Polycube Symmetry

Reference: [1] W.F.Lunnon: Symmetry of cubical and general polyominoes, pp. 101-108 of R. C. Read, editor, Graph Theory and Computing. Academic Press, NY, 1972.

For each of the 33 different symmetry classes, [1] gives an illustration of one example of a polycube that satisfies said class. Lunnon states explicitly that the examples are intended to be of minimal size but are not so guaranteed.

The scope of the current document was initially to give the coordinate set for each of Lunnon's examples. In addition, further examples were found, and it was possible to confirm minimality in most cases.

The following table shows, for each symmetry class (I, A, B, …):

1. Its name (defined in [1]);
2. The size of the minimally sized known example;
3. A picture of all known examples, including those identified in [1];
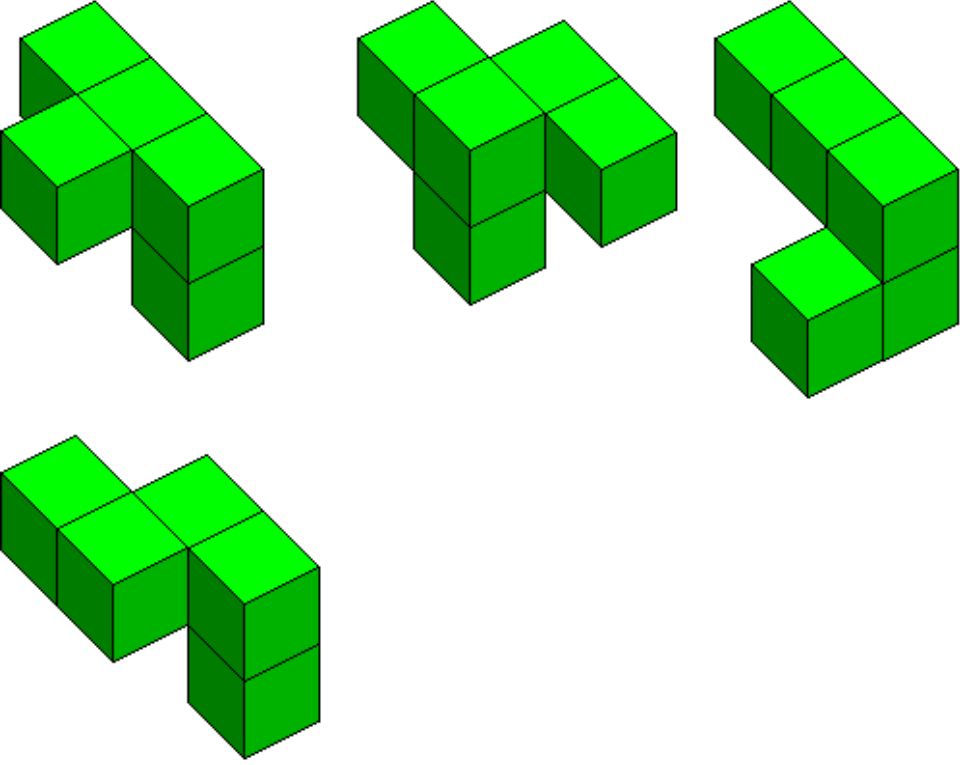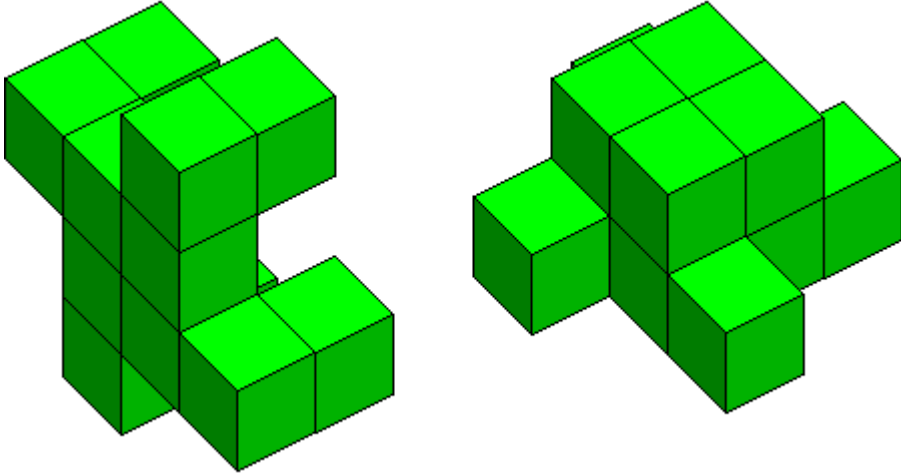4. The corresponding coordinate sets.

