analysis is unsolved.

APPENDIX A

Numbers of labeled minimally 2–connected graphs

Table A.1: Numbers of labeled minimally 2–connected graphs by number of nodes n.

Number	**n**
1	3
3	4
22	5
255	6
3951	7
76468	8
1773108	9
48018645	10
1488656845	11
51989095026	12
2019934308294	13
86440374376447	14
4040615973961035	15
204882667497974040	16
11202622636253431336	17
657174516317371860777	18
41178215707421481322713	19
2745345678762751634410750	20
194081220845989008686536410	21
14504640300344903152845107091	22
1142846984946965832542305604791	23
94703983496842972291899949276452	24
8235561093842703896290537856790300	25
750066475067436117339309831792670525	26
71417536329697522519356284181113464101	27
7097353643588240286414769732416396689418	28
735065083282003207986728325800611884229678	29
79232100959353125695397056832339487386710055	30
8877330773862605274056094273907124701282216035	31
1032704969774192945458565159543500839470584872496	32
124604172637434430548155045068041842541942822955344	33
15579045289365398297870857981186846948774712752298577	34

Table A.2: Numbers of labeled minimally 2–connected graphs by number of edges m and nodes n.

Number	**m**	**n**
1	3	3
3	4	4
12	5	5
10	6	5
60	6	6
180	7	6
15	8	6
360	7	7
2520	8	7
1050	9	7
21	10	7
2520	8	8
33600	9	8
36120	10	8
4200	11	8
28	12	8
20160	9	9
453600	10	9
967680	11	9
317520	12	9
14112	13	9
36	14	9
181440	10	10
6350400	11	10
23436000	12	10
15876000	13	10
2131920	14	10
42840	15	10
45	16	10
1814400	11	11
93139200	12	11
545529600	13	11
648648000	14	11
187276320	15	11
12127500	16	11
121770	17	11
55	18	11
19958400	12	12
1437004800	13	12

Table A.2: Numbers of labeled minimally 2–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
12573792000	14	12
23782429440	15	12
12330050040	16	12
1783782000	17	12
61747620	18	12
330660	19	12
66	20	12
239500800	13	13
23351328000	14	13
291632140800	15	13
821966745600	16	13
685348503840	17	13
182453871600	18	13
14650693200	19	13
290656080	20	13
868296	21	13
78	22	13
3113510400	14	14
399567168000	15	14
6870739075200	16	14
27537442732800	17	14
34336713771360	18	14
14923440211680	19	14
2260017559800	20	14
108046979040	21	14
1291145856	22	14
2222220	23	14
91	24	14
43589145600	15	15
7192209024000	16	15
165366321120000	17	15
909532928304000	18	15
1611406258862400	19	15
1056310749093600	20	15
265489532708400	21	15
24534143033400	22	15
734749535520	23	15
5487562080	24	15
5571930	25	15
105	26	15

Table A.2: Numbers of labeled minimally 2–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
653837184000	16	16
135998134272000	17	16
4080379919616000	18	16
29940076438272000	19	16
72564844883731200	20	16
67854999820608000	21	16
26003961109526400	22	16
4056322949078400	23	16
240715687212000	24	16
4692175488000	25	16
22529248560	26	16
13737360	27	16
120	28	16
10461394944000	17	17
2697296329728000	18	17
103445760337920000	19	17
989412754672742400	20	17
3185727294010060800	21	17
4078777655272320000	22	17
2244920861325542400	23	17
540356594518540800	24	17
55064140902650880	25	17
2181218503464000	26	17
28508990240640	27	17
89961886560	28	17
33390720	29	17
136	30	17
177843714048000	18	18
56020769925120000	19	18
2698066985822208000	20	18
32988497284334592000	21	18
137834871720363417600	22	18
234176933148238080000	23	18
177144128659802304000	24	18
61746183650511360000	25	18
9830563226914510080	26	18
680363414131040640	27	18
18542873455142400	28	18
166388916657600	29	18
351163205856	30	18

Table A.2: Numbers of labeled minimally 2–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
80174448	31	18
153	32	18
3201186852864000	19	19
1216451004088320000	20	19
72449794385160192000	21	19
1113615277330203648000	22	19
5922452893162027622400	23	19
13026202032200336179200	24	19
13096797818887246464000	25	19
6309032369529451161600	26	19
1467774883667990177280	27	19
160731725741454067200	28	19
7788683694293873280	29	19
149635707810039360	30	19
939579298713792	31	19
1345077539604	32	19
190460826	33	19
171	34	19
60822550204416000	20	20
27572889426001920000	21	20
2003596174650470400000	22	20
38157939208993443840000	23	20
254123956542714306048000	24	20
709234366063367079936000	25	20
923200706113477279488000	26	20
593139291045356030976000	27	20
191957088401688030220800	28	20
31008051841270491187200	29	20
2408159680224883603200	30	20
83728247760549100800	31	20
1156564279485681480	32	20
5161922595748080	33	20
5070333643740	34	20
448201260	35	20
190	36	20
1216451004088320000	21	21
651409512689295360000	22	21
5706858240579944480000	23	21
1329518300241823211520000	24	21
10933887595476176787456000	25	21

Table A.2: Numbers of labeled minimally 2–connected graphs by number of edges m and nodes n (continued).

Number	m	n
380822979286854116812800000	26	21
628371584870644893358080000	27	21
523661699705078998103040000	28	21
227068186311023870292480000	29	21
51388047144199154188800000	30	21
5944259630603786978304000	31	21
335553646027757425632000	32	21
8540325881112263112000	33	21
86221415521306044000	34	21
277097416163920800	35	21
188521243041600	36	21
10458747600	37	21
2100	38	21
255454710858547200000	22	22
16017010370830909440000	23	22
167394150052463093760000	24	22
4716562763920411231027200	25	22
47318459188994125061529600	26	22
202800569109658671191040000	27	22
416878913604388444886476800	28	22
440689416207742440952012800	29	22
248630310615488251223854080	30	22
75791063454442541056051200	31	22
12374243452975513343794560	32	22
1050664341293204365728000	33	22
43989301437627431738160	34	22
833339832540243324912	35	22
6234088884343875324	36	22
14584078715510304	37	22
6926226160392	38	22
242211354	39	22
231	40	22
5620003638888038400000	23	23
409323598365678796800000	24	23
50550387230797211197440000	25	23
1705216797358361042595840000	26	23
20646011824839707926523904000	27	23
107572873567896984509982720000	28	23
271502361707781471669642240000	29	23
357490482629086888453532160000	30	23

Table A.2: Numbers of labeled minimally 2–connected graphs by number of edges m and nodes n (continued).

Number	m	n
256334930236439867613517516800	31	23
102064402465687763031144614400	32	23
22574658521298513295201536000	33	23
2726147200765237199689939200	34	23
173385481724476730186829120	35	23
5475502677818245114294560	36	23
78298717296075523576560	37	23
439107030013252015800	38	23
754638143589200640	39	23
251808440170056	40	23
5570967402	41	23
253	42	23
12926008369442488320000	24	24
10857847030331690188800000	25	24
1571065835247148357877760000	26	24
62869049862323618280407040000	27	24
909866919563717502475984896000	28	24
5702691317068763708060418048000	29	24
17454169805376228895651000320000	30	24
28194428077998383358145916928000	31	24
25206552452306706702044964940800	32	24
12790906820294534874135857664000	33	24
3711603971923874134513858252800	34	24
610860555858398825116314777600	35	24
55688372653296120107653261440	36	24
2698308275326896743366042880	37	24
65193872292795404476422720	38	24
712153055952484387053120	39	24
3023934694836809946192	40	24
3847457431990041408	41	24
906963254993928	42	24
12733767288	43	24
276	44	24
310224200866619719680000	25	25
298590793334121480192000000	26	25
50230468523653509611520000000	27	25
2364762173456443940315136000000	28	25
40556881310736498011342438400000	29	25
302932069045266153033263923200000	30	25
1112402710275154397407442534400000	31	25

Table A.2: Numbers of labeled minimally 2–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
21763042527623400222724657152000000	32	25
23876863664673798960517553817600000	33	25
15135072411699873642463764710400000	34	25
5615847787021749348503214182400000	35	25
1218337619606949010204568332800000	36	25
152308816780213507941823000320000	37	25
10661818495408170086450805120000	38	25
399237535003258683608405760000	39	25
7470070110919560987057600000	40	25
62974587676605850963872000	41	25
204203774854479757896000	42	25
193629462630763104000	43	25
32394745576908000	44	25
28940532000	45	25
300	46	25
77556050216654929920000000	26	26
850272830541926881689600000	27	26
1651360906637288151072768000000	28	26
907720522912650235212103680000000	29	26
18304975841274500407888674816000000	30	26
161587755290573436357996503040000000	31	26
705264952471076670552404388864000000	32	26
1652781739613264218573655040000000000	33	26
2195676442521455837073004562688000000	34	26
1709968300232653117965602181888000000	35	26
794540980995905709108920734156800000	36	26
221237916441811820633463308928000000	37	26
36639282228536210975643271928960000	38	26
3539346980166144975547918598400000	39	26
193001635908045823515627690720000	40	26
5653797641478907336400624640000	41	26
82785725428536726772176000000	42	26
543368033165787673924080000	43	26
1355530569103633099320000	44	26
963344810208824472000	45	26
114835384769196000	46	26
65430947400	47	26
325	48	26
20164573056330281779200000000	27	27
250443997359622099697664000000	28	27

Table A.2: Numbers of labeled minimally 2–connected graphs by number of edges m and nodes n (continued).

Number	m	n
5579638187551870619713536000000	29	27
355630185880811370414625259520000 0	30	27
8372580196987442079171843194880000 0	31	27
86695819249566476715399477043200000 0	32	27
446033687532111854949249992355840000 0	33	27
124015665029500537163113555952640000 00	34	27
197224602099038586063456052363776000 00	35	27
186096203219343812449878440128512000 00	36	27
106428148611840117624076706538547200 00	37	27
372189783678730363399151159373312000 0	38	27
7942967721399156606099203814950400 00	39	27
1021685942748831690370909076208000 00	40	27
77374699361468901408123807860160 00	41	27
3327460766965212633391285409760 00	42	27
770626808777918853278476358400 0	43	27
8910946623895205303205792000 0	44	27
45884602327409820410230080 0	45	27
8863383701593253454336 00	46	27
474401903050849644000	47	27
40428716900867100	48	27
147219842250	49	27
351	50	27
5444434725209176080384000000	28	28
76222086152928465125376000000 00	29	28
1936599950249504489595469824000000	30	28
14221760745089082394181055283200000 0	31	28
388347041135654256922505009971200000 0	32	28
468491949842101854954254564474880000 00	33	28
282020172901906291686387754776576000 000	34	28
922529227463812280241864175623782400 000	35	28
173902587160537425883896687382778880 0000	36	28
196464438738636290918020800495820800 0000	37	28
136299941145371530357183091518940160 0000	38	28
587995244166420940462757923551252480 000	39	28
158110596002210711515484829355555200 000	40	28
263179999245286170779630115881664000 00	41	28
266793047733297593874112609550688000 0	42	28
160375250352447936733100903585856000	43	28
5497210960134035177086505502048000	44	28
1015759118834974736857630195200 00	45	28

Table A.2: Numbers of labeled minimally 2–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
934923117790382433442906176000	46	28
38017753717723332298986 24000	47	28
571854291071953580456 2200	48	28
231483671689358251800 0	49	28
14143965824693250 0	50	28
329772692340	51	28
378	52	28
152444172305856930250752000000	29	29
2394643873304502612688896000000 00	30	29
690114259881373303722053468160000 00	31	29
58051163084427606263183156183040000 00	32	29
182755604406249262166682820460544000000	33	29
2552683105296969396809024635379712000000	34	29
17860163842648077515533168269189120000000	35	29
68223519661001365835190371647247155200000	36	29
151130559901764854090231846440809062400000	37	29
202344477135669627060152969684658892800000	38	29
168207395719899659669146731013941657600000	39	29
88177991942911760278111524021510850560000	40	29
29325611726354350557093127171589521920000	41	29
6171829578491569642070885295761529600000	42	29
813176395299317276214199908938161920000	43	29
65783019450119967783942396242128128000	44	29
3172491335230608926196084144129408000	45	29
87480544587383385044479953688608000	46	29
1299956802473686574946150886464000	47	29
9590394660823547558684240544000	48	29
30973438458656849054182701600	49	29
36459028708733431729459440	50	29
11201823152223246138960	51	29
491970400325717040	52	29
735647060232	53	29
406	54	29
442088099686985097727180800000 0	30	30
77586461495065884651120230400000 00	31	30
25236322865569239938070036480000000 00	32	30
241847091140995717892397381058560000000	33	30
8729282589218458765545922947072000000000	34	30
140368680403443708598311576185290752000000	35	30
1134590884464887256001687893210009600000000	36	30

Table A.2: Numbers of labeled minimally 2–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
50273004960226578396245157956136960000000000	37	30
129878399594237660695023315352031139840000000	38	30
204254593916443791980569467331128514560000000	39	30
201312127603807465986776302834361034240000000	40	30
126613131902995506571015925889231774720000000	41	30
51275101679412750234028690457527322496000000	42	30
13384974350932771664019017876687542656000000	43	30
2237943953988632842436536285266345312000000	44	30
236359502889444411419322314981190144000000	45	30
15423891853028705670381278795898624000000	46	30
602344023385119337680492420753667200000	47	30
13470217849807139527683210051818400000	48	30
162095068113865123554644387860800000	49	30
964394194082483720406049802366640	50	30
2485866813252005748911758776000	51	30
229988146999748953023444600	52	30
5379971441564108385360	53	30
170209909699788336	54	30
1634771573820	55	30
435	56	30
13262642990609552931815424000000	31	31
2590194176066045687583552307200000000	32	31
9465275312996481214138821574656000000000	33	31
1028237245162401214238737209458688000000000	34	31
42331970368074314839088431458415411200000000	35	31
779539761812980631604144132034633728000000000	36	31
7239117657495780929542912986822414336000000000	37	31
36983724458487122858877904305437116416000000000	38	31
110675485090049783465043891345958666444800000000	39	31
202854648853745487344486373861604542566400000000	40	31
234874593221116142928397254288674458982400000000	41	31
175300542327847264097867679258961764480000000000	42	31
85314417213045306326396021485544855370240000000	43	31
27183400745228426224203144470989699633920000000	44	31
5654783711055836207708297516808906894720000000	45	31
760770140181845681992672824472700003520000000	46	31
65112289072953518282076549492101160192000000	47	31
3459874585734412538135989705379700480000000	48	31
110309015112345766482679791751713840000000	49	31
2014868406637398637080271227800088000000	50	31

Table A.2: Numbers of labeled minimally 2–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
197531042605747958056235421088699200	51	31
952884215808845670221082429909600	52	31
1968534362761538873838450404400	53	31
1437005725998978692672551800	54	31
256611654815255526699360	55	31
5859726726444231600	56	31
3619851719610	57	31
465	58	31
4111419327088961408862781440000000	32	32
8902593316256631103990876078080000000	33	32
3639346159952606859897156875059200000000	34	32
446070304832323635607462532490461184000000	35	32
20846407069935340775672958563973857280000000	36	32
437480897632066057594717856691333365760000000	37	32
4643846252514023857623591892746238033920000000	38	32
27205389369716861943963084675081859891200000000	39	32
93732812670269259153120236688501803311104000000	40	32
198843722656740651678893176319622589022208000000	41	32
268298471960638770520276792383863424688128000000	42	32
235387546126290071714139765264431204179968000000	43	32
136120860051079673114922447826611410506752000000	44	32
52223820665858819076418307900384427937382400000	45	32
13294846826269452946849483537675685726515200000	46	32
2232625635753792150322829926424063835648000000	47	32
244372949312349481223753391897244685299200000	48	32
17111198608693205113492020451933909401600000	49	32
746521360481468737126481897613698542080000	50	32
19570347234921544089249666189382589952000	51	32
293804989854201329539128439294491379200	52	32
2358836232687083241554958531500774400	53	32
9270317163327841444077162978451200	54	32
15402265705389771512919095120640	55	32
8901741463921848272193961920	56	32
1216247999789727939417600	57	32
20080105162144625760	58	32
7988638709280	59	32
496	60	32
131565418466846765083609006080000000	33	33
314770263681930885462534547046400000000	34	33
143377493302218734499580506254868480000000	35	33

Table A.2: Numbers of labeled minimally 2–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
19742133453642980644853667744291225600000000	36	33
1042628417324799133230274145337131139072000000	37	33
24822358147537214648303836936256162365440000000	38	33
299774939302605116327622889936115639255040000000	39	33
2003787898730453117186556247180992563380224000000	40	33
7904891511137439828633231496458487160635392000000	41	33
19289071137953510973784948694213813337391104000000	42	33
30114897941666945173708960910945329866670080000000	43	33
30800781218338022918782458766371564044845056000000	44	33
20958344962088102301984595871798165048701747200000	45	33
9569865754927247443915808766572451864834048000000	46	33
2939989431637733412888734070454461304993382400000	47	33
605901301007898107382719897745327413068185600000	48	33
83065487152810081541437175918621272886476800000	49	33
7468816113438640877622785718970174952908800000	50	33
431455083004986663012341508803656650240000000	51	33
15567991877716156259825406422173037356032000	52	33
337694922304618854593327861985834999398400	53	33
4189948357194343599258388321445421312000	54	33
27670190419923040204384051629473126400	55	33
88968738135946085075659049226777600	56	33
119213785106513089747305403438080	57	33
54716114803044270364388265600	58	33
5730994496221407836755200	59	33
68514750378755398080	60	33
17575005651456	61	33
528	62	33
43416588094059432477590972006400000000	34	34
114402709627846604578452211236864000000000	35	34
57848898753150167371485215768916787200000000	36	34
8912096676962508594923012846435224780800000000	37	34
529667668483260380587670631091003069562880000000	38	34
14244955967809981018880454327266330164592640000000	39	34
194875262037666225983537693747509219452518400000000	40	34
1479433669468045952745102081815337784521523200000000	41	34
6649278650220381960220218792826130133119139840000000	42	34
18559338009076278526653104188020349468256501760000000	43	34
33315201982178511068529454013791342118446694400000000	44	34
39433072987133059584473331741766991657224765440000000	45	34
31304089620784244531174179047460856472652775424000000	46	34

Table A.2: Numbers of labeled minimally 2–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
16841583274867243338197862755624193825780449280000000	47	34
6169484645595819337750948037237718175882997760000000	48	34
1538121824738507292917989509531732745349038080000000	49	34
259555934457013543755585781963411893844055367680000	50	34
29336443581600814358670772954312480613361868800000	51	34
2185444390405554682689004255352193082959052800000	52	34
104932059307058411885767502100614540822507520000	53	34
3151784773895671865120657816307801285015552000	54	34
56882344053951512319496376034261159717888000	55	34
586139277304510940733516227686739655168000	56	34
3195537901271090534099613558858964992000	57	34
84378095438986860192980712286025472 0	58	34
9137534234491588329364025277273600	59	34
333962904341413717652980051200	60	34
268587916005718808692934400	61	34
2328366774280927628160	62	34
38551625856480	63	34
561	64	34

APPENDIX B

Numbers of labeled minimally 2–edge–connected blocks

Table B.1: Numbers of labeled minimally 2–edge–connected blocks by number of nodes n.

Number	n
1	3
3	4
22	5
255	6
3321	7
52948	8
1064988	9
25071525	10
667694395	11
20114706546	12
678833013618	13
25302305856919	14
1033146095157645	15
45956558123679960	16
2213869047416018296	17
114892917344393371209	18
6396625360877830999983	19
380658930917828784087070	20
24132881460793575619262190	21
1625195405881640748456133371	22
115961457670364085439666479649	23
8746566274165870774746723908484	24
695955380463967965815163383212500	25
58309508637863044413394647168066925	26
5135502381327223110595219560106954851	27
474732269101673544118502355915523928778	28
45997070861549568002178965587483414637098	29
4665192623870399202541888749316926374752095	30
494710634467096629408878381298033606585860085	31
54789224790173497944934955086974023856699856816	32
6330699419463418491810363893792146723035820562928	33
762424809583409577519726492954589799897560423752849	34

Table B.2: Numbers of labeled minimally 2–edge–connected blocks by number of edges m and nodes n.

