# FAULT-FREE TILINGS WITH DOMINOES OR TROMINOES.

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Abstract. We list numerical tables of fault-free tilings for $m \times n$ floors with dominoes or trominoes for small $m$.

## 1. Fault-free Tilings

Tilings of $m \times n$ floors with polyominoes will be registered in the following by regarding $m$ as a fixed number of rows in the floors and $n$ as the variable number of columns.

$$T_m(x) \equiv \sum_{n\ge 0} T_{m,n} x^n \tag{1}$$

is the generating function for these tilings.

With this distinction, the tilings can be refined by their number of horizontal and vertical faults [7]. A vertical fault is a line of length $m$ which is not one of the two sides of the floor which splits the set of tiles in a left set with $n'$ columns and a right set with $n''$ columns, $n = n' + n''$, without cutting through a tile such the two subsets tile the $m \times n'$ and the $m \times n''$ subfloors. A horizontal fault is a line of length $n$ which is not one of the two sides of the floor which splits the set of tiles in a lower set with $m'$ rows and an upper set with $m''$ rows, $m = m' + m''$, without cutting through a tile such the two subsets tile the $m' \times n$ and the $m'' \times n$ subfloors.

With this definition the number of tiles, $T_{m,n}$ is decomposed in the number $T^{\perp}_{m,n}$ of tiles which have vertical and horizontal faults, the number $T^{p}_{m,n}$ of tiles which have neither vertical nor horizontal faults (called fault-free), the number $T^{|}_{m,n}$ of tiles which have only vertical faults, and the number $T^{-}_{m,n}$ of tiles which have only horizontal faults:

$$T_{m,n} = T^{\perp}_{m,n} + T^{-}_{m,n} + T^{|}_{m,n} + T^{p}_{m,n}. \tag{2}$$

The tilings can be rotated by 90° which turns horizontal faults to vertical faults and vice versa:

$$T^{\perp}_{m,n} = T^{\perp}_{n,m}; \quad T^{p}_{m,n} = T^{p}_{n,m}; \quad T^{|}_{m,n} = T^{-}_{n,m}; \quad T^{-}_{m,n} = T^{|}_{n,m}. \tag{3}$$

**Remark 1.** *With a standard poset diagram one may also introduce the number*

$$T^{\|}_{m,n} = T^{|}_{m,n} + T^{\perp}_{m,n} \tag{4}$$

*of tiles which have vertical faults (with or without horizontal faults) and the number*

$$T^{=}_{m,n} = T^{-}_{m,n} + T^{\perp}_{m,n} \tag{5}$$

*of tiles which have horizontal faults (with or without vertical faults).*

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| $n$ | $T^{\perp}_{2,n}$ | $T^{-}_{2,n}$ | $T^{\mid}_{2,n}$ | $T^{p}_{2,n}$ | $T_{2,n}$ |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 1 | 1 |
| 2 | 0 | 1 | 1 | 0 | 2 |
| 3 | 0 | 0 | 3 | 0 | 3 |
| 4 | 1 | 0 | 4 | 0 | 5 |
| 5 | 0 | 0 | 8 | 0 | 8 |
| 6 | 1 | 0 | 12 | 0 | 13 |
| 7 | 0 | 0 | 21 | 0 | 21 |
| 8 | 1 | 0 | 33 | 0 | 34 |
| 9 | 0 | 0 | 55 | 0 | 55 |
| 10 | 1 | 0 | 88 | 0 | 89 |
| 11 | 0 | 0 | 144 | 0 | 144 |
| 12 | 1 | 0 | 232 | 0 | 233 |
| 13 | 0 | 0 | 377 | 0 | 377 |

Table 1. Domino tilings $m = 2$ [8, A000045,A052952].