By generating all polycubes up to size 13, and of specific symmetries up to size 22, it is possible to state that, excluding classes BD, DEE and R, the table shows all minimal examples.

The examples shown for classes BD, DEE and R are probably minimal, and maybe unique, but the confirmation of these assertions is out of the reach of the programs used.
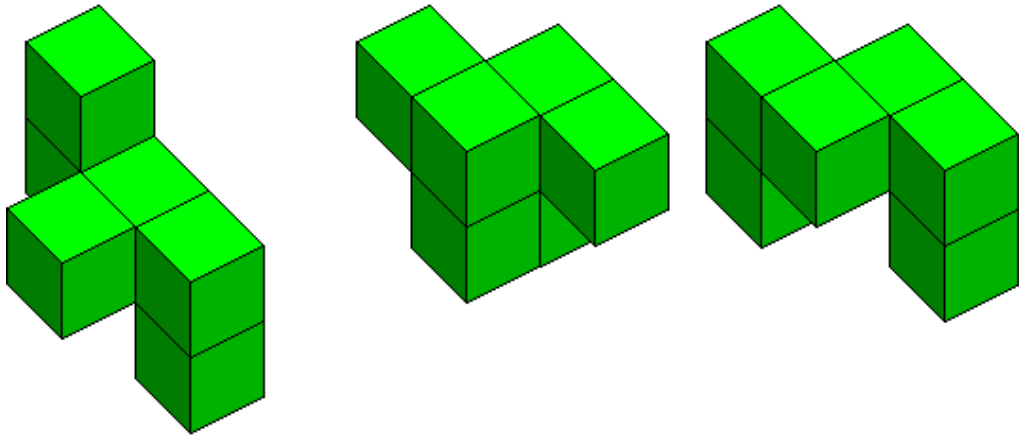

John Mason,

August 2024

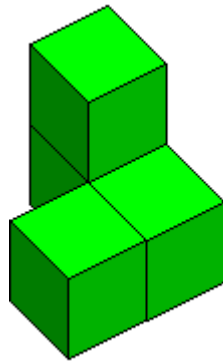| I 5	
	(1,1,2) (1,3,1) (1,2,2) (2,2,2) (1,3,2)
	(2,2,1) (2,1,2) (1,2,2) (2,2,2) (1,3,2)
	(1,1,2) (1,3,1) (1,2,2) (2,3,1) (1,3,2)
	(1,3,1) (2,1,2) (1,2,2) (2,2,2) (1,3,2)
A 12	
	(1,2,1) (2,2,1) (1,1,3) (2,2,2) (2,1,3) (2,3,2) (1,4,2) (2,2,3) (2,4,2) (2,3,3) (1,3,4) (2,3,4)
	(2,1,1) (2,2,1) (1,3,1) (3,2,1) (2,3,1) (2,2,2) (4,2,1) (3,3,1) (3,2,2) (2,3,2) (3,4,1) (3,3,2)