Number	**m**	**n**
1	3	3
3	4	4
12	5	5
10	6	5
60	6	6
180	7	6
15	8	6
360	7	7
2520	8	7
420	9	7
21	10	7
2520	8	8
33600	9	8
15960	10	8
840	11	8
28	12	8
20160	9	9
453600	10	9
514080	11	9
75600	12	9
1512	13	9
36	14	9
181440	10	10
6350400	11	10
14364000	12	10
3855600	13	10
317520	14	10
2520	15	10
45	16	10
1814400	11	11
93139200	12	11
370893600	13	11
177962400	14	11
22619520	15	11
1261260	16	11
3960	17	11
55	18	11
19958400	12	12
1437004800	13	12

Table B.2: Numbers of labeled minimally 2–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
9220780800	14	12
7666021440	15	12
1648979640	16	12
117089280	17	12
4866180	18	12
5940	19	12
66	20	12
239500800	13	13
23351328000	14	13
226248422400	15	13
309794284800	16	13
105262597440	17	13
13355662320	18	13
562779360	19	13
18429840	20	13
8580	21	13
78	22	13
3113510400	14	14
399567168000	15	14
5563064707200	16	14
11899836748800	17	14
6154514926560	18	14
1178329632480	19	14
101234853720	20	14
2575492920	21	14
68804736	22	14
12012	23	14
91	24	14
43589145600	15	15
7192209024000	16	15
138395537280000	17	15
441058585968000	18	15
338739423422400	19	15
95206866955200	20	15
11754391849200	21	15
743864121000	22	15
11373682320	23	15
253693440	24	15
16380	25	15
105	26	15

Table B.2: Numbers of labeled minimally 2–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
653837184000	16	16
135998134272000	17	16
3505003197696000	18	16
15981960234240000	19	16
17799442351891200	20	16
7132029019315200	21	16
1286630756611200	22	16
109392323040000	23	16
5398502709600	24	16
48841752960	25	16
924945840	26	16
21840	27	16
120	28	16
10461394944000	17	17
2697296329728000	18	17
90729934783488000	19	17
572072839040102400	20	17
901569322170316800	21	17
502837416643161600	22	17
126817170825945600	23	17
16121012203296000	24	17
974330862785280	25	17
39054897161280	26	17
204927253440	27	17
3337807200	28	17
28560	29	17
136	30	17
177843714048000	18	18
56020769925120000	19	18
2406758982211584000	20	18
20389559510960640000	21	18
44380934409421209600	22	18
33805631351178086400	23	18
11612694994687987200	24	18
2038595896969632000	25	18
193832911066354560	26	18
8427165851744640	27	18
282653669975040	28	18
842804857440	29	18
11932095456	30	18

Table B.2: Numbers of labeled minimally 2–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
36720	31	18
153	32	18
3201186852864000	19	19
1216451004088320000	20	19
65531229299407872000	21	19
727954692121552896000	22	19
2138735264676737126400	23	19
2188728317579374771200	24	19
1003484099881469606400	25	19
237820274910644025600	26	19
30794660547250279680	27	19
2283751833608044800	28	19
71367046238779200	29	19
2047342674424320	30	19
3405643989120	31	19
42287954580	32	19
46512	33	19
171	34	19
60822550204416000	20	20
27572889426001920000	21	20
1833293034078105600000	22	20
26151922596851251200000	23	20
101540401577777530368000	24	20
137499440869141198848000	25	20
82741800336849029376000	26	20
25863317712187667712000	27	20
4526512433537699404800	28	20
447293453760715161600	29	20
26705861667937670400	30	20
594490538686262400	31	20
14825767855310280	32	20
13546218136320	33	20
148682642940	34	20
58140	35	20
190	36	20
1216451004088320000	21	21
651409512689295360000	22	21
52725852321204142080000	23	21
948605523302129172480000	24	21
4775190819280396010496000	25	21

Table B.2: Numbers of labeled minimally 2–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
843214423515881031014400	26	21
656646096575253783398400	27	21
265778741934472935168000	28	21
61025656340163715200000	29	21
8240646025674088266240	30	21
633373114636973168640	31	21
31227440177639160000	32	21
888323901669728320	33	21
10717089775683480	34	21
5311786400928	35	21
51893857008	36	21
71820	37	21
210	38	21
25545471085854720000	22	22
160170103708309094400000	23	22
1559293426291314954240000	24	22
34831124652754590031872000	25	22
223447857938264113330176000	26	22
507355949190504030769152000	27	22
505048103849143705923072000	28	22
260751598210709416242432000	29	22
77021015744066620094668800	30	22
13611247354200521933107200	31	22
1461322607518641124857600	32	22
88144767348343036531200	33	22
3665204152245872359200	34	22
396083837235753374440	35	22
7722541654519230000	36	22
205600374441000	37	22
1798977715920	38	22
87780	39	22
231	40	22
5620003638888038400000	23	23
409323598365678796800000	24	23
474190617032997684019200000	25	23
1297155639143431128821760000	26	23
10442822954558350842630144000	27	23
30083950859481347938357248000	28	23
37858581757714125621717504000	29	23
24619467957342275959958016000	30	23

Table B.2: Numbers of labeled minimally 2–edge–connected blocks by number of edges m and nodes n (continued).

Number	m	n
920325928268760350691594240 0	31	23
2088879302994644856157862400	32	23
292716070663605981673881600	33	23
25540168527078028468444800	34	23
1211167305900929476249920	35	23
43239600903299832218880	36	23
317500814245567795920	37	23
5541181153099126920	38	23
786413592056400	39	23
6197395561896	40	23
106260	41	23
253	42	23
1292600836944248832000 0	24	24
1085784703033169018880000 0	25	24
14826519380001617377689600 00	26	24
490674853899685735715635200 00	27	24
488931250196688959255371776000	28	24
1764685547401096108877856768000	29	24
2778680577784223414722387968000	30	24
2251379112284691976637546496000	31	24
1050695311958065532506472524800	32	24
300770109757030252941316915200	33	24
54258151577717802720599193600	34	24
6144883459364913607190246400	35	24
443333894119594934273516160	36	24
16472584302971217096614400	37	24
512611240584049008937920	38	24
25167545224502416243 20	39	24
395637077474063298 72	40	24
2975412049727040	41	24
21225895542984	42	24
127512	43	24
276	44	24
31022420086661971968000 0	25	25
2985907933341214801920000 00	26	25
476517298539497331916800000 00	27	25
188732712832271619172761600000 0	28	25
22990853769884698612897382400000	29	25
102743979466274190701501644800000	30	25
200472575907980588692890009600000	31	25

Table B.2: Numbers of labeled minimally 2–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
200447667410347462809024921600000	32	25
115449218095206817806742517760000	33	25
41072757415505857718123166720000	34	25
9338280030211279083525388800000	35	25
1369975007458968585012898560000	36	25
126885878469199661349428352000	37	25
7681573630458049268513952000	38	25
222029823842643749730336000	39	25
6099961587801100222896000	40	25
19737821891279235744000	41	25
280979628115145652000	42	25
11145257269423200	43	25
72306698785200	44	25
151800	45	25
300	46	25
7755605021665492992000000	26	26
8502728305419268816896000000	27	26
1573727300370416566222848000000	28	26
73875996891162517803687936000000	29	26
1087999993973157883097569689600000	30	26
5954626389836716736799810355200000	31	26
14265117366934542795383451955200000	32	26
17450863533543669241008258662400000	33	26
12281854813352747268958376064000000	34	26
5363615838618027199226535598080000	35	26
1513298553209515015731893064960000	36	26
279994632901076558280089514240000	37	26
33953727203972248606156903680000	38	26
2589229485420168412578556800000	39	26
133283653494875643136064688000	40	26
2967569143629390268336368000	41	26
72750905138709056114856000	42	26
153227648494778731473600	43	26
1984588739826713935200	44	26
41363557070206800	45	26
245083100199600	46	26
179400	47	26
325	48	26
201645730563302817792000000	27	27
250443997359622099697664000000	28	27

Table B.2: Numbers of labeled minimally 2–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
53385858400932343487545344000000	29	27
2944540219424453471030604595200000	30	27
51903181118132011281722985676800000	31	27
344382894361826435795932565913600000	32	27
1004092888025178415928676589977600000	33	27
1491018169258493230549864442572800000	34	27
1271221385946738725205018350438400000	35	27
674307290020850726750506345789440000	36	27
232973000532453089158955234449920000	37	27
53525815195022953629653495969280000	38	27
8193393684956912821735764337920000	39	27
831466203989395989631474585728000	40	27
52359771606891427364745163104000	41	27
2320275150137198937202648128000	42	27
39339917399886207589268784000	43	27
868246290234946718846150400	44	27
1178077666079667529519200	45	27
13940917032134086394400	46	27
152206744915180800	47	27
826842542795100	48	27
210600	49	27
351	50	27
5444434725209176080384000000	28	28
7622208615292846512537600000000	29	28
1859463199062740882888589312000000	30	28
119555542628725206172477980672000000	31	28
2499427732050862908642030587904000000	32	28
19917968689131909386504384308838400000	33	28
70092141136051385354665369791283200000	34	28
125416815340311409180733137262592000000	35	28
128540050341252565723457742409697280000	36	28
82063379035554845887548656549683200000	37	28
34330331805625188395892502279326720000	38	28
9654400626288809686546389247549440000	39	28
1839416641530555343358130766657920000	40	28
235612374980687612589274197276672000	41	28
20210666871666449528809121767104000	42	28
1050986944293205713387509627712000	43	28
40563920797165650978375137856000	44	28
517314098869135263433850112000	45	28

Table B.2: Numbers of labeled minimally 2–edge–connected blocks by number of edges m and nodes n (continued).

Number	m	n
10354558644982091600476416000	46	28
8974930489963550924011200	47	28
97407394749390018547800	48	28
555662896090848000	49	28
2777448854243700	50	28
245700	51	28
378	52	28
152444172305856930250752000000	29	29
239464387330450261268889600000000	30	29
66469419414937166060274057216000000	31	29
4946307861042329114080035864576000000	32	29
121632228267379585102333781803008000000	33	29
1154142221035184815547544794947584000000	34	29
4863482110895890463096200796661350400000	35	29
10413822033062489640191503853150208000000	36	29
12741441222035248249239796252551782400000	37	29
9712417967322478555894239993793413120000	38	29
4872113878775826084314151386093506560000	39	29
1657123346435660616841632529044019200000	40	29
386772343632047279529651194308606464000	41	29
61910992478261412281574142126963200000	42	29
6688558751024433146754768019414272000	43	29
489264558827719368044240588514816000	44	29
20958241876698846319547793797952000	45	29
712288405472515543740012680736000	46	29
6748354239974873076575690265600	47	29
123255505845497932534890662400	48	29
67784300770730404370328000	49	29
677104251315832011475440	50	29
2013710155583162400	51	29
9292023875337840	52	29
285012	53	29
406	54	29
4420880996869850977271808000000	30	30
7758646149506588465112023040000000	31	30
2437425107117961899750203392000000000	32	30
208557735660338326112638791843840000000	33	30
5986977136298022298734786199633920000000	34	30
67106316466154492467640530889490432000000	35	30
336106688000238181909755907036434432000000	36	30

Table B.2: Numbers of labeled minimally 2–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
85560623883840517061420131758235238400000	37	30
124178961473665355645934750897889996800000	38	30
112220654746240325662987989612615782400000	39	30
66934505814928654049140299637477386240000	40	30
27252122824102172269369005737698590720000	41	30
7694822666878026222774686382648868736000	42	30
1511209299809500824632177581286015232000	43	30
205236191067099391177958888629680960000	44	30
18801698237591150216592984318463872000	45	30
1182531255856793252572881509292902400	46	30
41539661938049956223655001073184000	47	30
1255820057518376014747133304052800	48	30
8733940872972246203952712032000	49	30
146315142934628473299198532464O	50	30
5077894919720117351383800000	51	30
4683613000348395174461400	52	30
724792202653706460O	53	30
30969325825640160	54	30
328860	55	30
435	56	30
1326264299060955293181542400000000	31	31
259019417606604568758355230720000000	32	31
9164627774703100911115730681856000000 0	33	31
89627815829498568110022959205580800000000	34	31
298285559016838183529711224461361152000000	35	31
3920457239959195292642990012306718720000000	36	31
23175291324233076341071872910240604160000000	37	31
69703377126838224467226845094416461824000000	38	31
119299852239804846152811291659528116224000000	39	31
127003866298639061988543663703437766656000000	40	31
89402297803506105820302076192558647705600000	41	31
43184212712625847868833170061736181964800000	42	31
14590829730078689327935269353864781696000000	43	31
3470906690016878688648382456466806886400000	44	31
579382055865235623828818626532480263680000	45	31
67282361034186925321382764932418442880000	46	31
5244343141444881277035503167973556480000	47	31
285887362345788740592143832188949504000	48	31
8184678286539446098029388771125120000	49	31
222144659874518587244861790855888000	50	31

Table B.2: Numbers of labeled minimally 2–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
1121650256416215205609179849144000	51	31
173106563316912236422149757200000	52	31
37748760236702527833845388000	53	31
32245860451374966679756200	54	31
25921667728594204560	55	31
102852901319766480	56	31
377580	57	31
465	58	31
4111419327088961408862781440000000	32	32
8902593316256631103990876078080000000	33	32
35315940822282593592936810990796800000000	34	32
392589784509415843393257244075229184000000	35	32
15051362887912773827297440654920253440000000	36	32
230395100072996668607206200069665587200000000	37	32
1596858861852419451221527731099364884480000000	38	32
5640975834376217776259848649569911177216000000	39	32
11323121679796178117193262987586449563648000000	40	32
14118550534545337061146583819689355771904000000	41	32
11652107285152427632509076282747471380480000000	42	32
6625204040449461392639859440767612428288000000	43	32
2653463117543642962983240674149103945932800000	44	32
755751291358713212751527994443321986252800000	45	32
153119016402950672346879918682231999057920000	46	32
21890096673073458966373599496353052753920000	47	32
2188000204470733009621903420067653086720000	48	32
145351530298377316436431225390747914240000	49	32
6922767751287392201215248259874836992000	50	32
160320956481808100735203673261747712000	51	32
3938893070229145823427825656206579200	52	32
14296371497604258967459042393651200	53	32
204035461486712732312963753990400	54	32
27860175595907097845109926400	55	32
221024963063794666420797120	56	32
92157010345316125440	57	32
340456600875251040	58	32
431520	59	32
496	60	32
131565418466846765083609006080000000	33	33
314770263681930885462534547046400000000	34	33
139411387979826405342752570962083840000000	35	33

Table B.2: Numbers of labeled minimally 2–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
1752668916301455354816014666113744896000000	36	33
7695568206115381572274846014696031518720000000	37	33
13633066765598203677427222380257248542720000000	38	33
110101987428135038779477252907361891778560000000	39	33
454246653453510307440767343947284530069504000000	40	33
1063855465389628250758322972210117051940864000000	41	33
1545465397813379694679084953001926932496384000000	42	33
1486557082458150105502675312395992092311552000000	43	33
988054275118281021160095181192571395207987200000	44	33
465230428567407770854459286592430647710515200000	45	33
157085226391504837683522399310977448635187200000	46	33
38151295547345328072888804281427107319398400000	47	33
6638781858978347337137319625941754281123840000	48	33
817604087838611164106035690482392214835200000	49	33
70753452455558299587915753461327566141440000	50	33
4006642415849087421113083547405330135040000	51	33
168060686518511589564466293433729876992000	52	33
3121918698597419948142524500252743782400	53	33
69936701307059056694081820817741286400	54	33
1808917200280991667237498879687936000	55	33
23953339795308843355217096515200000	56	33
204227214660985291966459906560	57	33
1508647348160986065609836160	58	33
32581887820927524288 0	59	33
1123456254433191360	60	33
491040	61	33
528	62	33
434165880940594324775909720064000000	34	34
11440270962784660457845221123686400000000	35	34
56350592298024176356683551324975923200000000	36	34
79742898753337054412731403626642145280000000	37	34
3988290370932350279063577215581278751948800000	38	34
81293430700927482658999742291186929788518400000	39	34
760559581834642391823894767024149388328960000000	40	34
3645071718555521273036661835534769961959424000000	41	34
9911698465524378614792864886219623257433702400000	42	34
16693879839710801780525350822416035609562316800000	43	34
18614834344324281295678209754290973149251174400000	44	34
14373912084092779363597338933170588054195404800000	45	34
7899008238919185430738254460904656577333299200000	46	34

Table B.2: Numbers of labeled minimally 2–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
3134574198853208569688945935985359413399060480000	47	34
903210298180170646201643180883971627014348800000	48	34
188710407062907171223259642759565074137313280000	49	34
28394335272037441043381132656568732109041664000	50	34
3026051490134078804683773018095854292011008000	51	34
227959814100135492322173826401165470346240000	52	34
10990673249588646806519994325114984092364800	53	34
409244697853626157249144449525951883345920	54	34
6043410812389266975952723779961387776000	55	34
124223816425348609346874187373096494080	56	34
2272750960549281427355061390922252800	57	34
28005987785064590559307382177898240	58	34
148753265660199465723788787072	59	34
1025684097689863981149872256	60	34
114592986100510020576	61	34
36964178270665176 96	62	34
556512	63	34
561	64	34

APPENDIX C

Numbers of labeled minimally 2–edge–connected graphs

Table C.1: Numbers of labeled minimally 2–edge–connected graphs by number of nodes n.

Number	**n**
1	3
3	4
37	5
435	6
6996	7
134428	8
3094785	9
82061325	10
2473950880	11
83606473566	12
3132506471139	13
128918727009163	14
5784184164416490	15
281105943314654760	16
14716271275161095761	17
825944690635111256697	18
49490362937885427672552	19
3154393114011369835245010	20
213171722496912161137680315	21
15230405693148359301804179271	22
1147479043294292099586940655086	23
90955417408856323415723335773564	24
7569429918400141256529611758290825	25
660142844625731377054760088528884725	26
60231157026014894166150028194998381076	27
5740531119919758048246314821662300299718	28
570733234707632499276490471658370506890795	29
59118671423318556284328656356352166035596995	30
6372930098666987790442311642035174026044235650	31
714228276582597219255096380894497532397075600976	32
83142395080478847045844767776743586943721802489889	33
10044769351584623098171761088350297308507827007056113	34

Table C.2: Numbers of labeled minimally 2–edge–connected graphs by number of edges m and nodes n.

Number	**m**	**n**
1	3	3
3	4	4
12	5	5
25	6	5
60	6	6
360	7	6
15	8	6
360	7	7
4410	8	7
2205	9	7
21	10	7
2520	8	8
53760	9	8
74760	10	8
3360	11	8
28	12	8
20160	9	9
680400	10	9
1905120	11	9
482265	12	9
6804	13	9
36	14	9
181440	10	10
9072000	11	10
43848000	12	10
27518400	13	10
1428840	14	10
12600	15	10
45	16	10
1814400	11	11
128066400	12	11
973803600	13	11
1164032100	14	11
201694185	15	11
4518360	16	11
21780	17	11
55	18	11
19958400	12	12
1916006400	13	12

Table C.2: Numbers of labeled minimally 2–edge–connected graphs by number of edges m and nodes n (continued).

Number	m	n
21515155200	14	12
42816090240	15	12
16408549080	16	12
917088480	17	12
13590060	18	12
35640	19	12
66	20	12
239500800	13	13
30356726400	14	13
480518438400	15	13
1465344480600	16	13
1010933583660	17	13
141079483545	18	13
3993663960	19	13
40537926	20	13
55770	21	13
78	22	13
3113510400	14	14
508540032000	15	14
10946324188800	16	14
48322200326400	17	14
53467929234720	18	14
14768123610240	19	14
885999474360	20	14
16373316960	21	14
123231108	22	14
84084	23	14
91	24	14
43589145600	15	15
8990261280000	16	15
255677582160000	17	15
1566734649480000	18	15
2582981752150800	19	15
1215379829213550	20	15
149154340259925	21	15
5156453627325	22	15
65320565310	23	15
386411025	24	15
122850	25	15
105	26	15

Table C.2: Numbers of labeled minimally 2–edge–connected graphs by number of edges m and nodes n (continued).