| $n$ | $T^{\perp}_{3,n}$ | $T^{-}_{3,n}$ | $T^{\mid}_{3,n}$ | $T^{p}_{3,n}$ | $T_{3,n}$ |
|---|---|---|---|---|---|
| 2 | 0 | 3 | 0 | 0 | 3 |
| 4 | 7 | 2 | 2 | 0 | 11 |
| 6 | 23 | 2 | 16 | 0 | 41 |
| 8 | 65 | 2 | 86 | 0 | 153 |
| 10 | 175 | 2 | 394 | 0 | 571 |
| 12 | 463 | 2 | 1666 | 0 | 2131 |
| 14 | 1217 | 2 | 6734 | 0 | 7953 |
| 16 | 3191 | 2 | 26488 | 0 | 29681 |
| 18 | 8359 | 2 | 102410 | 0 | 110771 |
| 20 | 21889 | 2 | 391512 | 0 | 413403 |
| 22 | 57311 | 2 | 1485528 | 0 | 1542841 |
| 24 | 150047 | 2 | 5607912 | 0 | 5757961 |
| 26 | 392833 | 2 | 21096168 | 0 | 21489003 |
| 28 | 1028455 | 2 | 79169594 | 0 | 80198051 |

Table 2. Domino tilings $m = 3$ [8, A001835].

With the aid of the SEQ transformation [3, Thm I.1] (which is also known as the INVERT transformation [1]) a left-to-right stacking of two or more vertically fault-free tiles becomes a tiling for a floor with the sums of the columns of the individual tilings which does have vertical faults [6, 5]:

$$T_m(x) = \frac{1}{1 - [T^{-}_m(x) + T^{p}_m(x)]}. \tag{6}$$

## 2. Domino Tilings

The refinement for domino tilings [8, A099390] for various small $m$ is collected in Tables 1 to 7. Necessary and sufficient for $T^{p}_{m,n} \neq 0$ are (i) $2 \mid (nm)$, (ii) $n \geq 5$, $m \geq 5$ and (iii) $(n, m) \neq (6, 6)$ [4].

| $n$ | $T^{\perp}_{4,n}$ | $T^{-}_{4,n}$ | $T^{|}_{4,n}$ | $T^{p}_{4,n}$ | $T_{4,n}$ |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 1 |
| 2 | 1 | 4 | 0 | 0 | 5 |
| 3 | 7 | 2 | 2 | 0 | 11 |
| 4 | 30 | 3 | 3 | 0 | 36 |
| 5 | 62 | 2 | 31 | 0 | 95 |
| 6 | 210 | 3 | 68 | 0 | 281 |
| 7 | 439 | 2 | 340 | 0 | 781 |
| 8 | 1358 | 3 | 884 | 0 | 2245 |
| 9 | 3023 | 2 | 3311 | 0 | 6336 |
| 10 | 8794 | 3 | 9264 | 0 | 18061 |
| 11 | 20734 | 2 | 30469 | 0 | 51205 |
| 12 | 57850 | 3 | 87748 | 0 | 145601 |
| 13 | 142127 | 2 | 271222 | 0 | 413351 |
| 14 | 386174 | 3 | 788323 | 0 | 1174500 |
| 15 | 974167 | 2 | 2361482 | 0 | 3335651 |
| 16 | 2604978 | 3 | 6870920 | 0 | 9475901 |

TABLE 3. Domino tilings $m = 4$ [8, A005178].

| $n$ | $T^{\perp}_{5,n}$ | $T^{-}_{5,n}$ | $T^{|}_{5,n}$ | $T^{p}_{5,n}$ | $T_{5,n}$ |
|---|---|---|---|---|---|
| 2 | 0 | 8 | 0 | 0 | 8 |
| 4 | 62 | 31 | 2 | 0 | 95 |
| 6 | 880 | 169 | 128 | 6 | 1183 |
| 8 | 10195 | 907 | 3614 | 108 | 14824 |
| 10 | 107910 | 4729 | 72100 | 1182 | 185921 |
| 12 | 1091810 | 24109 | 1205840 | 10338 | 2332097 |

TABLE 4. Domino tilings $m = 5$ [8, A003775,A232621]. See [4, Fig 1] for an illustration of $T^{p}_{5,6}$.