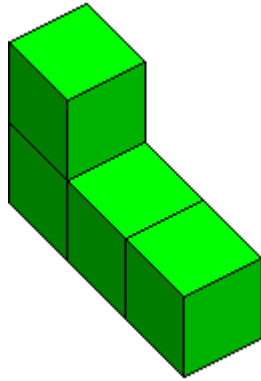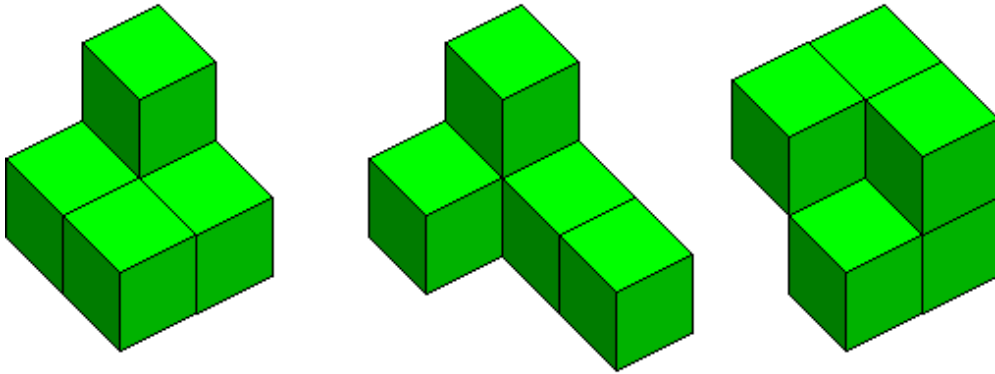| B 6	
(1,1,2) (1,3,1) (1,2,2) (1,1,3) (2,2,2) (1,3,2)	
(1,2,1) (2,2,1) (2,1,2) (1,2,2) (2,2,2) (1,3,2)	
(2,1,1) (1,3,1) (2,1,2) (1,2,2) (2,2,2) (1,3,2)	
C 4	
(1,1,1) (1,2,1) (1,1,2) (2,2,1)	
D 7	
(1,2,1) (1,2,2) (2,2,2) (2,1,3) (2,3,2) (2,2,3) (3,3,2)	
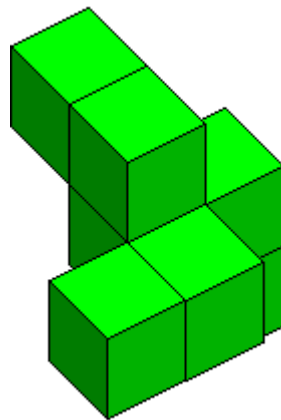
3

E 4	
	(1,1,1) (1,2,1) (1,1,2) (1,3,1)

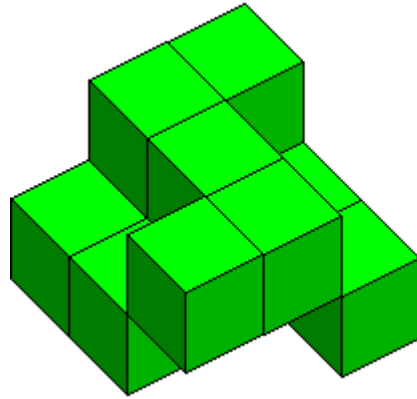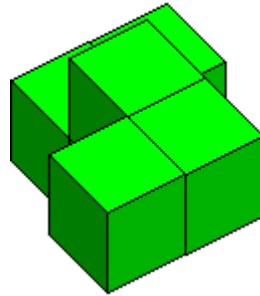| F 5	
	(1,1,1) (2,1,1) (1,2,1) (1,1,2) (2,2,1)
	(1,1,1) (2,1,1) (1,2,1) (1,1,2) (1,3,1)
	(1,2,1) (1,1,2) (2,2,1) (2,1,2) (1,2,2)
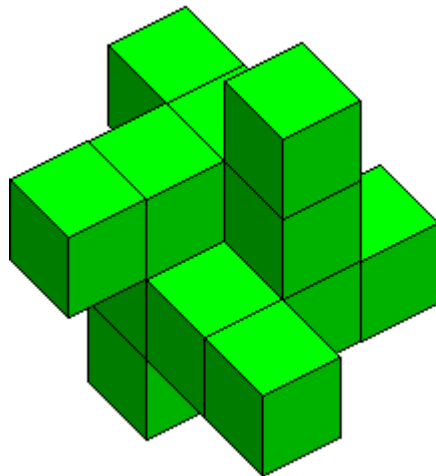
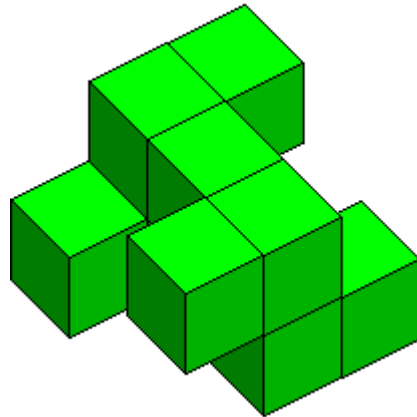| H 12	
	(3,2,1) (1,3,2) (2,1,3) (3,2,2) (2,3,2) (2,2,3) (3,3,2) (3,2,3) (2,3,3) (3,4,2) (4,2,3) (2,3,4)