Number	m	n
653837184000	16	16
167382319104000	17	16
6142146506496000	18	16
50573870290944000	19	16
118022700744748800	20	16
85984413215500800	21	16
18982755186595200	22	16
1203199834636800	23	16
28561855402080	24	16
258274336320	25	16
1249531920	26	16
174720	27	16
120	28	16
10461394944000	17	17
3275288400384000	18	17
152026734892032000	19	17
1638643274306438400	20	17
5211219179175609600	21	17
5511836236006900800	22	17
1966231744159082400	23	17
223947940325334225	24	17
8923387973540040	25	17
156004108500240	26	17
1020279482040	27	17
4138605120	28	17
242760	29	17
136	30	17
177843714048000	18	18
67224923910144000	19	18
3880638762384384000	20	18
53587156542087168000	21	18
225491401841561856000	22	18
330673738614752332800	23	18
176573945569012992000	24	18
33374789784251435520	25	18
2231543706512440320	26	18
63208787057095680	27	18
860211723964320	28	18
4034217096000	29	18
13925969424	30	18

Table C.2: Numbers of labeled minimally 2–edge–connected graphs by number of edges m and nodes n (continued).

Number	m	n
330480	31	18
153	32	18
3201186852864000	19	19
1444535567354880000	20	19
102199624253895168000	21	19
1775433682491460992000	22	19
9653867341700258476800	23	19
18961715932690619088000	24	19
14345822170576499803200	25	19
4172803727170559481000	26	19
456016635231952940145	27	19
20610146139042980520	28	19
441062998976461680	29	19
4861881191360466	30	19
15949478292480	31	19
47284442226	32	19
441864	33	19
171	34	19
60822550204416000	20	20
32438693442355200000	21	20
2777056271416627200000	22	20
59754404851909447680000	23	20
411718863886643529216000	24	20
1054313433644939500032000	25	20
1085033625990212650752000	26	20
457424265376705955328000	27	20
77684140833849027878400	28	20
5467916756005127424000	29	20
183787861922258630400	30	20
3090542119432185600	31	20
28416561639860520	32	20
62930842977600	33	20
161235882900	34	20
581400	35	20
190	36	20
1216451004088320000	21	21
759977764804177920000	22	21
77845565555627950080000	23	21
2046774679587675797760000	24	21
17578016208505612379520000	25	21

Table C.2: Numbers of labeled minimally 2–edge–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
5742433807044719748163200	26	21
7791655000428122936112000	27	21
4547075401773895173183000	28	21
1137222595251110357859870	29	21
122126817640909168029442	30	21
6153596919575413504560	31	21
163050712526507346450	32	21
2197876939254519642	33	21
172321534672702380	34	21
247366037226420	35	21
550446215550	36	21
754110	37	21
210	38	21
2554547108585472000	22	22
185460120083305267200	23	22
22503665404199140761600	24	22
714457293602921046097920	25	22
7540428554730082113822720	26	22
30866238826936085060167680	27	22
53871519076009643033625600	28	22
42003502139127757557288960	29	22
14769396064303440153834240	30	22
2335303629706571162166528	31	22
173622910255120655417088	32	22
6750674043576343905024	33	22
146777871766175727739	34	22
1593082215985343529	35	22
108257282579878740	36	22
967339040976000	37	22
1877766147180	38	22
965580	39	22
231	40	22
562000363888803840000	23	23
47072213812053061632000	24	23
670585156696214490931200	25	23
2543816903758999297228800000	26	23
32589211892359790662106112000	27	23
164644643327092266651977664000	28	23
362303240756010642342622368000	29	23
367030554492310414535283854400	30	23

Table C.2: Numbers of labeled minimally 2–edge–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
174721080375130067334820063800	31	23
39130786066139882560373177250	32	23
4205318609014738075609073325	33	23
235315124770833663259628925	34	23
7409103532460581164735060	35	23
135660899808380567703390	36	23
1176449720262521373900	37	23
7010603219139985080	38	23
3759758417823870	39	23
6393333789843	40	23
1221990	41	23
253	42	23
1292600836944248832000	24	24
124089680346647887872000000	25	24
20588158350596912121446400000	26	24
924417276282940290859008000000	27	24
14220417729458735143589560320000	28	24
87524678370462240835725557760000	29	24
238834105025219671276477808640000	30	24
307390511169498916016811233280000	31	24
192177013936718253331843100160000	32	24
58822973100534304954676127744000	33	24
8904791415684824395315227648000	34	24
702254569460671629742794854400	35	24
31459680884807182554255878400	36	24
826402168110645973504627200	37	24
12946202807353057747612800	38	24
88230934584563300006400	39	24
464843895108976928160	40	24
14514805963072800	41	24
21710009678808	42	24
1530144	43	24
276	44	24
31022420086661971968000	25	25
33930771969786531840000000	26	25
650889151443276499353600000	27	25
3430023000794388330748416000000	28	25
627492453745707502220556288000000	29	25
4652235226690730261728104960000000	30	25
15523100426385162036670083840000000	31	25

Table C.2: Numbers of labeled minimally 2–edge–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
2492018120979132599486152119600000	32	25
1996378592663607840389386765080000	33	25
809918929909794008742971182650000	34	25
167820987686098340260259512920000	35	25
18336207867082313530376545115625	36	25
1135420180216744592098511005500	37	25
42440365499480993158479634500	38	25
943694867892847839664203000	39	25
12770580557702815229151000	40	25
66905409819189401460000	41	25
313417679067444972900	42	25
556383059696079000	43	25
73494526512600	44	25
1897500	45	25
300	46	25
7755605021665492992000000	26	26
961177982351743431475200000	27	26
21177506776193272933908480000000	28	26
1299796616543433770822860800000000	29	26
28034905065500819528219713536000000	30	26
247892845570008779086737702912000000	31	26
999316490969353996663317789696000000	32	26
1970546181698354877618737928192000000	33	26
1982430126817349851055564652902400000	34	26
1038947234949135790710338517811200000	35	26
286651177022829722478342557990400000	36	26
42519353398710739571651337907200000	37	26
3580140923851378435859596366080000	38	26
182955429253253143824273256320000	39	26
5852318467750910944071323040000	40	26
110470818495939905897974080000	41	26
1299876234260446175485152000	42	26
5106806660599715687616000	43	26
21359928609205413084000	44	26
2117267130912648000	45	26
247977118030200	46	26
2332200	47	26
325	48	26
2016457305633028177920000000	27	27
281749497029574862159872000000	28	27

Table C.2: Numbers of labeled minimally 2–edge–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
708708873723884961853685760000000	29	27
5030968158764341199602727731200000	30	27
12693984663903044238990414120960000 0	31	27
132687248914076185821297901455360000 0	32	27
639499086880092413082056292664320000 0	33	27
1528888937983194471939863082034560000 0	34	27
1899426889741713255377086129569072000 0	35	27
1258671201112684034839579768438086000 0	36	27
451117653200128857611251897058745000 0	37	27
88888736722024059130521670625916750 0	38	27
10014135921436370237033818054619362 5	39	27
686868545651079345153694658764200 0	40	27
29904142689572878014044363456850 0	41	27
82931345700624413696580432806 25	42	27
1322424298578534166172559402 00	43	27
13607582433835331808586219 50	44	27
39084217128652343728509 00	45	27
14645115718296268716225	46	27
799851158752393800	47	27
833845033932300	48	27
2843100	49	27
351	50	27
5444434725209176080384000000	28	28
853687364912798809404211200000 0	29	28
243798883296346793319019315200000 0	30	28
1988990057835427576172420136960000 00	31	28
5829395468809156172227751458406400000	32	28
71461356657999475509683836688793600000	33	28
407987556574962519159555381730099200000	34	28
1169404303702400952545848266690232320000	35	28
1768906420976869616157537419795957760000	36	28
1455779685087762542389888893614407680000	37	28
663384329980947614832975003635573760000	38	28
169954830370938483845083653269806080000	39	28
25204000338825198040184913095817600000	40	28
2280230380775206516304663634947328000	41	28
132543365614441179540274671739200000	42	28
4999035086084186220706322481792000	43	28
120815163747846148499258878137600	44	28
1612046615367297592319316480000	45	28

Table C.2: Numbers of labeled minimally 2–edge–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
14591801687515095446398195200	46	28
29902055043627386828208000	47	28
100673508788502969828600	48	28
2999957529952000800	49	28
2794280347289340	50	28
3439800	51	28
378	52	28
152444172305856930250752000000	29	29
267094893560886829876838400000000	30	29
86161497001994438930035113984000000	31	29
8031450679951198283617429499904000000	32	29
271655873554458399589271249713152000000	33	29
3877623565502116952939760752048640000000	34	29
26010149046296520123198981341007667200000	35	29
88510962985576928171503406583188083200000	36	29
161058314129954105998303274120663687040000	37	29
162140432157369226303834127584649381280000	38	29
92225585414944716325338002419298534400000	39	29
30116461096655245507422568554884563893000	40	29
5779571428852779411267143381901052303500	41	29
680018949789763636225758037173961865625	42	29
51649506088454792615631968816240859000	43	29
2605247370345501536272651002453734250	44	29
85600582713909729355541023481259150	45	29
1805591304820920822744570557976450	46	29
19919688415156707442263711612900	47	29
159611199364019089338206905350	48	29
228189398689141778729499750	49	29
692220262485958504167870	50	29
11172965160539145600	51	29
9332226314415270	52	29
4132674	53	29
406	54	29
4420880996869850977271808000000	30	30
8620717943896209405680025600000000	31	30
3126529932505049920832446464000000000	32	30
3311974359192473333801033092300800000000	33	30
12851937190244725805424509593436160000000	34	30
21222161213169240870649752957775708160000000	35	30
1660200406002414599245291385155915776000000	36	30

Table C.2: Numbers of labeled minimally 2–edge–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
6649826293082831430438965120811909120000000	37	30
14404691017774525071736419732397016064000000	38	30
17511045704165344389848576720180222976000000	39	30
12237970812087395745883471944164007628800000	40	30
5003995833860410288839764468266200576000000	41	30
1221886700701762894419905079981798412800000	42	30
184386111552410643800271152202980943360000	43	30
18028484156676961529656000755136928640000	44	30
1183747800697651177468871179396608000000	45	30
52395090343253851596142401602740800000	46	30
1498092354325686810631540994634240000	47	30
27602995144490578479627162073872000	48	30
248459107893164033589880700544000	49	30
1773646225806050179920275415840	50	30
1734360625105425435468168000	51	30
475346407942678419480060 0	52	30
41330217023103808800	53	30
31064779341414420	54	30
4932900	55	30
435	56	30
13262642990609552931815424000000 0	31	31
2867714980644550582681790054400000 00	32	31
116421861921207719267689797058560000000	33	31
1394582566183729718444530783469568000000 0	34	31
6174593234919346524597581910945546240000 00	35	31
11724329877334948240042104472516881408000000	36	31
10626395531373423301520518289960565196800000 0	37	31
4971680014423553332081004901752964288000000 00	38	31
12703870200980445798733519893398346744000000 000	39	31
184417504113170528585045989257187986567840000 0	40	31
1562235489851333055359869155892443678439200000	41	31
78740654080834648345594905951192920204280000 0	42	31
24076394134649606741850824808326646011850000 0	43	31
4595786634702221892922906054055376250167375 0	44	31
5707826370383876861054830418473302818593125	45	31
4798047071684085516055566784508524774031 25	46	31
276547325301246437144544860304050040772 50	47	31
10776514814683545225784669322276871483 75	48	31
266975459916475024482540670506766657 50	49	31
4304488939615142027420780346039252 45	50	31

Table C.2: Numbers of labeled minimally 2–edge–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
31170435087976183385007895590 81750	51	31
1994676185966297239029724262 6625	52	31
1311673202430130989451429245 0	53	31
32568304911339644188061775	54	31
151888973184835581870	55	31
103078268530351605	56	31
5852490	57	31
465	58	31
4111419327088961408862781440000000	32	32
98235512455245584595761391206400000000	33	32
4446203980055161358463660007096320000000	34	32
59949337150562216860131297988077158400000 0	35	32
30132590925956860136777680956724346880000000	36	32
654338806374736330330761187486223302656000000	37	32
682948557578092786791152864125919861145600000 0	38	32
3706693008756631805380040624735621716377600000 0	39	32
11082924247963244329812672488741115176386560000 0	40	32
19026696237760304513639417720001187068641280000 0	41	32
19309080591840569690951784523504154265845760000 0	42	32
11833082752288777711516151251437839737651200000 0	43	32
4464713748393538185847726110927708305879040000 0	44	32
1063491181438698185877027900648540677873664000 0	45	32
165751309042860211596882918318634582136832000 0	46	32
17574361495388905487611529262569625888768000 0	47	32
1295897271121132130398341788544227261184000 0	48	32
65919890416597841439157450911880812134400 0	49	32
2260919198697046187866370290922839603200 0	50	32
48263466464536460633799057867867955200 0	51	32
683084040613493352284115984304177920 0	52	32
392187667413869475919653487917312 00	53	32
226266143461842963305532715027200	54	32
986554002247428587956349184 00	55	32
2225124241186116563225366 40	56	32
554709848745514573440	57	32
340985889456322080	58	32
6904320	59	32
496	60	32
1315654184668467650836090060800000000	33	33
346247290050123974008788001751040000000 0	34	33
17405782527888583245987016662319104000000 0	35	33

Table C.2: Numbers of labeled minimally 2–edge–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
26303609078967355052931822162617303040000000	36	33
149389514309787345578577512019308131123200000	37	33
3691245892727565044750503340372872101888000000	38	33
44120518798845165807969201504500247438950400000	39	33
276071628311331931900349324936951704013721600000	40	33
958963837302452444005822917817201932041932800000	41	33
1930414811585284240677523720801214848334638080000	42	33
2323094341152856730096330505646967760937390080000	43	33
1710299743002742723595595635476659376895144640000	44	33
785819288906678917675374306928015165314617664000	45	33
230567804787536342777060905660996716248755392000	46	33
44572417674884815020447729473403907073055372000	47	33
58885702193342459436293449677827651831132270625	48	33
5459802122549190570172055531299991575640250000	49	33
356340649626566612358091217783007560330472000	50	33
15996841660641965300712438771942053413434000	51	33
482430432109370707408447198348136173188000	52	33
8820865638047969509390098468051618282960	53	33
110084339713820138584051917224353111200	54	33
493776732830236468536514939230682800	55	33
2581478533942760521252185068008800	56	33
737757220543606422213655875120	57	33
1515506219748808162045188480	58	33
2013778690076627884080	59	33
1124693149133459136	60	33
8102160	61	33
528	62	33
4341658809405943247759097200640000000	34	34
12547393959183175986023790909849600000000	35	34
6981037861990599563965183691304468480000000	36	34
1177706417499973473960686041770712104960000000	37	34
75249091972147193785198685918784122781696000000	38	34
2105729155027822325960647127463245380583424000000	39	34
28677456388210898877459804318311930304921600000000	40	34
205706919023452633187841581172759449332875264000000	41	34
824780360984412391638016849991243535478947840000000	42	34
1932163291529207560461691362200184691865026560000000	43	34
2732657796563125709001016934677361574302544691200000	44	34
2391585578696384715685731078320972647573251686400000	45	34
1322378466120296289314098520125605321410843443200000	46	34

Table C.2: Numbers of labeled minimally 2–edge–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
4721779511239700908471031896243039982077707878400000	47	34
1119535282130515917796983181415185513760189644800000	48	34
182265676802872198323904074848136450665811148800000	49	34
20967966287252776797081004117702145359992545280000	50	34
1721057579897164495428512696022660259599482880000	51	34
99653591513997481553555623758352510295040000000	52	34
3939614419340886053277221343984030326968320000	53	34
10443578511607241236961295902314159122186240	54	34
1625254743783853849613500728674789313024000	55	34
17983766572789471502233714310445535872000	56	34
621027708677077570425553755304465920000	57	34
2955340185766390162463377536972288000	58	34
54849689550163386720492031180800	59	34
1028845869570565715748381984000	60	34
726923743800575162688000	61	34
369929481138192412800	62	34
9460704	63	34
561	64	34

APPENDIX D

Numbers of labeled 3–edge–connected blocks

Table D.1: Numbers of labeled 3–edge–connected blocks by number of nodes n.

Number	**n**
1	4
26	5
1858	6
236856	7
53448752	8
21492710960	9
15580155490586	10
20666523608722248	11
50987290998908713436	12
237747545131143536698656	13
2125708373867783657045636502	14
36886187089139407854559886683336	15
1253964423842424859542724072131638032	16
84096628290349161192945727511736213181184	17
11180321458143720676666717999589655484933660978	18
2956196065027153953890308976706797662291149659496840	19
1557906168375422162651067616394078933118378565625539732068	20
163857420008998096478258898700898620910814709375052236480449296 0	21

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n.

Number	**m**	**n**
1	6	4
15	8	5
10	9	5
1	10	5
70	9	6
537	10	6
735	11	6
395	12	6
105	13	6
15	14	6
1	15	6
5670	11	7
32375	12	7
63945	13	7
66090	14	7
42602	15	7
18732	16	7
5880	17	7
1330	18	7
210	19	7
21	20	7
1	21	7
16800	12	8
510720	13	8
2818980	14	8
7207396	15	8
11163523	16	8
11924808	17	8
9459226	18	8
5831560	19	8
2872737	20	8
1147576	21	8
373156	22	8
98112	23	8
20475	24	8
3276	25	8
378	26	8
28	27	8
1	28	8

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	m	n
3515400	14	9
58441320	15	9
332628660	16	9
1040637780	17	9
2158288104	18	9
3277818432	19	9
3872947050	20	9
3704885712	21	9
2948201280	22	9
1987998768	23	9
1149824529	24	9
574550928	25	9
248787882	26	9
93290260	27	9
30163059	28	9
8340552	29	9
1947540	30	9
376992	31	9
58905	32	9
7140	33	9
630	34	9
36	35	9
1	36	9
9238320	15	10
650745900	16	10
8492242500	17	10
51016379400	18	10
188457538500	19	10
490985823825	20	10
975944078145	21	10
1556477503290	22	10
2061536733630	23	10
2324010011625	24	10
2270132385381	25	10
1946802611250	26	10
1479734628330	27	10
1003586008995	28	10
610052393295	29	10
333216921144	30	10
163688109840	31	10

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	m	n
72270520875	32	10
28618931775	33	10
10128741210	34	10
3187559826	35	10
885933085	36	10
215540145	37	10
45379260	38	10
8145060	39	10
1221759	40	10
148995	41	10
14190	42	10
990	43	10
45	44	10
1	45	10
3830765400	17	11
131573395800	18	11
1540470670200	19	11
9857858873940	20	11
41950112984010	21	11
131755268803410	22	11
325480811982480	23	11
659834298980175	24	11
1131141132119955	25	11
1676207000291175	26	11
2183023317317095	27	11
2530480711778965	28	11
2636274835162485	29	11
2487008219768787	30	11
2136729421875675	31	11
1679082757997535	32	11
1210581893082825	33	11
802469996161515	34	11
489678723185949	35	11
275195549819355	36	11
142401250148685	37	11
67785468007455	38	11
29637473735265	39	11
11876328289221	40	11
4349381135715	41	11
1450561674705	42	11

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	m	n
438654417135	43	11
119646432405	44	11
29248154419	45	11
6358379665	46	11
1217565855	47	11
202927725	48	11
28989675	49	11
3478761	50	11
341055	51	11
26235	52	11
1485	53	11
55	54	11
1	55	11
9520156800	18	12
1249655299200	19	12
30624548668560	20	12
342562095219960	21	12
2341938014967600	22	12
11274977968919280	23	12
41483468972754270	24	12
122931660335292270	25	12
304244002678771980	26	12
645598282306244080	27	12
1197983981393559030	28	12
1973682941817384930	29	12
2921451961237286884	30	12
3921926777575992504	31	12
4811030448893456535	32	12
5425288217159867880	33	12
5651186040429998805	34	12
5458180112880550308	35	12
4902898794987512308	36	12
4105460933711974650	37	12
3210163350299355450	38	12
2346820709135704120	39	12
1605303827845094916	40	12
1027832616312111126	41	12
616000143780504450	42	12
345455329430793780	43	12
181166212358880660	44	12