| $n$ | $T^{\perp}_{6,n}$ | $T^{-}_{6,n}$ | $T^{|}_{6,n}$ | $T^{p}_{6,n}$ | $T_{6,n}$ |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 1 |
| 2 | 1 | 12 | 0 | 0 | 13 |
| 3 | 23 | 16 | 2 | 0 | 41 |
| 4 | 210 | 68 | 3 | 0 | 281 |
| 5 | 880 | 128 | 169 | 6 | 1183 |
| 6 | 5730 | 499 | 499 | 0 | 6728 |
| 7 | 22585 | 956 | 7864 | 124 | 31529 |
| 8 | 132180 | 3665 | 31182 | 62 | 167089 |
| 9 | 523649 | 6936 | 285760 | 1646 | 817991 |
| 10 | 2871619 | 26422 | 1313462 | 1630 | 4213133 |
| 11 | 11711572 | 49484 | 9222623 | 18120 | 21001799 |

TABLE 5. Domino tilings $m = 6$ [8, A028468].

| $n$ | $T^{\perp}_{7,n}$ | $T^{-}_{7,n}$ | $T^{\vert}_{7,n}$ | $T^{p}_{7,n}$ | $T_{7,n}$ |
|---|---|---|---|---|---|
| 2 | 0 | 21 | 0 | 0 | 21 |
| 4 | 439 | 340 | 2 | 0 | 781 |
| 6 | 22585 | 7864 | 956 | 124 | 31529 |
| 8 | 958609 | 183786 | 136788 | 13514 | 1292697 |
| 10 | 36655239 | 4161910 | 11593186 | 765182 | 53175517 |

Table 6. Domino tilings $m = 7$ [8, A028469].

| $n$ | $T^{\perp}_{8,n}$ | $T^{-}_{8,n}$ | $T^{\vert}_{8,n}$ | $T^{p}_{8,n}$ | $T_{8,n}$ |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 1 |
| 2 | 1 | 33 | 0 | 0 | 34 |
| 3 | 65 | 86 | 2 | 0 | 153 |
| 4 | 1358 | 884 | 3 | 0 | 2245 |
| 5 | 10195 | 3614 | 907 | 108 | 14824 |
| 6 | 132180 | 31182 | 3665 | 62 | 167089 |
| 7 | 958609 | 136788 | 183786 | 13514 | 1292697 |
| 8 | 10732974 | 1115168 | 1115168 | 25506 | 12988816 |

Table 7. Domino tilings $m = 8$ [8, A028470].

| $n$ | $T^{\perp}_{3,n}$ | $T^{-}_{3,n}$ | $T^{\vert}_{3,n}$ | $T^{p}_{3,n}$ | $T_{3,n}$ |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 1 | 1 |
| 2 | 0 | 0 | 1 | 0 | 1 |
| 3 | 0 | 1 | 1 | 0 | 2 |
| 4 | 0 | 0 | 3 | 0 | 3 |
| 5 | 0 | 0 | 4 | 0 | 4 |
| 6 | 1 | 0 | 5 | 0 | 6 |
| 7 | 0 | 0 | 9 | 0 | 9 |
| 8 | 0 | 0 | 13 | 0 | 13 |
| 9 | 1 | 0 | 18 | 0 | 19 |
| 10 | 0 | 0 | 28 | 0 | 28 |
| 11 | 0 | 0 | 41 | 0 | 41 |
| 12 | 1 | 0 | 59 | 0 | 60 |
| 13 | 0 | 0 | 88 | 0 | 88 |
| 14 | 0 | 0 | 129 | 0 | 129 |
| 15 | 1 | 0 | 188 | 0 | 189 |
| 16 | 0 | 0 | 277 | 0 | 277 |
| 17 | 0 | 0 | 406 | 0 | 406 |
| 18 | 1 | 0 | 594 | 0 | 595 |

TABLE 8. Straight 3-omino tilings $m = 3$ [8, A000930,A099560].