4

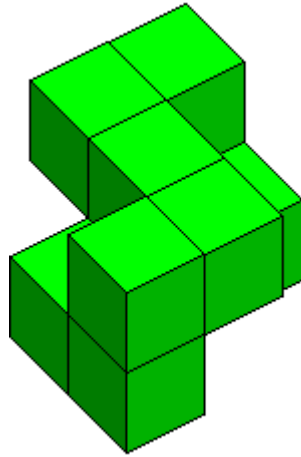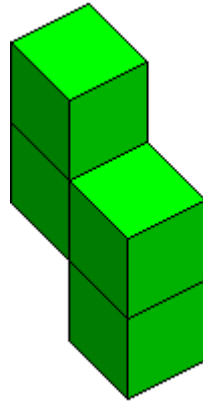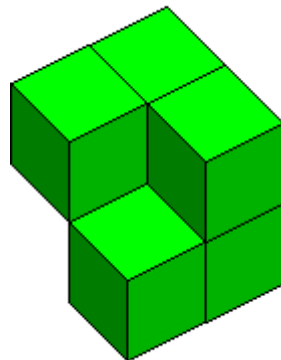| J 10	
(1,2,1) (1,1,2) (3,1,1) (2,2,1) (1,3,1) (2,1,2) (3,2,1) (2,2,2) (2,3,2) (3,3,2)	
K 6	
(1,1,1) (2,1,1) (2,2,1) (2,2,2) (2,3,2) (3,3,2)	
BB 10	
(2,1,1) (1,1,2) (3,1,1) (2,2,1) (1,3,1) (2,1,2) (2,3,1) (2,2,2) (2,3,2) (3,3,2)	

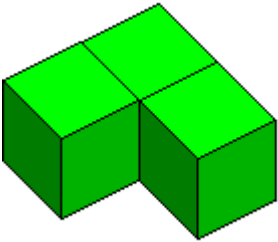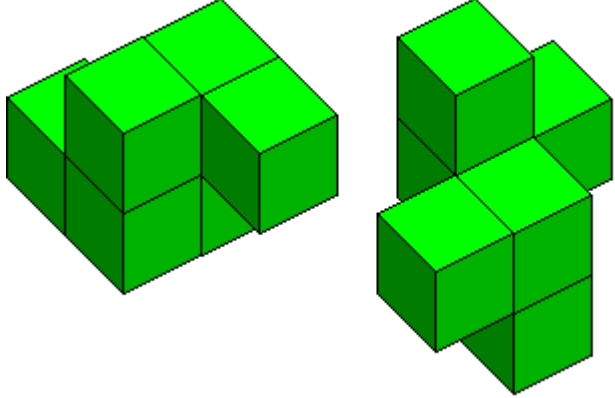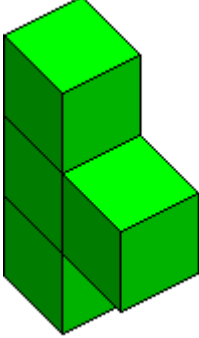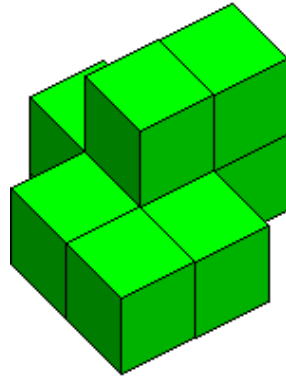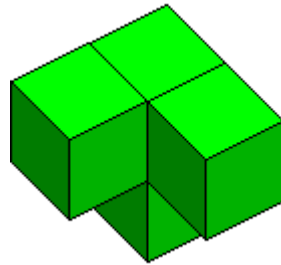| BC 10	
(1,1,1) (1,2,1) (1,1,2) (2,2,1) (2,1,2) (3,2,1) (2,2,2) (3,3,1) (2,3,2) (3,3,2)	
BE 4	
(1,2,1) (1,1,2) (1,2,2) (1,1,3)	
BF 6	
(1,1,1) (1,2,1) (1,1,2) (2,2,1) (2,1,2) (1,2,2)	