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
88761640548523866	45	12
40578264311362866	46	12
17282467989749160	47	12
6844596385501050	48	12
2515112321312250	49	12
855282836083104	50	12
268347662117964	51	12
77411290969980	52	12
20448655423020	53	12
4922861969740	54	12
1074081808596	55	12
210980512776	56	12
37014130780	57	12
5743572120	58	12
778789440	59	12
90858768	60	12
8936928	61	12
720720	62	12
45760	63	12
2145	64	12
66	65	12
1	66	12
6782977000800	20	13
407496100611600	21	13
8310933094464000	22	13
91846508423670000	23	13
670754076845612760	24	13
3603080647651785300	25	13
15205075114450054680	26	13
52680225949811322720	27	13
154614027850541512110	28	13
393434217152034445530	29	13
883536790182037069078	30	13
1775566022674161398538	31	13
3228562413554260230162	32	13
5359360877859649155742	33	13
8180884261744868474190	34	13
11551933190539279757934	35	13
15163633484072584083001	36	13
18578063963267510404560	37	13

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
21315350318382842863713	38	13
22965073452721201295670	39	13
23286111894863799733323	40	13
22261989022649875107180	41	13
20095184868092108587241	42	13
17146052495231126192154	43	13
13840128101369195531685	44	13
10574747870910762629584	45	13
7650778669554903898587	46	13
5242149680860798656522	47	13
3401431927211930498744	48	13
2089617560796542097750	49	13
1214944546882032291468	50	13
668180028450769800394	51	13
347354651313675041454	52	13
170538858496096748658	53	13
78995813324888146830	54	13
34482976595961931674	55	13
14165784135232385292	56	13
5468236651960764682	57	13
1980037677842167944	58	13
671229894401828190	59	13
212561426247966796	60	13
62723851677249048	61	13
17198583234300468	62	13
4367906528841856	63	13
1023729290804220	64	13
220495634692872	65	13
43430964297396	66	13
7778680447356	67	13
1258315963047	68	13
182364632450	69	13
23446881315	70	13
2641902120	71	13
256851595	72	13
21111090	73	13
1426425	74	13
76076	75	13
3003	76	13
78	77	13

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
1	78	13
16305064776000	21	14
3534109849670400	22	14
143200051416021600	23	14
2628880855673272200	24	14
29257842678545360880	25	14
227894844819081922020	26	14
1350965376255812176860	27	14
6434083131732181325400	28	14
25559506033604306774010	29	14
87037527428609666325417	30	14
259367099121334298397303	31	14
687294697159495024787187	32	14
1640281187640217383324427	33	14
3562050236687853980770792	34	14
7097943828033581706184092	35	14
13068470710563768903370331	36	14
22360462980107459881308635	37	14
35727066290887593625811808	38	14
53522684268880832289429992	39	14
75437941448213233274743358	40	14
100325118772671312468304356	41	14
126199847222320061028817979	42	14
150463920434281314201301961	43	14
170326642450982461675524111	44	14
183331934806211582808998035	45	14
187853764963101326547449446	46	14
183423221489473613068889286	47	14
170798574413315000148236240	48	14
151767333553230568532038394	49	14
128747455258283304699881625	50	14
104305571099742345212542663	51	14
80718415233844721994634435	52	14
59672182560808126287805827	53	14
42140261130915630603426506	54	14
28424796020509649962162598	55	14
18309673238834685868879914	56	14
11259431833289748807499710	57	14
6607498834062887715684366	58	14
3698603909832700110340098	59	14

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
1973683528368647520825400	60	14
1003409410002844032396208	61	14
485650728821582950746120	62	14
223594038651772082844264	63	14
97834164417179852023362	64	14
40641985058857641274662	65	14
16011273921836908092014	66	14
5974539478859586495770	67	14
2108699942445363265908	68	14
702907541579576040344	69	14
220915133603719104836	70	14
65341308740414807100	71	14
18150393963503626825	72	14
4724078999322017335	73	14
1149100725977359155	74	14
260462872358886197	75	14
54834292254323582	76	14
10682005207130790	77	14
1917282997610974	78	14
315502265886262	79	14
47325339894651	80	14
6426898010533	81	14
783768050065	82	14
84986896995	83	14
8093990190	84	14
666563898	85	14
46504458	86	14
2672670	87	14
121485	88	14
4095	89	14
91	90	14
1	91	14
18240218516520000	23	15
1735510121947602000	24	15
55797855012500000400	25	15
964337553067817728800	26	15
10931844685078219102200	27	15
90617425521712752663000	28	15
587539631238943537053900	29	15
3117829457467369817172000	30	15

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
1398596247671925377176101O	31	15
54325247157138051086367375	32	15
186126047527565117287279965	33	15
570731814987739335932286285	34	15
1584731889968111262908314785	35	15
4022740408225819414705426310	36	15
9409221568613910644949574590	37	15
20413024642349236831211840835	38	15
41303476122931851740455717075	39	15
78311872384020937176807133380	40	15
139689792068728184297594219640	41	15
235223813453837290548863422025	42	15
375016624705915196486601724605	43	15
567501920894013988488970520520	44	15
816911023853310499567115480000	45	15
1120689395845733305585471848825	46	15
1467568674687960664787677869705	47	15
1837027335637210657186146042415	48	15
2200656325301857713025426327455	49	15
2525503992488666563160361123003	50	15
2778920399974039839027040686155	51	15
2933931630902659883758461553200	52	15
2973927306917293823484192018060	53	15
2895544989858974939088961473115	54	15
2709069939179094548744372630763	55	15
2436304695839868027061221946950	56	15
2106496448123770380301953326270	57	15
1751344354116781328527247981265	58	15
1400226261957004250176879236675	59	15
1076582461618224414136267545925	60	15
795975194318642033774309934885	61	15
565863050786537275150817713275	62	15
386737245151958742990474245005	63	15
254053160726788141082436147075	64	15
160371311195109366295609439235	65	15
97250229068245232972638107560	66	15
56632261376250201978915240480	67	15
31657230323004515521235888490	68	15
16979417244752186922633112450	69	15
8733669778171822597940630700	70	15

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	m	n
430581666027878718664366920	71	15
2033462223609107236883156310	72	15
919285917438467873345401350	73	15
397543508773657173163536300	74	15
164321947673485149881136804	75	15
64864944876671077508415045	76	15
24429899009015235929436165	77	15
8769762033773631737342160	78	15
2997271811362022011045620	79	15
974115525833226351048474	80	15
300653327789211965874670	81	15
87996158961807349354920	82	15
24384487124224536298080	83	15
6386414547684871560575	84	15
1577820215758858386315	85	15
366934950084448145400	86	15
80135220507862920400	87	15
16391295227952832005	88	15
3130921568954395125	89	15
556608279411651740	90	15
91748617507304400	91	15
13961746143144975	92	15
1951641934004035	93	15
249145778809200	94	15
28848458598960	95	15
3005047770725	96	15
278818865325	97	15
22760723700	98	15
1609344100	99	15
96560646	100	15
4780230	101	15
187460	102	15
5460	103	15
105	104	15
1	105	15
42856575521760000	24	16
14186400089564102400	25	16
876358217163501072000	26	16
24362893224350737056000	27	16
407468024627650325539200	28	16

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
4737301195527923598576000	29	16
41686165629142246221609600	30	16
293430918493800174123036000	31	16
1717091035635560392668861000	32	16
8591604521827489534604232000	33	16
37550408914700352604061888760	34	16
145761359113077120054598156200	35	16
509239998590316835970408400120	36	16
1618585812720738114027755580600	37	16
4722128710697098098297749282160	38	16
12739221637775858504206376038240	39	16
31978401392955664001522999757240	40	16
75088962160587605322494709228600	41	16
165677648317160605022797490436880	42	16
344832075020939980457297699218080	43	16
679305148967898254805783734234280	44	16
1270280872730366073737144951686720	45	16
2260539068487544715363177635630680	46	16
3836727837710155695342988686300840	47	16
6222788510575756730087878464769160	48	16
9660903565997332114082110294146840	49	16
14378098175331236244471284066447040	50	16
20539912273148864740479341831013280	51	16
28196899428403901227382831711884920	52	16
37234015792800556714180376606494320	53	16
47335745382178653901017506342654280	54	16
57979627656469198531879026015378264	55	16
68466907330318037307257478505287120	56	16
77991633339590390228971608112829240	57	16
85740278857888513742387203450470720	58	16
91005354659315084418334965925147920	59	16
93291232104473367890893375717087688	60	16
92390399064201530643519646742278560	61	16
88414039773719367492391570056999360	62	16
81770789089609959872781318138758560	63	16
73099022795982208757828465260425495	64	16
63167900524839531891640836513331512	65	16
52767967523343644886399712564761180	66	16
42612256425623924028373728986081160	67	16
33264017808960861313887555469655910	68	16

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
250993240171823525042068086731555 60	69	16
1830432250575827490076175961008 7468	70	16
12900107024688045176801622261817560	71	16
8784390338439810217092811634652405	72	16
5778650548909561242124787525514480	73	16
3671494842257897106782715556353000	74	16
2252442328752895120327845276262800	75	16
1333947135828580963091366644194060	76	16
762368265154172089947809261836200	77	16
420325951492803210921327304423920	78	16
223482362263332449236532136302760	79	16
114541380038564039513029648782687	80	16
56566009054541097069367768435720	81	16
26904144068250457846554146129700	82	16
12317816256459371182380980375160	83	16
5425782949514800651140412764090	84	16
2298001270810349294086663235664	85	16
935239267855731988864663925340	86	16
365497409038994290677321946880	87	16
137061919975917568542218076405	88	16
49280780371783283747170022640	89	16
16974510463094455553087064896	90	16
5595996394271666770818823680	91	16
1763956128404406691273517160	92	16
531083696254281831287557560	93	16
152545338326121444942963480	94	16
41749253698986245634267864	95	16
10872201928109541453226975	96	16
2690029399133740248148200	97	16
631333436838024097027500	98	16
140296319960270438861000	99	16
29462227252550832347790	100	16
5834104411367434183800	101	16
1086744939716954163100	102	16
189916591427320965480	103	16
31044058215085159965	104	16
4730523156623397008	105	16
669413654240284920	106	16
87586833265105440	107	16
10542859559688820	108	16

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
1160681786387760	109	16
116068178638776	110	16
10456592670160	111	16
840261910995	112	16
59487568920	113	16
3652745460	114	16
190578024	115	16
8214570	116	16
280840	117	16
7140	118	16
120	119	16
1	120	16
70712366893616448000	26	17
9968435142862996800000	27	17
472466915278196254416000	28	17
11951614351258537677321600	29	17
196906140134750147957856000	30	17
2357585589974343218840750400	31	17
21966524156747962549892534400	32	17
166815462755154261986927589600	33	17
1067275579180416362613994737600	34	17
5896847209006624236554168220240	35	17
28677642187963614980531721924600	36	17
124620716781963918228424502230440	37	17
489820811617681710991460311876200	38	17
17587749055323812555805018136859000	39	17
5817059098263661192846474373062080	40	17
17845830032842007248824540948073440	41	17
51082425065806314945744119609714600	42	17
137119042215136195366043411325574040	43	17
346657721615839649628784867379413440	44	17
828539796766632382878460859154561408	45	17
1878261296206332207164924221017741288	46	17
4050144681669489739241227403099436600	47	17
8328105827589773293142182708454629320	48	17
16365963822144304098757941520931607360	49	17
30796557204408502822056499877406017016	50	17
55587293122221860374413218117515791096	51	17
96388149360216591167760810418311038040	52	17
160780531844445951027400998369212168040	53	17

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
25830084606034634752706708001122084l560	54	17
40009500437180563473909007707941950512 8	55	17
59807526615005175855690735181485717628 8	56	17
86351166715372163220244771688169959032 0	57	17
120510015256446899793257237145650327476 0	58	17
162669634202145714121916746753715265684 0	59	17
212505799482140187510944027441715492138 4	60	17
268805725985624075970215181755440346646 4	61	17
329385890863412426393053070682292920852 0	62	17
391147810260372574852250529185910347668 0	63	17
450292327444028014208416567402181957452 0	64	17
502682560099663032184209929705799077200 8	65	17
544312069173962348035273200893952882816 8	66	17
571806570787167841193029693149751884404 0	67	17
582872809266242394492313373844215708754 0	68	17
576612273891565625761754093854211472196 0	69	17
553640617523755791765359821985129701343 2	70	17
515990925604975850062611816039806021467 2	71	17
466821337477919410646522004643428359705 0	72	17
409984472329289954137222927201357343312 0	73	17
349538880986179251190769927379472668048 0	74	17
289286738506270126846492311295011852310 4	75	17
232407869776978196210891577358177538068 4	76	17
181233240265136139943110741569742257280 0	77	17
137169164617110662995732765814481027364 0	78	17
100754703782962714128503720728566444664 0	79	17
71814944532339858498643784579252284106 1	80	17
49664659177258718171941443468121484881 6	81	17
33319445593792643827711575820866492418 0	82	17
21681652997842091468133761248580449500 0	83	17
13682013853030324517563679890861394747 0	84	17
8371077168509771301819783449076579927 2	85	17
4964652505844495839968989991604467073 2	86	17
2853426682693095808592597696481745188 0	87	17
1588914438203092956337315669476900841 5	88	17
856972630133543292194259252082023396 0	89	17
447541749756218243691188763379547288 8	90	17
226234197119830566636670720504470332 8	91	17
110659561047404290072237657424344308 0	92	17
52355582422580214139617876506630996 0	93	17

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	m	n
23950064109989345742389389219147232 0	94	17
105885024260912898251890723283209632	95	17
45221887108838350937521677242649197	96	17
18648245784005168069486288514172640	97	17
7421252836070426828742038743175100	98	17
2848564831118839940874799080560200	99	17
1053969755786369333488831694964394	100	17
375672566724675986419893952964784	101	17
128907292570848624316912309465220	102	17
42551929988835570196737242267960	103	17
13502056310130729011649057710295	104	17
4114912694872560070599008646928	105	17
1203417914764785031968805910128	106	17
337406900276279005451418070320	107	17
90600002251072523252743298220	108	17
23273395249068666203487494040	109	17
5712560674784854050414374568	110	17
1338077277887479948968346288	111	17
298677964110143486995027525	112	17
63436027804941158139859320	113	17
12798496840458657226987140	114	17
2448408091426855123232472	115	17
443246292428128642726182	116	17
75768596997152950981720	117	17
12200028330096177771340	118	17
1845382436487107630280	119	17
261429178502407087543	120	17
34569147570567911728	121	17
4250305029168213960	122	17
483774556165488000	123	17
50718300243156000	124	17
4868956823342976	125	17
425067659180736	126	17
33469894423680	127	17
2353351951665	128	17
145944307080	129	17
7858539612	130	17
359933112	131	17
13633830	132	17
410040	133	17

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
9180	134	17
136	135	17
1	136	17
16332935130832320000	27	18
781362014683915304640000	28	18
69533678366534590528320000	29	18
2767347207327054820916448000	30	18
65802018766500160208352288000	31	18
1080648639143266774881432132000	32	18
13357386015539240705983436484000	33	18
131449812772440421584784354896000	34	18
1071171322629953325526696861310400	35	18
7439213643510823436111351798696400	36	18
45006549405725092822504187125806000	37	18
241292961463793500505402025713445600	38	18
1162211445102580487236397378138936400	39	18
5085636374648679628978924187890477050	40	18
20404882402129359030984741730598189850	41	18
75649883323771410627511548096436654200	42	18
260860425766781333370851057690871135000	43	18
841315811206016934106262180923259036700	44	18
2550042613504687683482297475094646635500	45	18
7294250851540470669374587159671239688240	46	18
19762110305340168871209549518111057975280	47	18
50872593905336845050101636369590186130790	48	18
124779923737611142717798211126794007762550	49	18
292338523511625600141549764170910157269640	50	18
655625479957418467597166209473802840176120	51	18
1410249988743512760011448476654320522010520	52	18
2914451198714757246088036319680554722048040	53	18
5795716301386042318256961357050783680693800	54	18
11105687717549808330074323292703508367808360	55	18
20530881143490859253081644431094096047123330	56	18
36658428979078079489015434613871827856964530	57	18
63281280836380281202537761102700415467203360	58	18
105705806828616632397945409993690743616448400	59	18
170998476295253129962580974057004820050004780	60	18
268082048036752552594429984193837683471403580	61	18
407573360933965526276654909443924567248994320	62	18
601254271057235698296899641244960271555598320	63	18

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	m	n
861094226348166227523684593995456866120472 65	64	18
11978034335506175892617836304392148089218173 7	65	18
16189881612560145807056562654587980248782662 8	66	18
21270927904199980019984228407655384715943338 0	67	18
27174244587139013051668181237615148735735383 4	68	18
33766356992984651549831419709041462596815121 0	69	18
40820577596391140443269191967493753141382574 0	70	18
48022101956443442096854605798793106803401346 8	71	18
54986563264696102094942583594628950069979031 1	72	18
61291479205965960067328131306998117138281358 3	73	18
66517475546291219063181472490583492423387528 0	74	18
70293898955045113120257322820271049297837929 6	75	18
72342012689392145352335928024704321770558832 4	76	18
72508827589774844448232785528875300784902586 0	77	18
70785895807315746252477667628878844113379647 2	78	18
67309876552940531600984520970150369425244140 0	79	18
62344852888480091103984792573758029052607393 3	80	18
56249538051659237864964291090757710800755834 9	81	18
49434966979496222933183962306052526551314286 4	82	18
42319520040467815465384186260845752475347888 8	83	18
35287980182304185011644973865997898333892583 0	84	18
28659932338855544662863647236842535562687381 4	85	18
22670600135515246387782981142431594175576710 0	86	18
17464743014553041973458517274515894671619058 8	87	18
13102050437312828324121197128671632790012416 7	88	18
9570961645420432295496740079433938744124741 5	89	18
6807181396290910938197334220887578656178680 4	90	18
4713305208074283313998854917303594038174241 2	91	18
3176699905850179439333217054808378691033191 2	92	18
2083818582160209891661314177515246950696466 4	93	18
1330185343655516663757918185798505042693924 0	94	18
826157881194276523141549209156087696653887 2	95	18
499157079296032771817473049339959422105497 1	96	18
293328174929599598427136432513709807241617 1	97	18
167620067412772504209763735091076395848315 2	98	18
93123936996363589242686214968423633950516 0	99	18
50287610742081436645670250008043616008610 2	100	18
26388818021610960500492210407133409037608 6	101	18
13453225461341109321578033977243651781667 6	102	18
6661343385495488348190603607454448055765 2	103	18

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	m	n
32025821994672498158260353466378961096145	104	18
14945428678238668259159309119777729350681	105	18
6767756028890467311094199189162309738808	106	18
2972757281807211618729466292539620448008	107	18
1266175796877110947251226112542105596116	108	18
522733532559414355693182081859637286420	109	18
209093525656324199153710774166303032968	110	18
81000224595081356732012903073798279144	111	18
30375091853504388333951072942544280919	112	18
11021052882822194831256421516745051103	113	18
3867036530237591667233255663465279820	114	18
1311429875188877551523242341020113452	115	18
429606358504726621297520350938630186	116	18
135858425082356316059029671548871946	117	18
41448333837884512677750028475241372	118	18
12190686559680351274977927743917740	119	18
3454027881761290209056710923786213	120	18
942007607819071216364409916955949	121	18
247083963262071491733367239835344	122	18
62273194070925981124738628810688	123	18
15066095349977084939785596007200	124	18
3495334122443253311398712894688	125	18
776740916239486463663180003264	126	18
165133895593137877982949200064	127	18
33542822543752913453983072503	128	18
6500547004724273804610026655	129	18
1200100985497012198722139764	130	18
210704753179524394307437812	131	18
35117458863294664770603102	132	18
5544861925785537787648254	133	18
827591332206895317140580	134	18
116475817125418604888212	135	18
15415916972481984887181	136	18
1912924003884628322421	137	18
221788290305464189512	138	18
23933988162460164600	139	18
2393398816246016460	140	18
220667975965944780	141	18
18647997968953080	142	18
1434461382227160	143	18