| $n$ | $T^{\perp}_{4,n}$ | $T^{-}_{4,n}$ | $T^{\vert}_{4,n}$ | $T^{p}_{4,n}$ | $T_{4,n}$ |
|---|---|---|---|---|---|
| 3 | 0 | 3 | 0 | 0 | 3 |
| 6 | 7 | 4 | 2 | 0 | 13 |
| 9 | 31 | 6 | 20 | 0 | 57 |
| 12 | 111 | 8 | 130 | 0 | 249 |
| 15 | 367 | 10 | 710 | 0 | 1087 |
| 18 | 1177 | 12 | 3556 | 0 | 4745 |
| 21 | 3731 | 14 | 16968 | 0 | 20713 |
| 24 | 11775 | 16 | 78626 | 0 | 90417 |
| 27 | 37101 | 18 | 357572 | 0 | 394691 |
| 30 | 116829 | 20 | 1606068 | 0 | 1722917 |

TABLE 9. Straight 3-omino tilings $m = 4$ [8, A049086].

## 3. Straight Tromino Tiling

The numerical results for tilings with straight trominoes are collected in Tables 8 to 14. The cases $m = 1$ and $m = 2$ are not interesting because there is exactly one tiling if $3 \mid n$. The absence of fault-free tilings for small $m$ is explained by a theorem of Robinson [7].

| $n$ | $T^{\perp}_{5,n}$ | $T^{-}_{5,n}$ | $T^{\vert}_{5,n}$ | $T^{p}_{5,n}$ | $T_{5,n}$ |
|---|---|---|---|---|---|
| 3 | 0 | 4 | 0 | 0 | 4 |
| 6 | 14 | 6 | 2 | 0 | 22 |
| 9 | 86 | 9 | 26 | 0 | 121 |
| 12 | 426 | 12 | 226 | 0 | 664 |
| 15 | 1970 | 15 | 1658 | 0 | 3643 |
| 18 | 8877 | 18 | 11092 | 0 | 19987 |
| 21 | 39532 | 21 | 70104 | 0 | 109657 |
| 24 | 174914 | 24 | 426686 | 0 | 601624 |
| 27 | 770795 | 27 | 2529938 | 0 | 3300760 |
| 30 | 3387379 | 30 | 14721936 | 0 | 18109345 |

TABLE 10. Straight 3-omino tilings $m = 5$ [8, A236576].

| $n$ | $T^{\perp}_{6,n}$ | $T^{-}_{6,n}$ | $T^{\vert}_{6,n}$ | $T^{p}_{6,n}$ | $T_{6,n}$ |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 1 |
| 2 | 1 | 0 | 0 | 0 | 1 |
| 3 | 1 | 5 | 0 | 0 | 6 |
| 4 | 7 | 2 | 4 | 0 | 13 |
| 5 | 14 | 2 | 6 | 0 | 22 |
| 6 | 48 | 8 | 8 | 0 | 64 |
| 7 | 77 | 4 | 74 | 0 | 155 |
| 8 | 163 | 6 | 152 | 0 | 321 |
| 9 | 495 | 14 | 274 | 0 | 783 |
| 10 | 774 | 10 | 1104 | 0 | 1888 |
| 11 | 1667 | 14 | 2552 | 0 | 4233 |
| 12 | 4534 | 26 | 5352 | 0 | 9912 |
| 13 | 7720 | 24 | 15750 | 0 | 23494 |
| 14 | 16609 | 32 | 37536 | 0 | 54177 |
| 15 | 41491 | 52 | 84476 | 0 | 126019 |
| 16 | 76673 | 56 | 218952 | 0 | 295681 |
| 17 | 164762 | 74 | 522854 | 0 | 687690 |
| 18 | 387955 | 110 | 1212120 | 0 | 1600185 |
| 19 | 760254 | 130 | 2977948 | 0 | 3738332 |
| 20 | 1633112 | 172 | 7079708 | 0 | 8712992 |

TABLE 11. Straight 3-omino tilings $m = 6$ [8, A236577].

| $n$ | $T^{\perp}_{7,n}$ | $T^{-}_{7,n}$ | $T^{\vert}_{7,n}$ | $T^{p}_{7,n}$ | $T_{7,n}$ |
|---|---|---|---|---|---|
| 3 | 0 | 9 | 0 | 0 | 9 |
| 6 | 77 | 74 | 4 | 0 | 155 |
| 9 | 1721 | 768 | 340 | 32 | 2861 |
| 12 | 30609 | 7372 | 13810 | 1026 | 52817 |
| 15 | 484169 | 66530 | 402480 | 19378 | 972557 |
| 18 | 7179281 | 578466 | 9845852 | 288682 | 17892281 |