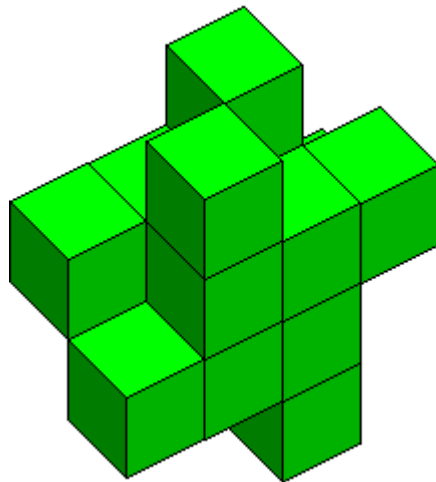| CE 3	
(1,1,1) (2,1,1) (1,2,1)	
CK 6	
(2,1,1) (1,2,1) (2,2,1) (1,2,2) (2,2,2) (1,3,2)	
(1,1,2) (2,2,1) (2,1,2) (2,2,2) (2,1,3) (3,2,2)	
EE 4	
(1,1,1) (1,1,2) (1,2,2) (1,1,3)	

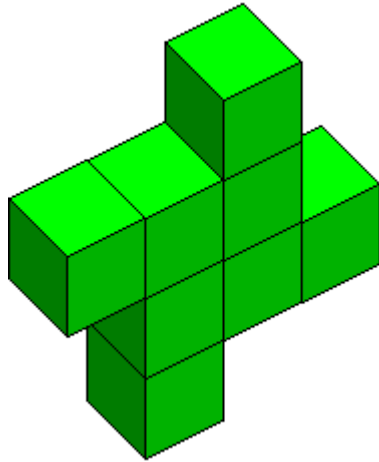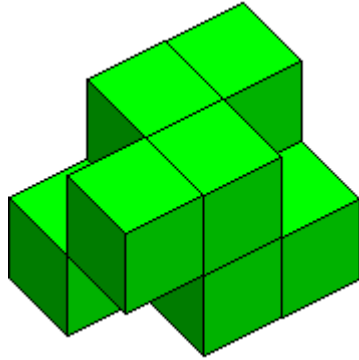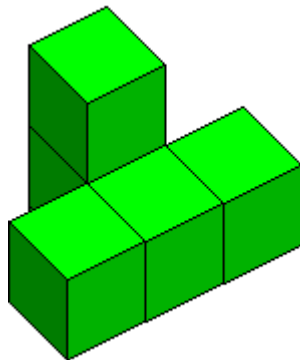| CD 10	
(2,1,1) (2,2,1) (2,1,2) (1,2,2) (2,2,2) (1,2,3) (3,2,2) (2,3,2) (2,2,3) (3,3,2)	
FF 4	
(1,1,1) (1,1,2) (2,1,2) (1,2,2)	
AB 16	
(1,1,2) (3,1,1) (2,2,1) (2,1,2) (3,1,2) (2,2,2) (2,1,3) (1,2,3) (3,2,2) (3,1,3) (2,2,3) (2,1,4) (4,2,2) (4,1,3) (3,2,3) (3,2,4)	

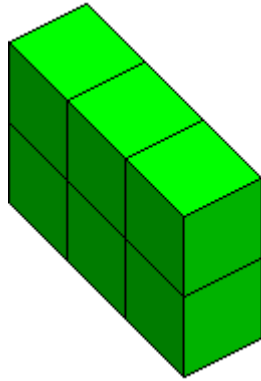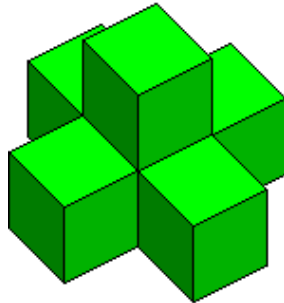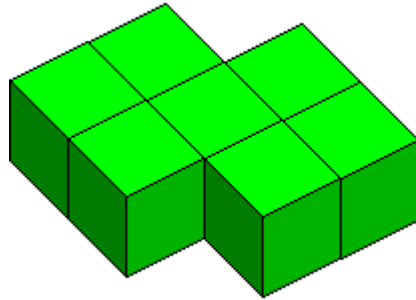| AE 8	
(1,1,2) (3,1,1) (2,1,2) (3,1,2) (2,1,3) (3,1,3) (2,1,4) (4,1,3)	
BFF 8	
(2,1,1) (1,2,1) (1,1,2) (3,1,1) (2,2,1) (2,1,2) (2,2,2) (3,2,2)	
CJ 6	
(2,1,1) (2,1,2) (1,2,2) (2,2,2) (2,1,3) (3,2,2)	