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
99615373765775	144	18
6183023199255	145	18
338795791740	146	18
16133132940	147	18
654045930	148	18
21947850	149	18
585276	150	18
11628	151	18
153	152	18
1	153	18
379538942835190057920000	29	19
75533424977671986989664000	30	19
5028159431631017611657728000	31	19
177491349808842089578934376000	32	19
4054309707369154654507511224800	33	19
66909908508268404551075972668800	34	19
854948601237581881097396923994400	35	19
8865034610253378235097442712809120	36	19
77157410389720070764400647308948240	37	19
578111261773559871743357732465664960	38	19
3802480877103689708969686485266986560	39	19
22298121793001738280080360731424710140	40	19
118046163028789226179460110808271051990	41	19
570016205638429328250843695718703540140	42	19
2532208270266349663558748805007177634370	43	19
10423886163746258310154547373608246624970	44	19
40008558881844718302503920996612858226850	45	19
143935083054400802637681862715279580200046	46	19
487594485176328729964622498623535717890902	47	19
1561565897751854448562836571866436449847365	48	19
4744464107895546027033308243250323197539261	49	19
13717451720148507042159671943582370221587079	50	19
37844118662364260280040990848865315909030215	51	19
99863883601137814958398980956256988043825496	52	19
252601163541399173204033462725984161121336920	53	19
613636572589201959366961381204301448196613304	54	19
1434106273626529143494254152725229987567417984	55	19
3229346179435734531633219022030468087490938731	56	19
7016375089468623385966283506180566534489762123	57	19
14727133780112845826595733945315992218123199021	58	19

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
29896505117534067613822099508148804728267401365	59	19
58757301409908650358877345394769866268181762134	60	19
111903301818933873908753121229020542424049715742	61	19
206693253286439185452756232210567465859276084202	62	19
370544892637539343916889768630608174967370726602	63	19
645185673426110032722872334373581202629925716117	64	19
1091763451800070638540253447415139598135911386229	65	19
1796461541590424504010157399561891160153476654287	66	19
2875912786318415396351084676194029443480250172583	67	19
4481301298130391640514916479990100826951512475236	68	19
6799651285377572003142116743023950599090407279384	69	19
10050541180294625008751884546964670977980403650728	70	19
14476484501071869431321334530023183296940357035228	71	19
20325619587127330410866365214111696363448528666849	72	19
27826236363660668826690916449311216786389818647325	73	19
37154035263552936232068036952369940670184824647967	74	19
48394726608222116961241655569069490664855328287219	75	19
61506360208383843098918156523620168590151043843900	76	19
76287230465987121333499493895846806453644437846780	77	19
92355893126175798049228362110258295891189116668812	78	19
109149381003778441951314638071958076173629182152268	79	19
125943926700538667998000669989230181379363880467899	80	19
141899481946732475221162574096345904284402443469439	81	19
156125478255592368174203800522025815407134039864937	82	19
167761290381132301431835357657259820444173444292285	83	19
176061574714680337041620113581273151561497215226378	84	19
180474841777123634776789684654294828235235135751482	85	19
180703812827363540942739868048984877194589526016962	86	19
176738419602547960806444215930209944710828296181658	87	19
168856374811233773345043584320430354719673444030997	88	19
157591301378545096398477695081002165042200805664649	89	19
143673458202042843616247727352889487117663901041743	90	19
127952142035240787809193615795422888152846390609683	91	19
111311124562551991389872074529233057619492512377972	92	19
94588658579888841428740504176602128195115408485404	93	19
78511784651850284679853578106095265514679825126960	94	19
63651421427663119615703051310236904449450459103072	95	19
50400805103942102685067089275485097750563939451787	96	19
38976073155696676024968311641862748706621415301609	97	19
29434833513824873237648526946802848582064425339689	98	19

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
21706818366034639263915872473497557608517110587563	99	19
15630273568296942578952365041264677201597773724726	100	19
10988389975346349274738054277330910901428107336850	101	19
7541476591909071208024729566960888146834400068790	102	19
5052284220704802060395136989347950260691096297138	103	19
3303534625192424978377582335649906769702862793749	104	19
2108029307321777312469237689984714267099197295943	105	19
1312575779369940387072711076220402895343756924527	106	19
797373039683339175286959167172088629618619310325	107	19
472523793353567676947118871351908502779355331284	108	19
273112970306024789039656802242669048426629529452	109	19
153937658435684562700103078680359797619270845076	110	19
84596901414094176758808401952764137199411314188	111	19
45319985698509399899525422116538524618809272305	112	19
23662733465234979439913181563794155181649186547	113	19
12038967618126694923454555306099553192293629531	114	19
5967152756865949910648607952382182348326016449	115	19
2880698853411431390373902544377005800929639730	116	19
1354176211976380719646560150104504922623237510	117	19
619708274948745057867843795250893918938218998	118	19
276004693732223842313388655593992366963231154	119	19
119602086609356216251455111713909140090651791	120	19
50410812767951053768933822690106936946948645	121	19
20660173814842353692703639655690104725385825	122	19
8230477053504230571998638709139107163216147	123	19
3185991470829523802577423828899941070434656	124	19
1197932884970855376806960693056454685559968	125	19
437340600040556750053243580692120925209744	126	19
154963210256529769588541729182059506122080	127	19
53268604795799145098647528413695882016717	128	19
17756201878659974699030243234108834690031	129	19
5736619127572512994340748225356891848155	130	19
1795430425882570953662047647167863569441	131	19
544069828315577882051110792990482817950	132	19
159539273384410785244117055711464519146	133	19
45242480585133651635558169552320200938	134	19
12399790987283913756780281457927750014	135	19
3282297616177378773060534985849429467	136	19
838543186895629088326467555489423465	137	19
206597596810644291371285315373958533	138	19

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
49048350327109976586161853479039199	139	19
11211051503974951228756151042945820	140	19
2464841110876165284043464061912884	141	19
520741079770409691399410345304660	142	19
105604834359812607107161781338620	143	19
20534273347812616027074013202005	144	19
3823623313046951644224346911111	145	19
680919220132105906970158694211	146	19
115802588457872659758629667225	147	19
18778798128305617695825376770	148	19
2898740650678154128012409910	149	19
425148628766134274670348822	150	19
59126630490654637013653122	151	19
7779819801401932638891195	152	19
966121413245991812818889	153	19
112923282067713332282997	154	19
12385134162265333264047	155	19
1270270170488752129944	156	19
121363392084912623880	157	19
10753718286004916040	158	19
879234828415496280	159	19
65942612131162221	160	19
4505395859893071	161	19
278110855548955	162	19
15355814110065	163	19
749064102930	164	19
31778477094	165	19
1148619654	166	19
34389810	167	19
818805	168	19
14535	169	19
171	170	19
1	171	19
864876880105205071104000	30	20
573707328841844079442560000	31	20
70512303542044939991816400000	32	20
3853317088640902009001434560000	33	20
125024310219120980694229803600000	34	20
2785166454891160315505661715584000	35	20
46451226232442423436419641229964000	36	20

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
613951123671050110755189076212516000	37	20
6692527434683625164122423509237859200	38	20
61960130961358449529203903433015992000	39	20
498215335609097534764752873244517129400	40	20
3540968488342456236372042878631161072400	41	20
22559753113745298054430725373314588348600	42	20
130328761439452211988797596056895507518000	43	20
689244285573339758791509161928452208498450	44	20
3363590971181574065555928671721610106633730	45	20
15250190795880060913171737403827989326296480	46	20
64611844661359898743041455857743259761667200	47	20
257094113180481799072313841203998879803272280	48	20
964960565471023889468972146870435544224882620	49	20
3429429904430593215715526540894960167299903660	50	20
11579436471629772591143549865523131085493412900	51	20
37255905731969114536765121088967663949115933270	52	20
114522157934371227308004088117507640961505883970	53	20
337124752540197703600279626450683148799250960200	54	20
952380977614556258541287168229055283056330963000	55	20
2586831203611262892814376861393752884872023657260	56	20
6767043194993488821691239043138670669951481980400	57	20
17075254363565115619801937831057157950861332643340	58	20
41617199167977959016391155004185353222215538572500	59	20
98097829590235329685169580160364434612442530850730	60	20
223882371694171220101376705538519149646113208660870	61	20
495224846635465320710709713958395693566369839028440	62	20
1062709968679943332694976337241828384427320723248680	63	20
2214259161428759347220874604610162614282113874153185	64	20
4483141671591444350075731036250738857760239458238890	65	20
8826454515251403113145615046794717694130209481141175	66	20
16909188054129219203630844686585707354603709383274880	67	20
31539187828685250977483846634924597921400012185702555	68	20
57306804996386757137838639294826934228248219220721720	69	20
101486074338946181227677023489634961897477975265427675	70	20
175246612927132979582072381014363358362568822696215920	71	20
295200703416024231847777541425536522092273540519730305	72	20
485261776158976561916003117262450634940867817537307250	73	20
778714149084199083097349609512371078537759094835885535	74	20
1220289318155176661473986042551104636762784320437283056	75	20
1867918631314653575238171187714819451734895833163620935	76	20

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
2793703031979210712041377042224933982360243160649870860	77	20
4083535703793326835049814715140322044864111979512060355	78	20
5834769741772865569641395871078337726898705130079736640	79	20
8151380587927391387853763161312593445389105855076777633	80	20
11136266085168583054453230579339977946183005871676286630	81	20
14880671914926055278769121785279834444438510180145594675	82	20
19451213344541030960724779373939338095473812736044796360	83	20
24875533897633074716990679751219341108858866839040300535	84	20
31128207109791251896326084707585911862984480019152152128	85	20
38118929131640656300730931171185217608363757943305750255	86	20
45685239645648367298200544119152343982457749344224910800	87	20
53591839842441807851875059616553602678595632182570712485	88	20
61537995886313684861836574489139176235586549405957389210	89	20
69173550350772319480654094795894096797742209426600212571	90	20
76122828433476704683595223334343903237358251296844210320	91	20
82014415845327390760086636838185926912576473592390210595	92	20
86513643460388034858338109127520115348353944424735509620	93	20
89353879594395680444649408070003477750471129812827888655	94	20
90362585092558731878123925066324201114685790939413507320	95	20
89478604340042466877641699426809202814992531784593055185	96	20
86758290614561075466273477381849276003710317832687466750	97	20
82369614124751135270431853710727018862051040632847032795	98	20
76575102001099723558451509777446505428343298148731085960	99	20
69706005347231820087483902685610773850119547558022461521	100	20
62131207262164572545001656717691762604476831287756231640	101	20
54224897509659322407258115480877166522695671957586299845	102	20
46336890895455583748637083179471885637319998420626807680	103	20
38768734553917008537885820395739923855233190941099817465	104	20
31757615659706787999792826513823800103459028494599607298	105	20
25468783742907383418548109355967944505806319493912864375	106	20
19995980419618129170778028615280858221321511412945296720	107	20
15368416084570571053651640153885645588185327498946338345	108	20
11562260851401910981527197280602530318258573700958950380	109	20
8514449896040427192512632777678158383500157409568164393	110	20
6136787967271035527242016722221803509690792202849426640	111	20
4328769002398350394352296232262637259983582178169841905	112	20
2988079045581333673518238317726428932636721360322719670	113	20
2018307061273997779533245672457116242752933437766729915	114	20
1333860622308193740432771964820563571727773258811953136	115	20
862421745053789026119629131477423747403537741164280625	116	20

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	m	n
545469308378061571493376026611527699933621218528635960	117	20
337454291882244670778936328124461564594034987703510125	118	20
204175432768394545583070666928754155289862384379611000	119	20
120804453819917844521204807623948172580717549691235625	120	20
6988717187804056577776551394117238572484354031651050	121	20
39526482840169786266885102222520882138259360038154415	122	20
21852095827624249471274046256795822319068386232075800	123	20
11807204446256049568087425449703719199942755682459225	124	20
6234213557083853432000510596135100735798190319528236	125	20
3216066347066724703962703582746849079737574379739665	126	20
1620696310597007794261419353565947967640615230122240	127	20
797687009495553188355791534765785230372943341684075	128	20
383384651284551310729793049262950902395522736379690	129	20
179895944137836553848621788555182032189368685881313	130	20
82395112559594605589857302308320262235641889070200	131	20
36828126313345928995688664549453473202250663362675	132	20
16060388418805268456838979839869609161584930568140	133	20
6831658539240488115039079768429603388306599852755	134	20
2833873406145830958506609633537211470778092705848	135	20
1146051813335544573332902740876629065650265011235	136	20
451728471171658605551887729219532633580141327390	137	20
173489925311751842161206410489287804422409282965	138	20
64902707244919994354963787280807435831807089840	139	20
23643129416027290005752011639689172729764506811	140	20
8384088530588289214193527111343093564740566960	141	20
2893100991904910278564771290717255269285267655	142	20
971110827015707886833181373270513672884163920	143	20
316959784812806478250222439933690428323861875	144	20
100552759528724030189069499407975896746600770	145	20
30992288937893468686250272509494649191175825	146	20
9276603499735886222022123746034558867303000	147	20
2695229396695598425456303986593657354831355	148	20
759729091951703203541259195769455666174300	149	20
207659285178350625587549873109233193052587	150	20
55009082173680955938289344844581255146280	151	20
14114172400934803046208636109992065827035	152	20
3505480727191115405440465578131148356470	153	20
842225889022785014922233035551393774805	154	20
195613754872726317809379984880587093936	155	20
43887701414112765527528342532541207395	156	20

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	m	n
9504342981442034315639366237253101880	157	20
1985084293596881038710716406995862615	158	20
399513820095434256267013465866053760	159	20
77405802643534487574332714136250311	160	20
14423441486376769244789625775814130	161	20
2581974093240601400072982866989685	162	20
443529292090433577334434515627240	163	20
73020066380743710559420890884655	164	20
11506192278177894811576271389362	165	20
1732860282858122066725537011525	166	20
249033813105359120670657680400	167	20
34093914889424176327534433895	168	20
4438261109865869502878723250	169	20
548255784159901538587510245	170	20
64123483527473864440675600	171	20
7083408064081415262894775	172	20
737001995106736848219930	173	20
72005942050658197814925	174	20
6583400416060178085936	175	20
561085262732401541415	176	20
44379625300867918530	177	20
3241208589389230005	178	20
217287726662965140	179	20
13278694407181203	180	20
733629525258630	181	20
36278383117185	182	20
1585940245560	183	20
60334683255	184	20
1956800538	185	20
52602165	186	20
1125180	187	20
17955	188	20
190	189	20
1	190	20
27312537864129554029593600000	32	21
739551256437352201985862720000	33	21
6664379496124665290616793152000	34	21
31658265353129956235447195963520000	35	21
967485725582708581866235214275680000	36	21
21246779956547164334305276485959760000	37	21

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
35952586715004293183063481945005328000	38	21
491613693755146489494110483619230952000	39	21
5621739971895373926789759661496364960000	40	21
55163571393864004653660958288035737510400	41	21
473838030095603001089748207787482123708000	42	21
3619767642242245408023774236918481031696800	43	21
24910168579855764215465471730060794964917400	44	21
156065793597883806351053498991176138157726180	45	21
898064785913901216004507911137623743550206480	46	21
4782078209963314978324109470627826312732416740	47	21
23713791215820781572130702813521699229991284860	48	21
110114104762257095387087944614418477613340138300	49	21
481071836404601350719622582164851656845568616400	50	21
1985674341943380837919989309837595283336126644260	51	21
7771849868658077837199218629922418664982097587370	52	21
28937512837173335255009395701756133956985195406730	53	21
102792847611500492776545313205973691212762978082690	54	21
349251826385598758153364678171034305580627085528310	55	21
1137577313500300553848332266556141048778152027769440	56	21
3559430006451358303169770031458233188813507185450240	57	21
10718635032187948140589819430327035733862216324055500	58	21
31115900690204819225576060924437459016164691470589280	59	21
87209721773226435163879517482447266114832835431399350	60	21
236309985684770691422111799369977351532994868594138070	61	21
619833327399740329251228958410604814967770388020786430	62	21
1575565000474614361363711493388382093824341172394268930	63	21
3885223974745124506349278069707854652604474355371052160	64	21
9303047412742880141091611809510929110240356606206833460	65	21
21649114511164318773057429051470411374653023202903690740	66	21
49001078439410612744797942025790984893006806167310468920	67	21
107953860349905822088085919466391577379401895565679227475	68	21
231647925598938267964559200061113074739986110386811998980	69	21
484444008051051301756934776535826714319873314740057261205	70	21
987937792326575865300041714486266760678953203274949948070	71	21
1965680313905932617413733396055846113797519217634677872875	72	21
3817702752202351873118119424778240650223665193714222743730	73	21
7240851493984278639920650554021390641907568646167974820905	74	21
13416922800053019089099885143993653019478736485731443304180	75	21
24297169825358871742060723882625941201872900537223439685795	76	21
43017887336774364358894056226078607689969913894891186071260	77	21

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	m	n
74485650337228890715789115558270538739768083362362406529165	78	21
126169716133116906420984819969634582052515918701656721474630	79	21
209129628844597699665897132540399529967835201884392512830334	80	21
339284299501510094379360771162959551299034518045216272289680	81	21
538891208193425047811502347549462116575905824652139435871140	82	21
838148815556899971348769177151006893541600751212011506618660	83	21
1276762699087847963277824460705765964970176730377561494855275	84	21
1905231833869620441190958896362871778086121821739119784807808	85	21
2785527101731222915117765848374693520122564316526337382202005	86	21
3990766930746586521837746833377441154514140411290723425292450	87	21
5603467253459840831292899163450366438072009905613940661419665	88	21
7711978858092029398695419620052931925124633735775446998382730	89	21
10404845580965441830433823255152593418921790063905542628617639	90	21
13763032838778253451460176720795024182006014528319911772405200	91	21
17850282573434418556632515386029927589539126118449021537481095	92	21
22702221240413513078565084697414683182614408355997625231466380	93	21
28315232632285843328843594296004942965608977419091830184071305	94	21
34636438669272574127247726132487762277237264070688997028284070	95	21
41556331110261745477729472437342749618457923520072427165205225	96	21
48905593408908385070135561374915903112531581539297767170405950	97	21
56457395926848852522966868128706024447110612716574239679534375	98	21
63935930385134561795854501651366492484648365679299354151631380	99	21
71031212077411478994812794863992559870293645329964420951843811	100	21
77419313483467784634946664057057170628776148968713792074060700	101	21
82786333238369339380678675303862789288730516281135617107538145	102	21
86853700765542871912311581934592089273378649365987332016724830	103	21
89402008766554702921322723785795626617327975448428548535548095	104	21
90290550869642313340773107837229007003412410939352169632159166	105	21
89470151832127795239807325038104510082699880221767549749355265	106	21
86987667220651535392783679388104333120273886193687118066245920	107	21
82981580655626819332468521278159000440851115174559306389800255	108	21
77669270608739995625964379502103281333295812400405712826953200	109	21
71327568250184155937821149217998541218875744176434262524983109	110	21
64269014639298185052400507615123694445905436619346011552514250	111	21
56816632815596815283650235246529469216363253186943279115181475	112	21
49280013612637430631808318130839016724877218977997617050369290	113	21
41935105902834029365446252818010717609843530312092283052596625	114	21
35009400552940790187942148140893512095040934005114763015461404	115	21
28673341887619123210071331555531459781538371864074794401266605	116	21
23037940971288912296697838071501144954136267763195490882994500	117	21