TABLE 12. Straight 3-omino tilings $m = 7$ [8, A236578].

| $n$ | $T^{\perp}_{8,n}$ | $T^{-}_{8,n}$ | $T^{\vert}_{8,n}$ | $T^{p}_{8,n}$ | $T_{8,n}$ |
|---|---|---|---|---|---|
| 3 | 0 | 13 | 0 | 0 | 13 |
| 6 | 163 | 152 | 6 | 0 | 321 |
| 9 | 5383 | 1936 | 766 | 48 | 8133 |
| 12 | 135391 | 22720 | 44922 | 1942 | 204975 |
| 15 | 2995691 | 253216 | 1862670 | 46646 | 5158223 |

TABLE 13. Straight 3-omino tilings $m = 8$.

| $n$ | $T^{\perp}_{9,n}$ | $T^{-}_{9,n}$ | $T^{\vert}_{9,n}$ | $T^{p}_{9,n}$ | $T_{9,n}$ |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 1 |
| 2 | 1 | 0 | 0 | 0 | 1 |
| 3 | 1 | 18 | 0 | 0 | 19 |
| 4 | 31 | 20 | 6 | 0 | 57 |
| 5 | 86 | 26 | 9 | 0 | 121 |
| 6 | 495 | 274 | 14 | 0 | 783 |
| 7 | 1721 | 340 | 768 | 32 | 2861 |
| 8 | 5383 | 766 | 1936 | 48 | 8133 |
| 9 | 27898 | 4623 | 4623 | 16 | 37160 |
| 10 | 77296 | 6480 | 58307 | 1336 | 143419 |
| 11 | 262013 | 16172 | 187657 | 2974 | 468816 |
| 12 | 1237297 | 77900 | 558500 | 3158 | 1876855 |
| 13 | 3330876 | 122596 | 3768994 | 41002 | 7263468 |
| 14 | 11520265 | 310712 | 13561788 | 104098 | 25496863 |

TABLE 14. Straight 3-omino tilings $m = 9$ [8, A251073].

| $n$ | $T^{\perp}_{4,n}$ | $T^{-}_{4,n}$ | $T^{\vert}_{4,n}$ | $T^{p}_{4,n}$ | $T_{4,n}$ |
|---|---|---|---|---|---|
| 3 | 0 | 4 | 0 | 0 | 4 |
| 6 | 16 | 0 | 0 | 2 | 18 |
| 9 | 64 | 0 | 16 | 8 | 88 |
| 12 | 256 | 0 | 164 | 48 | 468 |
| 15 | 1024 | 0 | 1360 | 288 | 2672 |
| 18 | 4096 | 0 | 10248 | 1728 | 16072 |
| 21 | 16384 | 0 | 73312 | 10368 | 100064 |
| 24 | 65536 | 0 | 508624 | 62208 | 636368 |
| 27 | 262144 | 0 | 3462592 | 373248 | 4097984 |
| 30 | 1048576 | 0 | 23291424 | 2239488 | 26579488 |

TABLE 15. Right 3-omino tilings $m = 4$ [8, A046984,A084477].

| $n$ | $T^{\perp}_{5,n}$ | $T^{-}_{5,n}$ | $T^{\vert}_{5,n}$ | $T^{p}_{5,n}$ | $T_{5,n}$ |
|---|---|---|---|---|---|
| 3 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 64 | 0 | 8 | 72 |
| 9 | 0 | 0 | 0 | 384 | 384 |
| 12 | 2048 | 0 | 3136 | 3360 | 8544 |
| 15 | 0 | 0 | 55296 | 21504 | 76800 |
| 18 | 65536 | 0 | 939008 | 163968 | 1168512 |
| 21 | 0 | 0 | 11649024 | 1136640 | 12785664 |