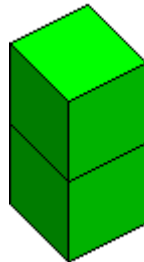| EEE 6	
(1,1,1) (1,2,1) (1,1,2) (1,3,1) (1,2,2) (1,3,2)	
EF 6	
(2,1,1) (1,2,1) (2,2,1) (3,2,1) (2,3,1) (2,2,2)	
EFF 7	
(2,1,1) (1,2,1) (3,1,1) (2,2,1) (1,3,1) (3,2,1) (2,3,1)	
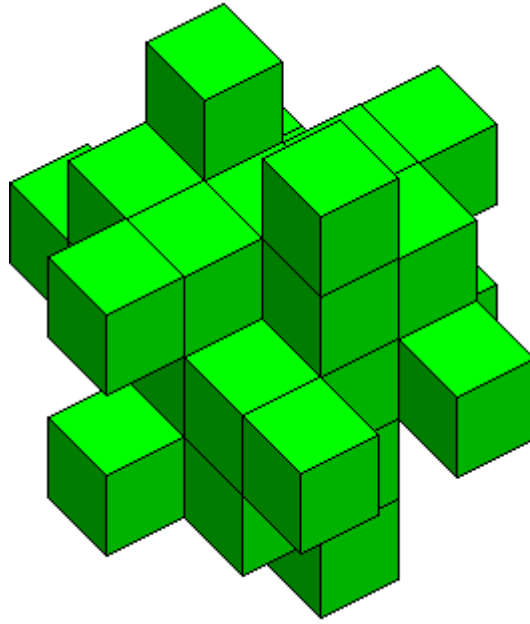
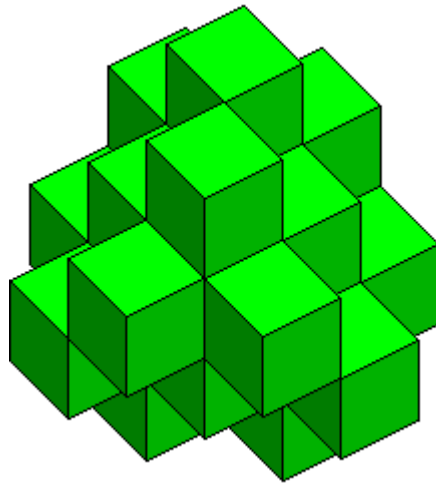|	
BD 34	
(3,2,1) (2,2,2) (1,3,2) (2,1,3) (3,2,2) (2,3,2) (2,2,3) (3,4,1) (3,3,2) (4,1,3) (3,2,3) (2,3,3) (1,3,4) (4,3,2) (3,4,2) (4,2,3) (2,4,3) (3,2,4) (2,3,4) (5,3,2) (4,4,2) (4,3,3) (3,4,3) (2,5,3) (4,2,4) (3,3,4) (2,4,4) (3,2,5) (4,4,3) (4,3,4) (3,4,4) (4,5,3) (5,3,4) (3,4,5)	
CF 6	
(2,1,1) (1,2,1) (1,1,2) (2,2,1) (2,1,2) (1,2,2)	
BBC 2	
(1,1,1) (1,1,2)	
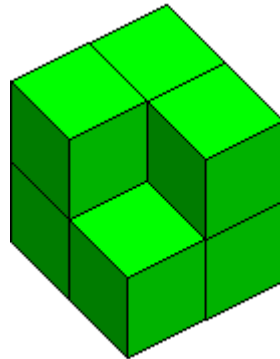
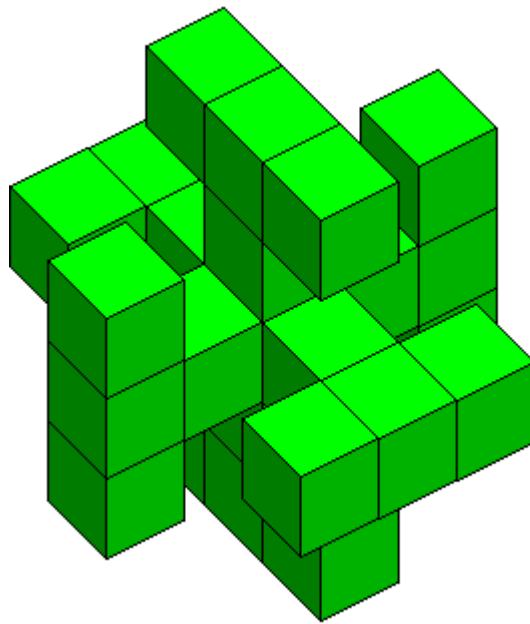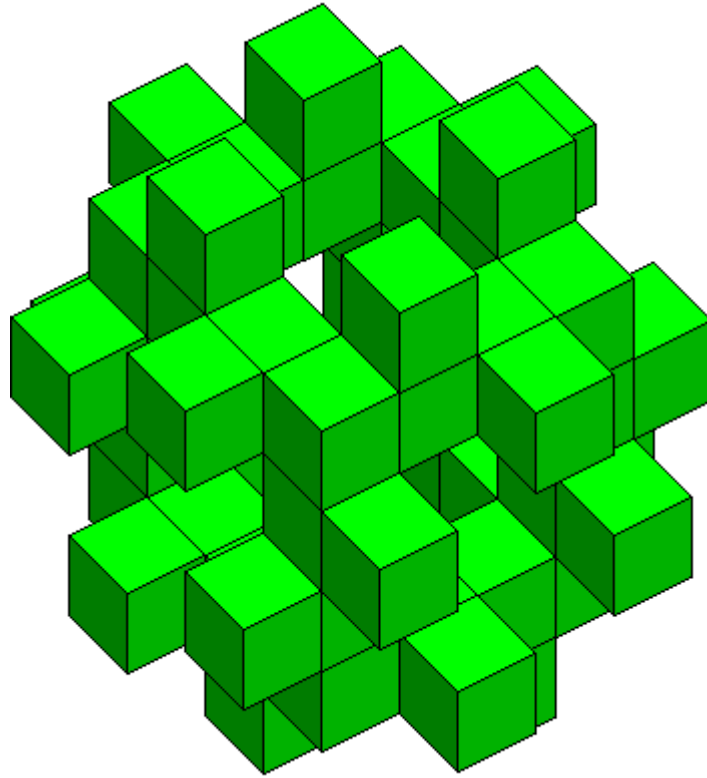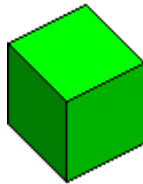|	
CCC 20	
