The size of an initial Collatz F(n) is never 2, 4, 5, 7 or 10.

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$|F(n)| = 1$: if $n \not\equiv 1 \bmod 3$

$|F(n)| = 3$: if $n \equiv 1 \bmod 3$ and $\frac{2n-2}{3} \not\equiv 1 \bmod 3$

If $n \equiv 1 \bmod 3$ and $\frac{2n-2}{3} \equiv 1 \bmod 3$ hold then

$|F(n)| = 6$: if also $\frac{4n-10}{9} \equiv 0 \bmod 3$

$|F(n)| = 8$: if also $\frac{8n-20}{9} \equiv 1 \bmod 3$ and $\frac{16n-58}{27} \not\equiv 1 \bmod 3$

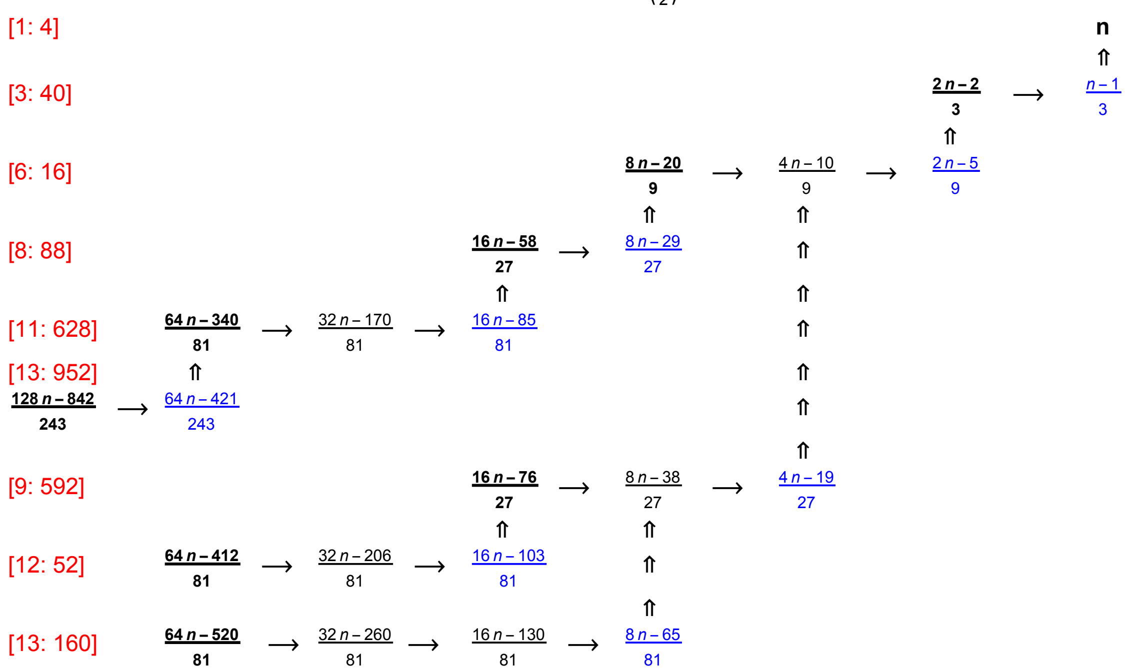
$|F(n)| = 9$: if also $\frac{4n-10}{9} \equiv 1 \bmod 3$ and $\frac{8n-38}{27} \equiv 0 \bmod 3$

Fans of size 9 are the smallest that are the union of two different maximal trajectories to n.

The bold face numbers, all even, are the initial numbers for individual trajectories leading to maximum n depending on which sets of modular conditions are satisfied. The numbers in blue are odd.

The pairs [x:y] signify size x and least maximum y for sets F(y) of size x.

Arrow "⇑" signifies $(3 \times k + 1)$-action and arrow "⟶" signifies $\left(\frac{k}{2}\right)$-action.



If $n \equiv 1 \bmod 3$ and $\frac{2n-2}{3} \equiv 1 \bmod 3$ hold then

$|F(n)| = 11$: if also $\frac{8n-20}{9} \equiv 1 \bmod 3$, $\frac{16n-58}{27} \equiv 1 \bmod 3$ and $\frac{32n-170}{81} \equiv 0 \bmod 3$

$|F(n)| = 12$: if also $\frac{4n-10}{9} \equiv 1 \bmod 3$, $\frac{16n-76}{27} \equiv 1 \bmod 3$ and $\frac{32n-206}{81} \equiv 0 \bmod 3$

$|F(n)| = 13$: either if also $\frac{4n-10}{9} \equiv 1 \bmod 3$, $\frac{8n-38}{27} \equiv 1 \bmod 3$ and $\frac{16n-130}{81} \equiv 0 \bmod 3$

or if also $\frac{8n-20}{9} \equiv 1 \bmod 3$, $\frac{16n-58}{27} \equiv 1 \bmod 3$ and $\frac{64n-340}{81} \equiv 1 \bmod 3$

There are two types of fans of size 13, one is a single trajectory to the maximum and the other consists of the union of maximal trajectories to n starting at three different initial values.

There are 6 possible extension points for fans in the partial tree drawn above since numbers $\frac{64n-520}{81}$ and $\frac{16n-130}{81}$ form the first pair on a single $\left(\frac{k}{2}\right)$-leg that are extendable in a fan simultaneously.