TABLE 16. Right 3-omino tilings $m = 5$ [8, A084478,A084479]

## 4. Right Tromino Tiling

The numerical results for tilings with right trominoes look as follows [8, A351322]. For $m = 1$ there are no tilings. For $m = 2$ and $3 \mid n$ these are simply $T_{2,n} = 2^{n/3}$. For $m = 3$ and $2 \mid n$ these are simply $T_{3,n} = 2^{n/2}$.

| $n$ | $T^{\perp}_{6,n}$ | $T^{-}_{6,n}$ | $T^{\vert}_{6,n}$ | $T^{p}_{6,n}$ | $T_{6,n}$ |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 4 | 0 | 0 | 4 |
| 3 | 0 | 8 | 0 | 0 | 8 |
| 4 | 16 | 0 | 0 | 2 | 18 |
| 5 | 0 | 0 | 64 | 8 | 72 |
| 6 | 128 | 16 | 16 | 2 | 162 |
| 7 | 0 | 0 | 480 | 40 | 520 |
| 8 | 256 | 0 | 1140 | 118 | 1514 |
| 9 | 768 | 128 | 3200 | 216 | 4312 |
| 10 | 1024 | 0 | 11208 | 1010 | 13242 |
| 11 | 0 | 0 | 36032 | 3056 | 39088 |
| 12 | 13440 | 1536 | 95924 | 7686 | 118586 |
| 13 | 0 | 0 | 333856 | 27856 | 361712 |
| 14 | 16384 | 0 | 1003096 | 84466 | 1103946 |
| 15 | 119808 | 18432 | 3028032 | 237352 | 3403624 |

TABLE 17. Right 3-omino tilings $m = 6$ [8, A351323]

| $n$ | $T^{\perp}_{7,n}$ | $T^{-}_{7,n}$ | $T^{\vert}_{7,n}$ | $T^{p}_{7,n}$ | $T_{7,n}$ |
|---|---|---|---|---|---|
| 3 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 480 | 0 | 40 | 520 |
| 9 | 0 | 6144 | 0 | 16512 | 22656 |
| 12 | 158208 | 125952 | 112192 | 1399008 | 1795360 |

TABLE 18. Right 3-omino tilings $m = 7$ [8, A351324]

| $n$ | $T^{\perp}_{8,n}$ | $T^{-}_{8,n}$ | $T^{\vert}_{8,n}$ | $T^{p}_{8,n}$ | $T_{8,n}$ |
|---|---|---|---|---|---|
| 3 | 0 | 16 | 0 | 0 | 16 |
| 6 | 256 | 1140 | 0 | 118 | 1514 |
| 9 | 28288 | 35648 | 16064 | 124184 | 204184 |

TABLE 19. Right 3-omino tilings $m = 8$

| $n$ | $T^{\perp}_{2,n}$ | $T^{-}_{2,n}$ | $T^{\vert}_{2,n}$ | $T^{p}_{2,n}$ | $T_{2,n}$ |
|---|---|---|---|---|---|
| 3 | 0 | 1 | 0 | 2 | 3 |
| 6 | 1 | 0 | 8 | 2 | 11 |
| 9 | 1 | 0 | 38 | 2 | 41 |
| 12 | 1 | 0 | 150 | 2 | 153 |
| 15 | 1 | 0 | 568 | 2 | 571 |
| 18 | 1 | 0 | 2128 | 2 | 2131 |
| 21 | 1 | 0 | 7950 | 2 | 7953 |
| 24 | 1 | 0 | 29678 | 2 | 29681 |
| 27 | 1 | 0 | 110768 | 2 | 110771 |
| 30 | 1 | 0 | 413400 | 2 | 413403 |

TABLE 20. Mixed 3-omino tilings $m = 2$ [8, A001835].

