A049982

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A049982 is the number of arithmetic progressions of 2 or more positive integers, strictly increasing with sum n.

When I stumbled on this sequence (and it's brothers and sisters, with various slightly different qualifications), I noticed a complete lack of any formulas or generating functions that help understand the sequence. So I did some amateur investigation on my own.

I started by considering the number of arithmetic progressions of 2 positive integers, strictly increasing with sum n. By convention, I like to start with n=0, so this sequence is 0, 0, 0, 1, 1, 2, 2, 3, 3, 4, 4, ... which has generating function $x^3/(x^3-x^2-x+1)$ which I will rewrite as $x^3/(x^3-x-x^2+1)$ for reasons that will become clear later.

The sum of an arithmetic progression of 3 positive integers is always three times its middle term, hence a multiple of 3. This sequence is 0, 0, 0, 0, 0, 1, 0, 0, 2, 0, 0, 3, 0, 0, 4, ... which has generating function $x^6/(x^6-2x^3+1)$, which I will write as $x^6/(x^6-x^3-x^3+1)$ for reasons that will become clear later.

The sum of an arithmetic progression of 4 positive integers follows a pattern that's a little harder to discern. But if I give you enough terms, I think you can start to pick up the rhythm: 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 2, 0, 1, 0, 2, 0, 2, 0, 2, 0, 2, 0, 3, 0, 2, 0, 3, 0, 3, 0, 3, 0, 3, 0, 4, 0, 3, ... This has generating function $x^{10}/(x^{10}-x^6-x^4+1)$

As you can imagine, I kept going. As the going got tougher, I started inventing little tools to help, such as PuzzleGeneratingFunction.xls, an Excel spreadsheet that guesses the generating function for a given series. (. . . . . . maybe I'll write a page about that spreadsheet some day.)

After a while, I had a little table that shows the

generating function for the sequence of the number of arithmetic progressions of k positive integers, strictly increasing with sum n:

k	Generating Function
2	$x^3/(x^3-x-x^2+1)$
3	$x^6/(x^6-x^3-x^3+1)$
4	$x^{10}/(x^{10}-x^6-x^4+1)$
5	$x^{15}/(x^{15}-x^{10}-x^5+1)$
6	$x^{21}/(x^{21}-x^{15}-x^6+1)$
7	$x^{28}/(x^{28}-x^{21}-x^7+1)$
8	$x^{36}/(x^{36}-x^{28}-x^8+1)$
9	$x^{45}/(x^{45}-x^{36}-x^9+1)$
10	$x^{55}/(x^{55}-x^{45}-x^{10}+1)$
11	$x^{66}/(x^{66}-x^{55}-x^{11}+1)$

Now, maybe you can see why I wrote the terms for k=2 and k=3 in such a funny way. In general, the generating function for the sequence of the number of arithmetic progressions of k positive integers, strictly increasing with sum n is:

$x^{t(k)}/(x^{t(k)}-x^{t(k-1)}-x^k+1)$, where t(k) is the k'th triangular number

Summary

A049982 has generating function $x^3/(x^3-x-x^2+1) + x^6/(x^6-x^3-x^3+1) + x^{10}/(x^{10}-x^6-x^4+1) + \ldots$ which is the
sum k=2,3,... of $x^{t(k)}/(x^{t(k)}-x^{t(k-1)}-x^k+1)$, where t(k) is the k'th triangular number
Term k of this generating function generates the number of arithmetic progressions of k positive integers, strictly increasing with sum n.

Internet References

A049982 -- The Online Encyclopedia of Integer Sequences.

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