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
181578331264939164547799809094242932865325888920552597166908095	118	21
140385091356033906702278782938498380511140876896576696605405 30	119	21
1064619589274038494682981881618867972212956092938676218775460 5	120	21
791885840001920593684176629100400674653790879853723034914485 0	121	21
577699230756449271452898571252361180663905986025764023734171 5	122	21
413320324602094020338930181485498589909401853856223595684720 0	123	21
289994920733048601786705813824266361385661080631808956031812 5	124	21
199518776500873222324228507927227537937951260052340710848021 6	125	21
134597245591120908579242351421034609841878070027631549125089 5	126	21
89025617551652932054962508915085149145677038058605925650363 0	127	21
57727898679173675100390885721078842786878102907260346808708 5	128	21
36695432476670759358971013230861473271782925024701292813343 0	129	21
22864166695807017193788800505092803653623820017290968792281 1	130	21
13962893623313025508521748965941034312910549299351128408478 0	131	21
8356601134172502463215619265168901879385898487229024027609 5	132	21
4900873537430748428102160945236022065279834996956901116296 0	133	21
2816178015999407081551818248438785073999534685001775928076 5	134	21
1585405879374635065324692905890201796926912525123013427547 4	135	21
874305561020682104983912234170211910487691821659917465723 5	136	21
472252996937983759520517563246724172632757695217135341123 0	137	21
249815138618493530569454808929219769236637345985062157692 5	138	21
129400706557379830828030717194658734300898494492795836616 0	139	21
65624667213484900105589716326043750196702880495738984783 9	140	21
32579630857791465773388610202439480889905789949064375778 0	141	21
15830950671259557365933472898476543460957780944279618112 5	142	21
7528005702451790217546206110421988268621223897641114239 0	143	21
3502614182491624457320029161494975490156577905406925142 5	144	21
1594293495328955672176103679350036014463076196479683547 0	145	21
709788247745890999219690201046942961678234347041664515 5	146	21
309023470286681633159661500596220133977483454356767210 0	147	21
131543779393451771274281826451656032001754785612117697 5	148	21
54736339228057912979884882388132513450541829193890560 0	149	21
22259445063573993590772016322702850657225352466424302 1	150	21
8844812736235610229170454020908469033297737134410301 0	151	21
3433183926936021988643016193566314631465423906090169 5	152	21
1301468426494759879410887385516060452114335554860315 0	153	21
481712342242055463030501457381103448320260377745762 5	154	21
174038008173697368480820471302296931462430950760116 0	155	21
61359554325190499034059317645694643755598735928929 5	156	21
21104560124899065495698927067989374431019680886842 0	157	21

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	m	n
70793777230726872639829010453227931116527318912 05	158	21
2315268204246411605844123750766893024101732824407 0	159	21
737991740530169894942531424271109666307429359084 7	160	21
229189981619660684122629012114482304462882490485 0	161	21
69322895692794331962559278054005301840647543342 5	162	21
20414104256503413550851184377583645114118858934 0	163	21
5850383537556399961251014158424391895888959859 5	164	21
1631016016643261431243641533374411694119000112 8	165	21
442142896097181878203357691489179942985453008 5	166	21
116492739094613369806552338973933105843879077 0	167	21
29816593935422692706093001568826768890525702 5	168	21
7410041096450878675783391129430783199794945 0	169	21
1787127558566321097369657209464049363331289 5	170	21
418041534168992435038141127581713096516140 0	171	21
94788487398968491614991631689314377675017 5	172	21
20820592607888671990818025532061247579860 0	173	21
4427367393634145968499777367705308732742 5	174	21
910772720976457873729761245496055712428 2	175	21
181119575194212038232218751002306071264 5	176	21
34791330828269640941080482266978040759 0	177	21
6450078187263738387850809320276901895 5	178	21
1153086603309741106846831521545984934 0	179	21
198587137236679405815114933340646752 3	180	21
32914995122101687584605176760603859 0	181	21
5244697024950277475351664942160150 5	182	21
802467304363977402608906288283520 0	183	21
117753354444714104018302827125737 5	184	21
16549120084121983594741735421037 0	185	21
2224344097328223662681538866332 5	186	21
285477317304157049934121004970 0	187	21
34925416478700064694353854139 5	188	21
4065392394346039278590336191 0	189	21
449332843585614867676200540 9	190	21
47050559537760719129073108 0	191	21
4656044954257571163821633 5	192	21
434242534593970367610345 0	193	21
38052180866481939429772 5	194	21
3122230224942107850648 0	195	21
238946190684344988570 0	196	21
16980947561324009340 0	197	21

Table D.2: Numbers of labeled 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
11149106984707682900	198	21
672307958876845200	199	21
36976937738226486	200	21
1839648643692860	201	21
81964543530870	202	21
3230129794320	203	21
110837787060	204	21
3244032792	205	21
78738660	206	21
1521520	207	21
21945	208	21
210	209	21
1	210	21

APPENDIX E

Numbers of labeled 3–edge–connected graphs

Table E.1: Numbers of labeled 3–edge–connected graphs by number of nodes n.

Number	n
1	4
26	5
1858	6
236926	7
53456032	8
21493860332	9
15580415345706	10
20666613177952152	11
50987341499153087708	12
237747594646361178843768	13
2125708461695379843278043006	14
36886187376770375742178892122216	15
1253964425604438617616987956626442896	16
84096628310785072553727510276872041548848	17
11180321458597627733541026925467194861999631890	18
2956196065046651841919049230432139901966019263184752	19
1557906168377054498924606426301069999862196714568060476388	20
1638574200090248844910881402835557024302703252287687573144674184	21

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n.

Number	**m**	**n**
1	6	4
15	8	5
10	9	5
1	10	5
70	9	6
537	10	6
735	11	6
395	12	6
105	13	6
15	14	6
1	15	6
5670	11	7
32445	12	7
63945	13	7
66090	14	7
42602	15	7
18732	16	7
5880	17	7
1330	18	7
210	19	7
21	20	7
1	21	7
16800	12	8
510720	13	8
2823180	14	8
7210196	15	8
11163803	16	8
11924808	17	8
9459226	18	8
5831560	19	8
2872737	20	8
1147576	21	8
373156	22	8
98112	23	8
20475	24	8
3276	25	8
378	26	8
28	27	8
1	28	8

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	m	n
3515400	14	9
58476600	15	9
332970183	16	9
1041102720	17	9
2158528134	18	9
3277877652	19	9
3872954925	20	9
3704886216	21	9
2948201280	22	9
1987998768	23	9
1149824529	24	9
574550928	25	9
248787882	26	9
93290260	27	9
30163059	28	9
8340552	29	9
1947540	30	9
376992	31	9
58905	32	9
7140	33	9
630	34	9
36	35	9
1	36	9
9238320	15	10
650745900	16	10
8498328300	17	10
51054633700	18	10
188531998200	19	10
491058742545	20	10
975987751425	21	10
1556495342370	22	10
2061542013030	23	10
2324011160325	24	10
2270132563041	25	10
1946802628890	26	10
1479734629170	27	10
1003586008995	28	10
610052393295	29	10
333216921144	30	10
163688109840	31	10

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	m	n
72270520875	32	10
28618931775	33	10
10128741210	34	10
3187559826	35	10
885933085	36	10
215540145	37	10
45379260	38	10
8145060	39	10
1221759	40	10
148995	41	10
14190	42	10
990	43	10
45	44	10
1	45	10
3830765400	17	11
131602363200	18	11
1541445485580	19	11
9863378808624	20	11
41963776682520	21	11
131775203950950	22	11
325500667271010	23	11
659849023698645	24	11
1131149711659539	25	11
1676211051229605	26	11
2183024889160555	27	11
2530481213598745	28	11
2636274965716755	29	11
2487008246868783	30	11
2136729426202305	31	11
1679082758496495	32	11
1210581893119785	33	11
802469996162835	34	11
489678723185949	35	11
275195549819355	36	11
142401250148685	37	11
67785468007455	38	11
29637473735265	39	11
11876328289221	40	11
4349381135715	41	11
1450561674705	42	11

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	m	n
438654417135	43	11
119646432405	44	11
29248154419	45	11
6358379665	46	11
1217565855	47	11
202927725	48	11
28989675	49	11
3478761	50	11
341055	51	11
26235	52	11
1485	53	11
55	54	11
1	55	11
9520156800	18	12
1249655299200	19	12
30634707494160	20	12
342738302056920	21	12
2342929207169160	22	12
11277941303787840	23	12
41489250498405270	24	12
122939892209494206	25	12
304253165755091916	26	12
645606617797166080	27	12
1197990357532982310	28	12
1973687116817158050	29	12
2921454325923337600	30	12
3921927942251535480	31	12
4811030948388119895	32	12
5425288403307762120	33	12
5651186100389612145	34	12
5458180129424647812	35	12
4902898798846312132	36	12
4105460934458569290	37	12
3210163350415991310	38	12
2346820709149841320	39	12
1605303827846342316	40	12
1027832616312182406	41	12
616000143780506430	42	12
345455329430793780	43	12
181166212358880660	44	12

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
88761640548523866	45	12
40578264311362866	46	12
17282467989749160	47	12
6844596385501050	48	12
2515112321312250	49	12
855282836083104	50	12
268347662117964	51	12
77411290969980	52	12
20448655423020	53	12
4922861969740	54	12
1074081808596	55	12
210980512776	56	12
37014130780	57	12
5743572120	58	12
778789440	59	12
90858768	60	12
8936928	61	12
720720	62	12
45760	63	12
2145	64	12
66	65	12
1	66	12
6782977000800	20	13
407534630302800	21	13
8313787610528400	22	13
91883864849016240	23	13
670971332939536915	24	13
3603842739948953502	25	13
15206946981344611911	26	13
52683737497059310356	27	13
154619345085382458567	28	13
393440959278732788280	29	13
883544129579976591727	30	13
1775573000313584992806	31	13
3228568276027251968811	32	13
5359365266481958885972	33	13
8180887205138356062315	34	13
11551934965335998709762	35	13
15163634447905023640846	36	13
18578064434824424044086	37	13

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	m	n
21315350526005472315522	38	13
22965073534788046770570	39	13
23286111923874608870490	40	13
22261989031773211427850	41	13
20095184870626776982500	42	13
17146052495847660978450	43	13
13840128101498986682460	44	13
10574747870934057797194	45	13
7650778669558398135762	46	13
5242149680861224782222	47	13
3401431927211971082144	48	13
2089617560796544929150	49	13
1214944546882032420168	50	13
668180028450769803254	51	13
347354651313675041454	52	13
170538858496096748658	53	13
78995813324888146830	54	13
34482976595961931674	55	13
14165784135232385292	56	13
5468236651960764682	57	13
1980037677842167944	58	13
671229894401828190	59	13
212561426247966796	60	13
62723851677249048	61	13
17198583234300468	62	13
4367906528841856	63	13
1023729290804220	64	13
220495634692872	65	13
43430964297396	66	13
7778680447356	67	13
1258315963047	68	13
182364632450	69	13
23446881315	70	13
2641902120	71	13
256851595	72	13
21111090	73	13
1426425	74	13
76076	75	13
3003	76	13
78	77	13

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	m	n
1	78	13
16305064776000	21	14
3534109849670400	22	14
143223499197979200	23	14
2629696759381692000	24	14
29267193024686897280	25	14
227952167712815391384	26	14
1351196785231551418160	27	14
6434771506637763603314	28	14
25561122402024879061392	29	14
87040661649952185890715	30	14
259372275377752837240941	31	14
687302138766295053626625	32	14
1640290648433616971236809	33	14
3562060997757089657618880	34	14
7097954875173535405561620	35	14
13068481014202790248531823	36	14
22360471755267206882155835	37	14
35727073140416229052326846	38	14
53522689182061344885259466	39	14
75437944692520106144025578	40	14
100325120746700950396620324	41	14
126199848329359129007868185	42	14
150463921006245243136927187	43	14
170326642722950917365847503	44	14
183331934925033293392193655	45	14
187853765010691444105894908	46	14
183423221506896462618508800	47	14
170798574419124502483088456	48	14
151767333554987167042969658	49	14
128747455258762398290985753	50	14
104305571099859457883361807	51	14
80718415233870181194710745	52	14
59672182560813001436053797	53	14
42140261130916443126587956	54	14
28424796020509766036831308	55	14
18309673238834699797838958	56	14
11259431833289750173083930	57	14
6607498834062887820729306	58	14
3698603909832700116286038	59	14

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	m	n
1973683528368647521045620	60	14
1003409410002844032400212	61	14
485650728821582950746120	62	14
223594038651772082844264	63	14
97834164417179852023362	64	14
40641985058857641274662	65	14
16011273921836908092014	66	14
5974539478859586495770	67	14
2108699942445363265908	68	14
702907541579576040344	69	14
220915133603719104836	70	14
65341308740414807100	71	14
18150393963503626825	72	14
4724078999322017335	73	14
1149100725977359155	74	14
260462872358886197	75	14
54834292254323582	76	14
10682005207130790	77	14
1917282997610974	78	14
315502265886262	79	14
47325339894651	80	14
6426898010533	81	14
783768050065	82	14
84986896995	83	14
8093990190	84	14
666563898	85	14
46504458	86	14
2672670	87	14
121485	88	14
4095	89	14
91	90	14
1	91	14
18240218516520000	23	15
1735588786733802000	24	15
55808397380298162600	25	15
964592773788834590400	26	15
10934602029592224533550	27	15
90635418339243481977975	28	15
587622015818830143025875	29	15
3118118232510545384860740	30	15

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	m	n
13986781584736406031535950	31	15
54327199158491161301974575	32	15
186130060411498651657734525	33	15
570739069918062347196319425	34	15
1584743592392506814025025125	35	15
4022757437376330451377654530	36	15
9409244118571234163811781560	37	15
20413052002278417075625844085	38	15
41303506705943972358934242625	39	15
78311904017220896176705712700	40	15
139689822450653220323308941210	41	15
235223840623471796310513009425	42	15
375016647376388125361815687125	43	15
567501938571512923568388284850	44	15
816911036748657262206915198260	45	15
1120689404651814370384442041065	46	15
1467568680319027975397262493485	47	15
1837027339008746071600025284945	48	15
2200656327191259943322818672275	49	15
2525503993478994093179963592243	50	15
2778920400459060694680055897685	51	15
2933931631124331869216550684165	52	15
2973927307011687248430891371535	53	15
2895544989896354056978423321615	54	15
2709069939192828730278125861313	55	15
2436304695844538195353762628595	56	15
2106496448125235613818692053005	57	15
1751344435411720400235926063515	58	15
1400226261957115900869920774325	59	15
1076582461618251293071974138895	60	15
795975194318647898270379454245	61	15
565863050786538427105046748885	62	15
386737245151958945087659084555	63	15
254053160726788172442340898250	64	15
160371311195109370547799796650	65	15
97250229068245233468726980840	66	15
56632261376250202027710867360	67	15
31657230323004515525171019690	68	15
16979417244752186922882962050	69	15
8733669778171822597952342400	70	15

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
4305816660278787186644027280	71	15
2033462223609107236883161770	72	15
919285917438467873345401350	73	15
397543508773657173163536300	74	15
164321947673485149881136804	75	15
64864944876671077508415045	76	15
24429899009015235929436165	77	15
8769762033773631737342160	78	15
2997271811362022011045620	79	15
974115525833226351048474	80	15
300653327789211965874670	81	15
87996158961807349354920	82	15
24384487124224536298080	83	15
6386414547684871560575	84	15
1577820215758858386315	85	15
366934950084448145400	86	15
80135220507862920400	87	15
16391295227952832005	88	15
3130921568954395125	89	15
556608279411651740	90	15
91748617507304400	91	15
13961746143144975	92	15
1951641934004035	93	15
249145778809200	94	15
28848458598960	95	15
3005047770725	96	15
278818865325	97	15
22760723700	98	15
1609344100	99	15
96560646	100	15
4780230	101	15
187460	102	15
5460	103	15
105	104	15
1	105	15
42856575521760000	24	16
14186400089564102400	25	16
876433875660815472000	26	16
24367418234551718726400	27	16
407557822991717428430400	28	16

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
4738254071320747142605440	29	16
41692812406009951312771008	30	16
293465029382102415441895200	31	16
1717229030264628017360091960	32	16
8592065111790500314602467840	33	16
37551718166957659564519259640	34	16
145764602250749233730306605992	35	16
509247120707596195970694774840	36	16
1618599863547486116890234682040	37	16
4722153874023702074259251343120	38	16
12739262888683576718728505236000	39	16
31978463715144803114568304564248	40	16
75089049419227224401094250450680	41	16
165677762058560671606750182699360	42	16
344832213573457746248785775525280	43	16
679305307183811412721326383954280	44	16
1270281042530799268599646355475136	45	16
2260539240115672877224041214556520	46	16
3836728001366003897403224809521960	47	16
6222788657992912196315857908400680	48	16
9660903691567019228582280907280760	49	16
14378098276552364690613717813208368	50	16
20539912350404759391535953023028320	51	16
28196899484250449878606296880831560	52	16
37234015831039827762098286283823760	53	16
47335745406977792503446464294697320	54	16
57979627671698101393671811205715576	55	16
68466907339169743574831985436819200	56	16
77991633344457438900031846544535320	57	16
85740278860418235005461011606623520	58	16
91005354660556937032709064883739280	59	16
93291232105048560075447992238027736	60	16
92390399064452596182346119479598240	61	16
88414039773822502170704399562093840	62	16
81770789089649770509284117367204160	63	16
73099022795996623843341317340154935	64	16
63167900524844418526934577222573336	65	16
52767967523345192348156136417150300	66	16
42612256425624380660331858174202440	67	16
33264017808960986519898793854292470	68	16

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	m	n
25099324017182384302606118699144200	69	16
18304322505758282353515168953297436	70	16
12900107024688046782010210404657240	71	16
8784390338439810533270258156906085	72	16
5778650548909561298753582671754160	73	16
3671494842257897115943255828923960	74	16
2252442328752895121655459802158640	75	16
1333947135828580963262059940189100	76	16
762368265154172089967042309269800	77	16
420325951492803210923197184035520	78	16
223482362263332449236685825037960	79	16
114541380038564039513040033156687	80	16
56566009054541097069368322269000	81	16
26904144068250457846554167991540	82	16
12317816256459371182380980943000	83	16
5425782949514800651140412771370	84	16
2298001270810349294086663235664	85	16
935239267855731988864663925340	86	16
365497409038994290677321946880	87	16
137061919975917568542218076405	88	16
49280780371783283747170022640	89	16
16974510463094455553087064896	90	16
5595996394271666770818823680	91	16
1763956128404406691273517160	92	16
531083696254281831287557560	93	16
152545338326121444942963480	94	16
41749253698986245634267864	95	16
10872201928109541453226975	96	16
2690029399133740248148200	97	16
631333436838024097027500	98	16
140296319960270438861000	99	16
29462227252550832347790	100	16
5834104411367434183800	101	16
1086744939716954163100	102	16
189916591427320965480	103	16
31044058215085159965	104	16
4730523156623397008	105	16
669413654240284920	106	16
87586833265105440	107	16
10542859559688820	108	16

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
1160681786387760	109	16
116068178638776	110	16
10456592670160	111	16
840261910995	112	16
59487568920	113	16
3652745460	114	16
190578024	115	16
8214570	116	16
280840	117	16
7140	118	16
120	119	16
1	120	16
70712366893616448000	26	17
9968670036123454656000	27	17
472517935807055155617600	28	17
11953639405017549266227200	29	17
196942039465543283221630320	30	17
2357968859047219685610186480	31	17
21969384905024210876341368585	32	17
166831751318377954482670534720	33	17
1067350391542554447418689928420	34	17
5897135148989618217082377202536	35	17
28678596534871906212265905607586	36	17
124623496355431262240855771286800	37	17
489828036856357045043214856521540	38	17
1758791873094014039340101024258640	39	17
5817095449792232764267930303156947	40	17
17845901650374677718593713745837232	41	17
51082555671422990666152840785464400	42	17
137119263894440068681249984904385720	43	17
346658073419129620961253411016850280	44	17
828540320803992583308940921656745744	45	17
1878262031274280206833197117617701464	46	17
4050145655295183879396214461370988280	47	17
8328107048171624781385596376556068975	48	17
16365965273269178624075141055878656360	49	17
30796558843193398721557015699498283508	50	17
55587294882642452305189456812748762768	51	17
96388151161089328970068975695113804350	52	17
160780533600450815709723067933635085920	53	17

