

W. Lang, Oct 19 2007

A035206 tabf, a partition number array a(n,k):

a(n,k) enumerates the number of distributions of n identical objects into n distinguishable boxes with m boxes occupied. The occupation of the m=m(n,k):=A036043(n,k) boxes is specified by the k-th partition of n in the Abramowitz-Stegun order. m=m(n,k) is the number of parts of the k-th partition of n.

For the Abramowitz-Stegun (A-St) order of partitions see pp. 831-2 of the reference given in A117506.

n\k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	...
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	3	6	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	4	12	6	12	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	5	20	20	30	30	20	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	6	30	30	15	60	120	20	60	90	30	1	0	0	0	0	0	0	0	0	0	0	0	0
7	7	42	42	42	105	210	105	105	140	420	140	105	210	42	1	0	0	0	0	0	0	0	0
8	8	56	56	56	28	168	336	336	168	168	280	840	420	840	70	280	1120	560	168	420	56	1	
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n\k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	...

The next two rows, for n=9 and n=10, are:

n=9: [9 72 72 72 72 252 504 504 252 252 504 84 504 1512 1512 1512 1512 504 630 2520 1260 3780

630 504 2520 1680 252 756 72 1]

n=10: [10 90 90 90 90 45 360 720 720 720 360 720 360 360 840 2520 2520 1260 2520 5040 840 840

1260 1260 5040 5040 7560 7560 5040 252 1260 6300 3150 12600 3150 840 5040 4200 360 1260 90 1]

The row sums give, for $n \geq 1$: A001700(n-1) = [1, 3, 10, 35, 126, 462, 1716, 6435, 24310, 92378, ...] =

$\text{binomial}(2*n-1, n-1)$.

They coincide with the row sums of triangle A103371.

Example: $a(5,5)$ relates to the partition $(1, 2^2)$ of $n=5$. Here $m=3$, and 5 indistinguishable (identical) balls are put into boxes b_1, \dots, b_5 with $m=3$ boxes occupied, one with one ball and two with two balls. Therefore, $a(5,5) = \text{binomial}(5,3) * 3! / (1! * 2!) = 10 * 3 = 30$.

Note added, July 11 2012

$a(n,k)$ is the order of the color class of necklaces or bracelets with signature determined from the k -th partition of n in A-St order.

E.g., $a(5,6)$ relates to the partition $1^3, 2$, which, in its reversed form $[2, 1, 1, 1]$, is the signature of a color class for multinomials of the form $c[.]^2 * c[.]^1 * c[.]^1 * c[.]$. The $n=5$ colors $c[1], c[2], \dots, c[5]$ can be distributed $a(5,6) = 5 * \text{binomial}(4,3) = 20$ times.

See A212360 and A213941 for the necklace and bracelet numbers, where the present numbers are multiplied by the number of color class representatives A212359 and A231939, respectively.

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