| $n$ | $T^{\perp}_{3,n}$ | $T^{-}_{3,n}$ | $T^{\vert}_{3,n}$ | $T^{p}_{3,n}$ | $T_{3,n}$ |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 1 | 1 |
| 2 | 0 | 0 | 1 | 2 | 3 |
| 3 | 0 | 5 | 5 | 0 | 10 |
| 4 | 0 | 0 | 21 | 2 | 23 |
| 5 | 0 | 0 | 60 | 2 | 62 |
| 6 | 17 | 4 | 149 | 0 | 170 |
| 7 | 0 | 0 | 439 | 2 | 441 |
| 8 | 0 | 0 | 1171 | 2 | 1173 |
| 9 | 77 | 4 | 3046 | 0 | 3127 |
| 10 | 0 | 0 | 8264 | 2 | 8266 |
| 11 | 0 | 0 | 21935 | 2 | 21937 |
| 12 | 301 | 4 | 57929 | 0 | 58234 |
| 13 | 0 | 0 | 154388 | 2 | 154390 |
| 14 | 0 | 0 | 409571 | 2 | 409573 |
| 15 | 1137 | 4 | 1085426 | 0 | 1086567 |
| 16 | 0 | 0 | 2882019 | 2 | 2882021 |
| 17 | 0 | 0 | 7645044 | 2 | 7645046 |
| 18 | 4257 | 4 | 20275568 | 0 | 20279829 |

TABLE 21. Mixed 3-omino tilings $m = 3$ [8, A134438].

## 5. Tromino Tiling

The numerical results for tilings with both types of trominoes in any mix or rotations are collected in Tables 20 to 25. [8, A233320]. For $m = 1$ there is only one tiling if $3 \mid n$.

| $n$ | $T^{\perp}_{4,n}$ | $T^{-}_{4,n}$ | $T^{\mid}_{4,n}$ | $T^{p}_{4,n}$ | $T_{4,n}$ |
|---|---|---|---|---|---|
| 3 | 0 | 21 | 0 | 2 | 23 |
| 6 | 255 | 174 | 274 | 236 | 939 |
| 9 | 6133 | 1680 | 24894 | 9106 | 41813 |
| 12 | 122485 | 16934 | 1472282 | 283444 | 1895145 |
| 15 | 2321975 | 175488 | 75446438 | 8265056 | 86208957 |

TABLE 22. Mixed 3-omino tilings $m = 4$ [8, A233339].

| $n$ | $T^{\perp}_{5,n}$ | $T^{-}_{5,n}$ | $T^{\mid}_{5,n}$ | $T^{p}_{5,n}$ | $T_{5,n}$ |
|---|---|---|---|---|---|
| 3 | 0 | 60 | 0 | 2 | 62 |
| 6 | 2350 | 2438 | 1494 | 2060 | 8342 |
| 9 | 221062 | 104717 | 575018 | 369103 | 1269900 |

TABLE 23. Mixed 3-omino tilings $m = 5$ [8, A233340].

| $n$ | $T^{\perp}_{6,n}$ | $T^{-}_{6,n}$ | $T^{\mid}_{6,n}$ | $T^{p}_{6,n}$ | $T_{6,n}$ |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 1 |
| 2 | 1 | 8 | 0 | 2 | 11 |
| 3 | 17 | 149 | 4 | 0 | 170 |
| 4 | 255 | 274 | 174 | 236 | 939 |
| 5 | 2350 | 1494 | 2438 | 2060 | 8342 |
| 6 | 30672 | 21554 | 21554 | 6312 | 80092 |
| 7 | 152969 | 41512 | 304114 | 115986 | 614581 |
| 8 | 1157667 | 218262 | 3047886 | 848108 | 5271923 |
| 9 | 11769903 | 3035480 | 26709232 | 4318146 | 45832761 |

TABLE 24. Mixed 3-omino tilings $m = 6$ [8, A233290].

| $n$ | $T^{\perp}_{7,n}$ | $T^{-}_{7,n}$ | $T^{\mid}_{7,n}$ | $T^{p}_{7,n}$ | $T_{7,n}$ |
|---|---|---|---|---|---|
| 3 | 0 | 439 | 0 | 2 | 441 |
| 6 | 152969 | 304114 | 41512 | 115986 | 614581 |

TABLE 25. Mixed 3-omino tilings $m = 7$ [8, A233343].

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