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	m	n
258300847693654835747620927367500644100	54	17
400095005821788002072838076190859354352	55	17
598075267379180213017633035678702208227	56	17
863511668148902484669211642533607391240	57	17
1205100153334227288265830026389312438880	58	17
1626696342590288396675829367064051546760	59	17
2125057995222983255059984847703582970512	60	17
2688057260127052317463012652569067900712	61	17
3293858908808532598527972614138574194760	62	17
3911478102710961283836862707971659708840	63	17
4502923274503203225312567800955725699605	64	17
5026825601031849020548370532897146237040	65	17
5443120691758415997117403165560099790660	66	17
5718065707881231962953669294777536377920	67	17
5828728092667047707475988503302026932510	68	17
5766122738917784988514641446580211804920	69	17
5536406175238489332864940507950280990332	70	17
5159909256050145421613071369392093669192	71	17
4668213374779346536151641542926038734765	72	17
4099844723292956419542287215582700754480	73	17
3495388809861812586851203450250256108000	74	17
2892867385062707960168588735183045743200	75	17
2324078697769784065225322145081606852480	76	17
1812332402651362021481081255327515988640	77	17
1371691646171106802749184428316463709880	78	17
1007547037829627186258283403765090821280	79	17
718149445323398595925878419152837064997	80	17
496646591772587184199021150545803473312	81	17
333194455937926438799138235610588862620	82	17
216816529978420914783030303187509507000	83	17
136820138530303245193889333114935616150	84	17
83710771685097713021201416065322132952	85	17
49646525058444958400140437151959258372	86	17
28534266826930958085987161033880448760	87	17
15889144382030929563380618166605658155	88	17
8569726301335432921943401596079626360	89	17
4475417497562182436911964688582081688	90	17
2262341971198305666366713550733012288	91	17
1106595610474042900722377016965883240	92	17
523555824225802141396178790510128360	93	17

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
2395006410998934574238938933348009520	94	17
1058850242609128982518907233221940032	95	17
452218871088383509375216772435155517	96	17
186482457840051680694862885141821160	97	17
74212528360704268287420387431751000	98	17
28485648311188399408747990805602000	99	17
10539697557863693334888316949643940	100	17
3756725667246759864198939529647840	101	17
1289072925708486243169123094652200	102	17
425519299888355701967372422679600	103	17
135020563101307290116490577102950	104	17
41149126948725600705990086469280	105	17
12034179147647850319688059101280	106	17
3374069002762790054514180703200	107	17
906000022510725232527432982200	108	17
232733952490686662034874940400	109	17
57125606747848540504143745680	110	17
13380772778874799489683462880	111	17
2986779641101434869950275250	112	17
634360278049411581398593200	113	17
127984968404586572269871400	114	17
24484080914268551232324720	115	17
4432462924281286427261820	116	17
757685969971529509817200	117	17
122000283300961777713400	118	17
18453824364871076302800	119	17
2614291785024070875430	120	17
345691475705679117280	121	17
42503050291682139600	122	17
4837745561654880000	123	17
507183002431560000	124	17
48689568233429760	125	17
4250676591807360	126	17
334698944236800	127	17
23533519516650	128	17
1459443070800	129	17
78585396120	130	17
3599331120	131	17
136338300	132	17
4100400	133	17

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	m	n
9180	134	17
136	135	17
1	136	17
163329351308323200000	27	18
781362014683915304640000	28	18
6953703225516891908064000	29	18
276766168169367910716883200	30	18
6581183464258601664606940800	31	18
108081206871398955999139935600	32	18
1335916687554119457761485034160	33	18
13146402203939110099523568713280	34	18
107126029114033281918285937327488	35	18
743967140511998118109254947843904	36	18
4500854858132989187746295765455344	37	18
24130055062023111477980141405491368	38	18
116223694303213527530420212091401680	39	18
508571327089372202604850371954630705	40	18
2040509291667334354490081229950953905	41	18
7565041129626191538560385398157727140	42	18
26086164805305605333505362195168216820	43	18
84131843968513240447450846024421585010	44	18
255004789201186244472043432128701518410	45	18
729426079522134485982024208295396967720	46	18
1976212794454463431857481833270430745640	47	18
5087262346728545840786995381984038887055	48	18
12477997067666525122002558788831435499015	49	18
29233859429592329936338102118397204984012	50	18
65562558154695000560158031075415214668516	51	18
141025012775571915880805262995929275074496	52	18
291445138035940904849058887404200696888312	53	18
579571652833961019361808288734697755680420	54	18
1110568798900127190234473536959985188327300	55	18
2053088145459883485947111450593363285573125	56	18
3665842932101993936351939692969922004446205	57	18
6328128119705565148336284543027420071281440	58	18
10570580719391976138944043293628453515609880	59	18
17099847665069330492855896369466762014760262	60	18
26808204836911949869509077504580878262173406	61	18
40757336123273184617986380420572185529424816	62	18
60125427131546048384056998558316147053377712	63	18

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	m	n
86109422656278870339320611425751024624336305	64	18
119780343372216846763648861422107634515615913	65	18
161898816138788691158312794171279347169974324	66	18
212709279051748230579124582427076016899744804	67	18
271742445878319431844729240899774389460379138	68	18
337663569934581828821037623469674123689645170	69	18
408205775967021841859172293957326977823474380	70	18
480221019566397760622650152832822197608109180	71	18
549865632648151543241741445345844495388743275	72	18
612914792060352857168256891616252603151021475	73	18
665174755463299707024395846607612629272943520	74	18
702938989550658971709839616705111567973977504	75	18
723420126894028358242062935739140646364693812	76	18
725088275897801149289289299304449579405409252	77	18
707858958073182352635823413086809530645847344	78	18
673098765529416568236217030577915320617971520	79	18
623448528884805777021515239262610365811463425	80	18
562495380516594389963893020624516363506574865	81	18
494349669794963023282304690326716438671284440	82	18
423195200404678453676667761549662348582275840	83	18
352879801823041957458533078438552857929845670	84	18
286599323388555483315284956947454203334577190	85	18
226706001355152475801012004942211123746385540	86	18
174647430145530423414583386608092878390233940	87	18
131020504373128284318285204688081786587978375	88	18
95709616454204323253433561115065379354519095	89	18
68071813962909109460143061556888646681352948	90	18
47133052080742833159301069279631490934805484	91	18
31766999058501794397823454412303640253923224	92	18
20838185821602098917593996881705349678288648	93	18
13301853436555166637779811312237379369034040	94	18
8261578811942765231453814571615074327808392	95	18
4991570792960327718181543378742808127422291	96	18
2933281749295995984272487328215556651075971	97	18
1676200674127725042097808242683563070169152	98	18
931239369963635892426886037781508763950360	99	18
502876107420814366456705549624768789020942	100	18
263888180216109605004922457176467341689326	101	18
134532254613411093215780376554221231490676	102	18
66613433854954883481906039487287392135652	103	18

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	m	n
3202582199467249815826035374497021918414 5	104	18
14945428678238668259159309139476101134681	105	18
6767756028890467311094199190344212045848	106	18
2972757281807211618729466292598130463208	107	18
1266175796877110947251226112544400106516	108	18
522733532559414355693182081859704116820	109	18
209093525656324199153710774166304318168	110	18
81000224595081356732012903073798291384	111	18
30375091853504388333951072942544280919	112	18
11021052882822194831256421516745051103	113	18
3867036530237591667233255663465279820	114	18
1311429875188877551523242341020113452	115	18
429606358504726621297520350938630186	116	18
135858425082356316059029671548871946	117	18
41448333837884512677750028475241372	118	18
12190686559680351274977927743917740	119	18
3454027881761290209056710923786213	120	18
942007607819071216364409916955949	121	18
247083963262071491733367239835344	122	18
62273194070925981124738628810688	123	18
15066095349977084939785596007200	124	18
3495334122443253311398712894688	125	18
776740916239486463663180003264	126	18
165133895593137877982949200064	127	18
33542822543752913453983072503	128	18
6500547004724273804610026655	129	18
1200100985497012198722139764	130	18
210704753179524394307437812	131	18
35117458863294664770603102	132	18
5544861925785537787648254	133	18
827591332206895317140580	134	18
116475817125418604888212	135	18
15415916972481984887181	136	18
1912924003884628322421	137	18
221788290305464189512	138	18
23933988162460164600	139	18
2393398816246016460	140	18
220667975965944780	141	18
18647997968953080	142	18
1434461382227160	143	18

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
99615373765775	144	18
6183023199255	145	18
338795791740	146	18
16133132940	147	18
654045930	148	18
21947850	149	18
585276	150	18
11628	151	18
153	152	18
1	153	18
379538942835190057920000	29	19
75534411357981998341920000	30	19
5028482932251937711042089600	31	19
177510849158865445447365741600	32	19
4054834094099428699398222483600	33	19
66918365407315426129035584637600	34	19
855043453859519858363361477526680	35	19
8865841878569892783209270783906752	36	19
77162925407856837607376532093139200	37	19
578142699229813512998115248698831428	38	19
3802634643533878723722344894830535364	39	19
22298780817661351419980483468176396620	40	19
118048678503661019300039085875310969012	41	19
570024867507685506800931515519507894880	42	19
2532235460038154159662164977239331038320	43	19
10423964638029736890839025401249103776622	44	19
40008768622065409726685346565992020399202	45	19
143935605310169898420515587765289689391466	46	19
487595702910936310726100432221544301471510	47	19
1561568568231699038984767278533302208584629	48	19
4744469636647473706831661091397577503751223	49	19
13717462561370095699560368276991589907061819	50	19
37844138853834307778714819424095639971370889	51	19
99863919407706939871014941070757884742531680	52	19
252601224130293117173760781463270920730852688	53	19
613636670599614887479951488582541340404156568	54	19
1434106425440620609947168495017977752185587248	55	19
3229346404930214441291760498141028470532242591	56	19
7016375411048959618536966643462346661206229165	57	19
14727134220921748935904969226043432567296868213	58	19

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	m	n
29896505698881562320787890943648098574911845223	59	19
58757302148169090655562114692647059701578796266	60	19
111903302722349996856016103786344954535189965014	61	19
206693254352406785166981952042665666304363481010	62	19
370544893850965437580030488127961847665948754494	63	19
645185674759313996370131036849408887070163550875	64	19
1091763453214442002299217041822657305274396052641	65	19
1796461543039714589289828587268867824467174671825	66	19
2875912787753203961169060662640431022522671329515	67	19
4481301299503021468122788324477965384817539578372	68	19
6799651286646750075695027091346062078909526426980	69	19
10050541181428972415127272570448512160700349298312	70	19
14476484502051937816939807368232868022718379732016	71	19
20325619587945924660951631711650033372381231709945	72	19
27826236364321639488821708833994489686436585577843	73	19
37154035264068854476256028938636237636685825687375	74	19
48394726608611372175979924379101800095460354042765	75	19
61506360208667699521241654452953588029742213336348	76	19
76287230466187161106048014712676412337138691909220	77	19
92355893126312010568960794845285559092274019110516	78	19
109149381003868043970556014120361244186015933821860	79	19
125943926700595595638982437870491316774527817433739	80	19
141899481946767399321462724124126797078112506606625	81	19
156125478255613050690035509705471354084320961466685	82	19
167761290381144121617315193399297824224305132655871	83	19
176061574714686853950394343035332756960321335077698	84	19
180474841777127099715906657713295894795259273092966	85	19
180703812827365316817919132274865225332225634110202	86	19
176738419602548837814981670654549020635228957587830	87	19
168856374811234190469995245423891566102528991986441	88	19
157591301378545287374903749848880784075985677311659	89	19
143673458202042927737894753526175474831103553551991	90	19
127952142035240823437495225195901719069638405129029	91	19
111311124562552005889846546209820199185190244381180	92	19
94588658579888847095418840712085246597501346465564	93	19
78511784651850286804863194067844524046922561718072	94	19
63651421427663120379752645150781351027877999107912	95	19
50400805103942102948239981714370323252581527077683	96	19
38976073155696676111728656658088658597630671640985	97	19
29434833513824873264996906020310603455982742806193	98	19

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
21706818366034639272149794685881750044831696575939	99	19
15630273568296942581317428064552754755234146962106	100	19
10988389975346349275385334721833644430333633085766	101	19
7541476591909071208193292187817999053224837991890	102	19
5052284220704802060436843205435600328116598545958	103	19
3303534625192424978387370529288522026874334595729	104	19
2108029307321777312471412844133045842072608477795	105	19
1312575779369940387073167858592083111841305899979	106	19
797373039683339175287049619126913317440783731025	107	19
472523793353567676947135720245456217628375068284	108	19
273112970306024789039659746709502691487342472772	109	19
153937658435684562700103559987438372856336488876	110	19
84596901414094176758808475294795157880232008420	111	19
45319985698509399899525432495127819974351610885	112	19
23662733465234979439913182921740418124223291807	113	19
12038967618126694923454555469556047805715780911	114	19
5967152756865949910648607970377392764481905629	115	19
2880698853411431390373902546176526842545222834	116	19
1354176211976380719646560150266623935381398150	117	19
619708274948745057867843795263921339606285478	118	19
276004693732223842313388655594914662231766834	119	19
119602086609356216251455111713965772256263631	120	19
50410812767951053768933822690109891668632741	121	19
20660173814842353692703639655690232084079105	122	19
8230477053504230571998638709139111517359507	123	19
3185991470829523802577423828899941181133216	124	19
1197932884970855376806960693056454687420448	125	19
437340600040556750053243580692120925225248	126	19
154963210256529769588541729182059506122080	127	19
53268604795799145098647528413695882016717	128	19
17756201878659974699030243234108834690031	129	19
5736619127572512994340748225356891848155	130	19
1795430425882570953662047647167863569441	131	19
544069828315577882051110792990482817950	132	19
159539273384410785244117055711464519146	133	19
45242480585133651635558169552320200938	134	19
12399790987283913756780281457927750014	135	19
3282297616177378773060534985849429467	136	19
838543186895629088326467555489423465	137	19
206597596810644291371285315373958533	138	19

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	m	n
49048350327109976586161853479039199	139	19
11211051503974951228756151042945820	140	19
2464841110876165284043464061912884	141	19
520741079770409691399410345304660	142	19
105604834359812607107161781338620	143	19
20534273347812616027074013202005	144	19
3823623313046951644224346911111	145	19
680919220132105906970158694211	146	19
115802588457872659758629667225	147	19
18778798128305617695825376770	148	19
2898740650678154128012409910	149	19
425148628766134274670348822	150	19
59126630490654637013653122	151	19
7779819801401932638891195	152	19
966121413245991812818889	153	19
112923282067713332282997	154	19
12385134162265333264047	155	19
1270270170488752129944	156	19
121363392084912623880	157	19
10753718286004916040	158	19
879234828415496280	159	19
65942612131162221	160	19
4505395859893071	161	19
278110855548955	162	19
15355814110065	163	19
749064102930	164	19
31778477094	165	19
1148619654	166	19
34389810	167	19
818805	168	19
14535	169	19
171	170	19
1	171	19
864876880105205071104000	30	20
573707328841844079442560000	31	20
70514301697861055717002320000	32	20
3853593224070868785774627360000	33	20
125037021193303137988869399600000	34	20
2785477467950672496177966517728000	35	20
46456180860462765753772199673612000	36	20

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	m	n
614008593417573177033422196428724000	37	20
6693047714955929957249065688091268800	38	20
61963982575158378574053941497821336000	39	20
498239434773761976624335179085610295800	40	20
3541099084439005677364564748953766472000	41	20
22560377643064780282273733811213951783000	42	20
130331436056366431081312104116374964740800	43	20
689254665542282779166528509060500063025050	44	20
3363627831842688851496727452804152435046370	45	20
15250311537306462167887524770561700647390800	46	20
64612211953111259866160704212235534345036800	47	20
257095156743394035842956890612860603225981040	48	20
964963348447280321702853726903128170277607900	49	20
3429436899935387621949994779636212358189499260	50	20
11579453107182907465772910847468659469611754740	51	20
37255943276637326701081152271633533439306584510	52	20
114522238576563868123094784786663428848591012530	53	20
337124917790127480891718097539506773307319293400	54	20
952381301371952056556165490276563504523338939080	55	20
2586831811217439051915284316035653728025962920460	56	20
6767044289145939393576441199431972383276827106880	57	20
17075256256921640918170151088426939673309035196020	58	20
41617202320502349403824787440656225743933577076900	59	20
98097834646899817954015027390197142506207939512010	60	20
223882379515860763214984922023969721693893910982470	61	20
495224858313459421994784211861543838621272861076880	62	20
1062709985523063337008829834825115877574057932740120	63	20
2214259184913105821656558519346574810783645144503825	64	20
4483141703266319476186749132334663199823983689934250	65	20
8826454556601653926662431975700349779273947496894015	66	20
16909188106403148071390764131995569959802197905204880	67	20
31539187892706817493253749916357366143992918456153275	68	20
57306805072378767132429710059086553723810371526871400	69	20
101486074426395105844358383791814147755592374802453875	70	20
175246613024724117030934906712804160561810914163223920	71	20
295200703521667506661044366248893667350097069585845585	72	20
485261776269929386634911747981154571507089400253558210	73	20
778714149197276002782545395937621480431235752276616175	74	20
1220289318267019589770142266849969905014460329508140736	75	20
1867918631422025005416650115771321640847663772392065215	76	20

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
2793703032079268075157119333500520943670019942956238940	77	20
4083535703883840157898303914639776069740276265915366555	78	20
5834769741852351711712311057083802709541589668581129440	79	20
8151380587995153665326600681865577456735360882702396633	80	20
11136266085224661358084565012099116446122062033271652390	81	20
14880671914971105356068004084810908869529073225522976915	82	20
19451213344576159877076388784140026902562473691486509640	83	20
24875533897659661683450675060889902308227381047366181455	84	20
31128207109810780253326902248756309944753685401126342128	85	20
38118929131654575220235433118686355591403043542981272515	86	20
45685239645657992953992619278350695811533607037286266000	87	20
53591839842448265499463940686918509219410244565216682805	88	20
61537995886317886938085725327158607935556489009287576490	89	20
69173550350774971138954474367886245445930420167438125371	90	20
76122828433478327037915522061969661431697995471821809040	91	20
82014415845328352927686012053226813898545138222260233475	92	20
86513643460388587860262357077400591540461429019245948900	93	20
89353879594395988379557069841114360574720481083490152675	94	20
90362585092558897960747790831975310873912615854943464760	95	20
89478604340042553611748012997806863400063020566506275025	96	20
86758290614561119310652767135355818823272767002073410510	97	20
82369614124751156716323350275406207757609807291224857155	98	20
76575102001099733704986869211217271492317529584754528200	99	20
69706005347231824729014032215903690844157265314426323281	100	20
62131207262164574597055845659896529821844138019913069000	101	20
54224897509659323283659013468065081376529777371659543505	102	20
46336890895455584110040294442318036049950747246147869280	103	20
38768734553917008681709757190323850212105491459746086265	104	20
31757615659706788054998027733407200834769312865979686338	105	20
25468783742907383438974046885346332115337380492306791055	106	20
19995980419618129178058563819482418283420159465141156240	107	20
15368416084570571056149863675866047387612109770524733145	108	20
11562260851401910982351853724284949829368646393456263180	109	20
8514449896040427192774302637038668550336846160344135253	110	20
6136787967271035527321763731763973190000692948896854160	111	20
4328769002398350394375618471718344634390205804414363585	112	20
2988079045581333673524777263498071560198408474861657270	113	20
2018307061273997779535001500507594059395416070570287115	114	20
1333860622308193740433223003221478008068430254339392976	115	20
862421745053789026119739840903433127371756503296149265	116	20