(3,2,1) (2,3,1) (3,1,2) (2,2,2) (1,3,2) (2,1,3) (1,2,3) (3,2,2) (2,3,2) (2,2,3) (4,2,2) (3,3,2) (2,4,2) (3,2,3) (2,3,3) (2,2,4) (3,3,3) (4,3,3) (3,4,3) (3,3,4)	
DEE 25	
(3,2,1) (1,3,2) (2,1,3) (3,3,1) (3,1,3) (1,3,3) (3,4,1) (3,3,2) (4,1,3) (3,2,3) (2,3,3) (1,3,4) (3,3,3) (5,3,2) (4,3,3) (3,4,3) (2,5,3) (3,3,4) (3,2,5) (5,3,3) (3,5,3) (3,3,5) (4,5,3) (5,3,4) (3,4,5)	

R 56	

(2,3,1) (3,1,2) (2,2,2) (1,2,3) (4,2,1) (3,2,2) (2,3,2) (1,4,2) (2,2,3) (2,1,4) (4,2,2) (2,4,2) (2,2,4) (3,5,1)
(5,2,2) (2,5,2) (5,1,3) (2,2,5) (1,3,5) (5,4,1) (5,3,2) (3,5,2) (5,2,3) (2,5,3) (1,5,4) (4,1,5) (3,2,5) (2,3,5)
(6,3,2) (5,4,2) (4,5,2) (2,6,3) (5,2,4) (2,5,4) (4,2,5) (2,4,5) (3,2,6) (5,5,2) (4,6,2) (6,2,4) (5,2,5) (2,5,5)
(2,4,6) (5,5,3) (5,3,5) (3,5,5) (6,5,3) (5,5,4) (5,4,5) (4,5,5) (3,6,5) (5,3,6) (5,6,4) (6,4,5) (5,5,5) (4,5,6)

G 1	

(1,1,1)