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	m	n
545469308378061571493401958549189471491858883812195000	117	20
337454291882244670778942116503407871181884949035034225	118	20
204175432768394545583071896318973207160537253301498200	119	20
120804453819917844521205055658816958750107004793174905	120	20
69887171878040565777776598844266051956074884611261610	121	20
39526482840169786266885110812634029510891735259565975	122	20
21852095827624249471274047725191232131557265007387000	123	20
11807204446256049568087425686140268237647129705288425	124	20
6234213557083853432000510631898612354939804298191116	125	20
3216066347066724703962703587813346559115086539349405	126	20
1620696310597007794261419354235898047558248804591680	127	20
797687009495553188355791534848156141838223975908475	128	20
383384651284551310729793049272326453294009903897290	129	20
179895944137836553848621788556164952848081049238833	130	20
82395112559594605589857302308414622618878275945080	131	20
36828126313345928995688664549461711013485586026355	132	20
16060388418805268456838979839870257808138861486540	133	20
6831658539240488115039079768429648996267423120455	134	20
2833873406145830958506609633537214299178763916248	135	20
1146051813335544573332902740876629217948762691795	136	20
451728471171658605551887729219532640555645037950	137	20
173489925311751842161206410489287804686632908365	138	20
64902707244919994354963787280807435839753665040	139	20
23643129416027290005752011639689172729942415211	140	20
8384088530588289214193527111343093564743202640	141	20
2893100991904910278564771290717255269285287035	142	20
971110827015707886833181373270513672884163920	143	20
316959784812806478250222439933690428323861875	144	20
100552759528724030189069499407975896746600770	145	20
30992288937893468686250272509494649191175825	146	20
9276603499735886222022123746034558867303000	147	20
2695229396695598425456303986593657354831355	148	20
759729091951703203541259195769455666174300	149	20
207659285178350625587549873109233193052587	150	20
55009082173680955938289344844581255146280	151	20
14114172400934803046208636109992065827035	152	20
3505480727191115405440465578131148356470	153	20
842225889022785014922233035551393774805	154	20
195613754872726317809379984880587093936	155	20
43887701414112765527528342532541207395	156	20

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	m	n
9504342981442034315639366237253101880	157	20
1985084293596881038710716406995862615	158	20
399513820095434256267013465866053760	159	20
77405802643534487574332714136250311	160	20
14423441486376769244789625775814130	161	20
2581974093240601400072982866989685	162	20
443529292090433577334434515627240	163	20
73020066380743710559420890884655	164	20
11506192278177894811576271389362	165	20
1732860282858122066725537011525	166	20
249033813105359120670657680400	167	20
34093914889424176327534433895	168	20
4438261109865869502878723250	169	20
548255784159901538587510245	170	20
64123483527473864440675600	171	20
7083408064081415262894775	172	20
737001995106736848219930	173	20
72005942050658197814925	174	20
6583400416060178085936	175	20
561085262732401541415	176	20
44379625300867918530	177	20
3241208589389230005	178	20
217287726662965140	179	20
13278694407181203	180	20
733629525258630	181	20
36278383117185	182	20
1585940245560	183	20
60334683255	184	20
1956800538	185	20
52602165	186	20
1125180	187	20
17955	188	20
190	189	20
1	190	20
2731253786412955402959360000	32	21
739556899423843408026799680000	33	21
66646452785254367457868227792000	34	21
3166057119895525808207919266803200	35	21
96757476222615784250379950909664000	36	21
2124883182373698975861928049185485600	37	21

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	m	n
35955856740294544911309345191575082400	38	21
491653014224711035907711744631463299840	39	21
5622117503288161145366782353610967058531	40	21
55166581718003440710223397059681379930490	41	21
473858542361089250451934010876657763000945	42	21
3619889691558594112351039642542155271401640	43	21
24910813466312063871762434448093314855350170	44	21
156068860387973845054504172076155091175537572	45	21
898078056095561324372051257376854965770141250	46	21
4782130930273376024069762428395896404232275500	47	21
23713984976694843590498245738875854875465042935	48	21
110114767766398325628498366198924278467189152030	49	21
481073960130754698160066861261389100752822086697	50	21
1985680740070882539303406098298222689258199888500	51	21
7771868071822492490888460464946416664949125784530	52	21
28937561919005901731698229868572563864734073465950	53	21
102792973423892443563827204162876634486476314532950	54	21
349252133812094268112422607149017160750497275333542	55	21
1137578031330848459759786044354175288385942493038015	56	21
3559431611518880475739897012005187299576548356107490	57	21
10718638475527220235622022246560408492344820983803805	58	21
31115907789507693282271206114945767910889467105453720	59	21
87209735861452549275796804435370721507898922740265160	60	21
236310012630355870887470233736835177654450100652307150	61	21
619833377131562063912591416962859704846336472620804360	62	21
1575565089142639792381600967046796608701248690499398370	63	21
3885224127610046145819025959524766832564306105275504585	64	21
9303047667799094571122506515672261500195884711246664754	65	21
21649114923348986623064436679942678984366676544573138695	66	21
49001079085037450579582344785458543711408390879852296320	67	21
107953861330704657726439276916116297997173102357880345475	68	21
231647927044822819273475491397070828140455601333169008940	69	21
484444010120543758770910620315144210320088797409335764401	70	21
987937795203767644260525909926957782999079998192531181010	71	21
1965680317793032704924283942378974059299840856020405832290	72	21
3817702757307316230809482289181472997388015158070087736260	73	21
7240851500503721650971183725633628852709834640730252971310	74	21
13416922808151505194076592366235546067364720230009875731932	75	21
24297169835146623417243336433093688539113695977667228725095	76	21
43017887348286199868279238118389897368006126817763467483960	77	21

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
74485650350407609238402791072757895408574824907692211043925	78	21
126169716147804201574076506824828682615763241833320947067470	79	21
209129628860534936608395205829199771637526336866012771160217	80	21
339284299518349996977477088607357468210188626181719337496810	81	21
538891208210753714729180298738779112617152371982794326962785	82	21
838148815574266978836179500392030852878834924223806817761460	83	21
1276762699104800956417797312638414764586441996300509300007555	84	21
1905231833885739852959213245841603784050443033080394503732996	85	21
2785527101746152448268780972382579408674662316446293300555225	86	21
3990766930760055779872481555203687646705477126727651991648050	87	21
5603467253471677823630537265517563260903879076085231490154570	88	21
7711978858102162283254916276283116458861683144794535740963280	89	21
10404845580973890859945892683610926732733019274368580507520398	90	21
13763032838785115359787264352964306791289249019871076972330080	91	21
17850282573439846361660477707819434875481104645234354119203045	92	21
22702221240417694427088839241153865340100764151459862811982220	93	21
28315232632288980134589837224706986521423131328339607020078575	94	21
34636438669274865518048472977549160770374809908550301225662030	95	21
41556331110263375175290825409589283979098487377320412413786050	96	21
48905593408909513467551746119213300853853894544366774090347100	97	21
56457395926849613042090849885189845370993282677716215979995250	98	21
63935930385135060670901374670015308962017828858014111710412460	99	21
71031212077411797445630146068907704589413708192518121935011867	100	21
77419313483467982419298565658353133222057837234001793142932900	101	21
82786333238369458879894267424596814236147172890605795928403885	102	21
86853700765542942135535210375993724404428378513711832448003250	103	21
89402008766554743049767980299574680164892900580941788594505470	104	21
90290550869642335634728869739525918433783926832814985186941168	105	21
89470151832127807278696237641688085807425266906488675228946310	106	21
86987667220651541710280509306503405496941504984149116099584360	107	21
82981580655626822553175958617670541177178004750505439988066655	108	21
77669270608739997220691921521706955061265019830177815526250960	109	21
71327568250184156704520013970029905994536660711978115497386353	110	21
64269014639298185410194304493443689275638437135975416497930710	111	21
56816632815596815445670391594815830316016913870075708043472290	112	21
49280013612637430702976151804496796635774197757554122453096780	113	21
41935105902834029395758508792360310181711290639036873521549770	114	21
35009400552940790200456391068397325248994204789395096186040828	115	21
28673341887619123215077031163751744911605982949398966614344865	116	21
23037940971288912298636983610337107114247227268114679068101140	117	21

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	m	n
18157833126493916455507160650013169340372203811284169622459775	118	21
14038509135603390670491722310086938013218302743829517426032870	119	21
10646195892740384946922395673121053730219393758253123206807200	120	21
7918858400019205936873161922752875523264126693401150338322220	121	21
5776992307564492714539270488860679683954196125386041598022380	122	21
4133203246020940203392554265574670337793368849434218712464000	123	21
2899949207330486017868050411359212356375542523092062430062355	124	21
1995187765008732223242576924309329857349177067838424437318692	125	21
1345972455911209085792506203637976271937670114299084637811525	126	21
890256175516529320549647640813005618392956797451260903635610	127	21
577278986791736751003914772400872439723212121086119497031250	128	21
366954324766707593589711623128881297193939840094812382838420	129	21
228641666958070171937888365733250783245851899045323116170570	130	21
139628936233130255085217573337709243000625172633498953710780	131	21
83566011341725024632156211246866554569233634205844359697495	132	21
49008735374307484281021613405665681260775005860697920759200	133	21
28161780159994070815518183287403022448203076151272936872425	134	21
15854058793746350653246929214525113263347198151540703482078	135	21
8743055610206821049839122370432536697756284333986750139090	136	21
4722529969379837595205175637511513517451413500795631325360	137	21
2498151386184935305694548090132909657554047660795112884330	138	21
1294007065573798308280307172079331337512314113659826857520	139	21
656246672134849001055897163280250038460063065438403460629	140	21
325796308577914657733886102027183239961040472010595164000	141	21
158309506712595573659334728985134491661899029979374812115	142	21
75280057024517902175462061104265678086865237029717171730	143	21
35026141824916244573200291614955064513235691867840087180	144	21
15942934953289556721761036793500933124307371261152214240	145	21
7097882477458909992196902010469486914750004400050799300	146	21
3090234702866816331596615005962206622566179168285705300	147	21
1315437793934517712742818264516560766450619232857912175	148	21
547363392280579129798848823881325168846423782457116000	149	21
222594450635739935907720163227028508957045572616896521	150	21
88448127362356102291704540209084690480998946734267710	151	21
34331839269360219886430161935663146322765010315157295	152	21
13014684264947598794108873855160604521529582751186750	153	21
4817123422420554630305014573811034483218261637021825	154	21
1740380081736973684808204713022969314624834939130160	155	21
613595543251904990340593176456946437556001370796735	156	21
211045601248990654956989270679893744310197087242740	157	21

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
70793777723072687263982901045322793111652735554025	158	21
23152682042464116058441237507668930241017328268010	159	21
7379917405301698949425314242711096663074293590847	160	21
2291899816196606841226290121144823044628824904850	161	21
693228956927943319625592780540053018406475433425	162	21
204141042565034135508511843775836451141188589340	163	21
58503835375563999612510141584243918958889598595	164	21
16310160166432614312436415333744116941190001128	165	21
4421428960971818782033576914891799429854530085	166	21
1164927390946133698065523389739331058438790770	167	21
298165939354226927060930015688267688905257025	168	21
74100410964508786757833911294307831997949450	169	21
17871275585663210973696572094640493633312895	170	21
4180415341689924350381411275817130965161400	171	21
947884873989684916149916316893143776750175	172	21
208205926078886719908180255320612475798600	173	21
44273673936341459684997773677053087327425	174	21
9107727209764578737297612454960557124282	175	21
1811195751942120382322187510023060712645	176	21
347913308282696409410804822669780407590	177	21
64500781872637383878508093202769018955	178	21
11530866033097411068468315215459849340	179	21
1985871372366794058151149333406467523	180	21
329149951221016875846051767606038590	181	21
52446970249502774753516649421601505	182	21
8024673043639774026089062882835200	183	21
1177533544447141040183028271257375	184	21
165491200841219835947417354210370	185	21
22243440973282236626815388663325	186	21
2854773173041570499341210049700	187	21
349254164787000646943538541395	188	21
40653923943460392785903361910	189	21
4493328435856148676762005409	190	21
470505595377607191290731080	191	21
46560449542575711638216335	192	21
4342425345939703676103450	193	21
380521808664819394297725	194	21
31222302249421078506480	195	21
2389461906843449885700	196	21
169809475613240093400	197	21

Table E.2: Numbers of labeled 3–edge–connected graphs by number of edges m and nodes n (continued).

Number	**m**	**n**
11149106984707682900	198	21
672307958876845200	199	21
36976937738226486	200	21
1839648643692860	201	21
81964543530870	202	21
3230129794320	203	21
110837787060	204	21
3244032792	205	21
78738660	206	21
1521520	207	21
21945	208	21
210	209	21
1	210	21

APPENDIX F

Harmonic mean of symmetry numbers for minimally 2–connected graphs

Table F.1: Harmonic mean of symmetry numbers for minimally 2–connected graphs by the number n of nodes.

Number	n
6.000000	3
8.000000	4
10.909091	5
8.470588	6
7.653759	7
6.327353	8
5.730412	9
5.138804	10
4.933770	11
4.846302	12
4.938620	13
5.118324	14
5.408222	15
5.762475	16
6.191418	17
6.679780	18
7.231320	19
7.842934	20
8.518463	21
9.259792	22
10.071571	23
10.958193	24
11.925254	25
12.978600	26
14.124754	27
15.370604	28
16.723511	29
18.191187	30
19.781697	31
21.503371	32

Table F.2: Harmonic mean of symmetry numbers for minimally 2–connected graphs by number of edges m and nodes n.

Number	**m**	**n**
64/1	32	32
2.216749	33	32
1.230864	34	32
1.261272	35	32
1.434961	36	32
1.725866	37	32
2.187894	38	32
2.929392	39	32
4.160392	40	32
6.307381	41	32
10.293149	42	32
18.269761	43	32
35.712782	44	32
78.027649	45	32
193.904199	46	32
559.468933	47	32
1920.434784	48	32
8074.241694	49	32
43067.744390	50	32
304392.600800	51	32
3008863.832000	52	32
44732820.720000	53	32
1096642616.000000	54	32
51354063280.400000	55	32
5613581475000.400000	56	32
2087093409000000.400000	57	32
5516807878000000000.400000	58	32
46113384910000000000000.400000	59	32
5305057196243821172726169600000/1	60	32

APPENDIX G

Harmonic mean of symmetry numbers for minimally 2–edge–connected blocks

Table G.1: Harmonic mean of symmetry numbers for minimally 2–edge–connected blocks by the number n of nodes.

Number	n	Number	n
6.000000	3	6.157310	19
8.000000	4	6.516164	20
10.909091	5	6.913289	21
8.470588	6	7.348270	22
7.588076	7	7.817416	23
7.615019	8	8.320104	24
6.473988	9	8.854670	25
6.078992	10	9.420545	26
5.500040	11	10.016933	27
5.358038	12	10.643394	28
5.173643	13	11.299432	29
5.171648	14	11.984660	30
5.223629	15	12.698685	31
5.376321	16	13.441164	32
5.577104	17	14.211779	33
5.845744	18	15.010275	34

Table G.2: Harmonic mean of symmetry numbers for minimally 2–edge–connected blocks by number of edges m and nodes n.

Number	**m**	**n**
136/1	34	34
4.387097	35	34
2.428585	36	34
2.482473	37	34
2.807712	38	34
3.360715	39	34
4.249401	40	34
5.674871	41	34
7.997216	42	34
11.891705	43	34
18.711540	44	34
31.360138	45	34
56.528129	46	34
110.892688	47	34
239.999615	48	34
581.581796	49	34
1608.167525	50	34
5166.921282	51	34
19835.244950	52	34
95658.661220	53	34
549713.980200	54	34
5556767.849000	55	34
48464620.350000	56	34
2224299790.000000	57	34
32558121980.700000	58	34
24770430790000.700000	59	34
811132813300000.700000	60	34
1288180058000000000.700000	61	34
151752952300000000000.700000	62	34
106101143924876423454523392000000/1	63	34
105252334773477412066887204864000000/1	64	34

APPENDIX H

Harmonic mean of symmetry numbers for 3–edge–connected blocks

Table H.1: Harmonic mean of symmetry numbers for 3–edge–connected blocks by the number n of nodes.

Number	**n**
24.000000	4
13.846154	5
7.362756	6
3.170534	7
1.944759	8
1.420389	9
1.211484	10
1.114543	11
1.065076	12
1.037664	13
1.021859	14
1.012626	15
1.007238	16
1.004115	17
1.002321	18
1.001300	19
1.000723	20
1.000400	21
1.000220	22
1.000121	23
1.000066	24

Table H.2: Harmonic mean of symmetry numbers for 3–edge–connected blocks by number of edges m and nodes n.

Number	**m**	**n**
1.085285	32	21
1.090422	33	21
1.089486	34	21
1.084629	35	21
1.078073	36	21
1.071057	37	21
1.064190	38	21
1.057745	39	21
1.051825	40	21
1.046448	41	21
1.041595	42	21
1.037231	43	21
1.033315	44	21
1.029804	45	21
1.026660	46	21
1.023846	47	21
1.021328	48	21
1.019076	49	21
1.017063	50	21
1.015265	51	21
1.013658	52	21
1.012224	53	21
1.010943	54	21
1.009801	55	21
1.008781	56	21
1.007872	57	21
1.007061	58	21
1.006339	59	21
1.005694	60	21
1.005120	61	21
1.004608	62	21
1.004152	63	21
1.003745	64	21
1.003382	65	21
1.003059	66	21
1.002770	67	21
1.002513	68	21
1.002283	69	21
1.002078	70	21

Table H.2: Harmonic mean of symmetry numbers for 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
1.001895	71	21
1.001731	72	21
1.001584	73	21
1.001453	74	21
1.001336	75	21
1.001231	76	21
1.001137	77	21
1.001052	78	21
1.000976	79	21
1.000908	80	21
1.000847	81	21
1.000792	82	21
1.000743	83	21
1.000698	84	21
1.000658	85	21
1.000622	86	21
1.000590	87	21
1.000560	88	21
1.000534	89	21
1.000511	90	21
1.000490	91	21
1.000471	92	21
1.000454	93	21
1.000439	94	21
1.000426	95	21
1.000414	96	21
1.000404	97	21
1.000395	98	21
1.000388	99	21
1.000381	100	21
1.000377	101	21
1.000373	102	21
1.000370	103	21
1.000368	104	21
1.000368	105	21
1.000369	106	21
1.000370	107	21
1.000373	108	21
1.000377	109	21
1.000382	110	21

Table H.2: Harmonic mean of symmetry numbers for 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
1.000388	111	21
1.000396	112	21
1.000405	113	21
1.000415	114	21
1.000427	115	21
1.000440	116	21
1.000456	117	21
1.000473	118	21
1.000492	119	21
1.000513	120	21
1.000537	121	21
1.000564	122	21
1.000593	123	21
1.000626	124	21
1.000663	125	21
1.000704	126	21
1.000749	127	21
1.000800	128	21
1.000856	129	21
1.000919	130	21
1.000989	131	21
1.001068	132	21
1.001156	133	21
1.001255	134	21
1.001366	135	21
1.001490	136	21
1.001631	137	21
1.001789	138	21
1.001968	139	21
1.002170	140	21
1.002400	141	21
1.002660	142	21
1.002957	143	21
1.003295	144	21
1.003681	145	21
1.004122	146	21
1.004628	147	21
1.005208	148	21
1.005876	149	21
1.006645	150	21

Table H.2: Harmonic mean of symmetry numbers for 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
1.007533	151	21
1.008560	152	21
1.009750	153	21
1.011133	154	21
1.012741	155	21
1.014615	156	21
1.016805	157	21
1.019368	158	21
1.022374	159	21
1.025908	160	21
1.030073	161	21
1.034991	162	21
1.040814	163	21
1.047727	164	21
1.055956	165	21
1.065782	166	21
1.077554	167	21
1.091707	168	21
1.108792	169	21
1.129508	170	21
1.154752	171	21
1.185689	172	21
1.223848	173	21
1.271267	174	21
1.330700	175	21
1.405932	176	21
1.502269	177	21
1.627301	178	21
1.792137	179	21
2.013459	180	21
2.317018	181	21
2.743798	182	21
3.361247	183	21
4.284489	184	21
5.718050	185	21
8.041462	186	21
11.993337	187	21
19.087486	188	21
32.606293	189	21
60.115879	190	21

Table H.2: Harmonic mean of symmetry numbers for 3–edge–connected blocks by number of edges m and nodes n (continued).

Number	**m**	**n**
120.253151	191	21
262.347009	192	21
627.527823	193	21
1655.073887	194	21
4842.583087	195	21
15823.477320	196	21
58169.719040	197	21
242593.724400	198	21
1156239.005000	199	21
6373770.535000	200	21
40991648.540000	201	21
309794908.500000	202	21
2799607861.000000	203	21
31344762110.700000	204	21
409479367700.700000	205	21
7137540363000.700000	206	21
167894415360000/1	207	21
4656271785984000/1	208	21
243290200817664000/1	209	21
5109094217170944000/1	210	21

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