

Automated Proof (or Disproof) of Linear Recurrences Satisfied by Pisot Sequences

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[.pdf](#) [.ps](#) [.tex](#)

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Written: Sept. 18, 2016

Dedicated to Richard K. Guy (b. 30 Sept., 1916) on his (100- ϵ)-th birthday

As far as I know, Richard Guy is the longest-living mathematician of all time (in the sense of Paul Erdős, where you die when you stop doing math), since at almost one hundred years of age, he keeps producing great new math. This tribute, joint with Neil Sloane, is dedicated to him. Keep up the good work, Richard!

Added Sept. 21, 2016: Tomas Oliveira e Silva sent us many more dramatic instances of Pisot sequences that satisfy linear recurrences that break down after a VERY long time, and he kindly permitted us to post it [here](#) .

Added Oct. 6, 2016: Tomas Oliveira e Silva sent us even more dramatic instances of Pisot sequences that satisfy linear recurrences that break down after a VERY long time, and he kindly permitted us to post it [here](#) .

Added Oct. 5, 2016: Here is Richard Guy's response to this paper that has been dedicated to him.

Dear Neil & Doron,

What a magnificent present!

Sorry I haven't replied earlier. But I haven't replied to the Queen, nor the Lieutenant-Governor nor Justin Trudeau, either! All the best! R.

Added Nov. 10, 2016: Tomas Oliveira e Silva broke yet another record the linear recurrence with $a_0=605$, $a_1=36312$, and with $r=1/2$, fails for $a_{13138671}$

Added Sept. 25, 2017: Robert Dougherty-Bliss, wrote a [Python version](#), and here is the [documentation](#).

Maple package

- **[Pisot.txt](#)**, a Maple package to discover, prove, and disprove, linear recurrence relations satisfied by Pisot sequences, first discovered and discussed by David Boyd, D.G. Cantor, and his student P. Galyean.
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Sample Input and Output Files for the Maple package Pisot.txt

- If you want to see linear recurrences, with COMPLETE proofs of Pisot sequences of the form PISOT(a,b) (with parameter 1/2) for $2 \leq a < 20$, and $a+2 \leq b < 40$

the [input](#) file generates the [output](#) file.

- If you want to see PROVED linear recurrences, (w/o proofs, to save disk-space) of Pisot sequences of the form PISOT(a,b) (with parameter 1/2) for $2 \leq a < 30$, and $a+2 \leq b < 600$

the [input](#) file generates the [output](#) file.

- If you want to see PROVED linear recurrences, (w/o proofs, to save disk-space) of Pisot sequences of the form PISOT(a,b) (with parameter 1/2) for $2 \leq a < 40$, and $a+2 \leq b < 1000$

the [input](#) file generates the [output](#) file.

- If you want to see a detailed, fully rigorous proof that the Pisot Sequence E(4,7), alias [OEIS sequence A010901](#) does indeed satisfy (for ALL n, not just ≤ 50000) the recurrence

$$a(n) = 2a(n-1) - a(n-2) + a(n-3)$$

the [input](#) file generates the [output](#) file.

- If you want to see several examples of Generalized Pisot sequences that satisfy linear recurrences, along with their terse proofs

the [input](#) file generates the [output](#) file.

- If you want to see many infinite families of Pisot sequences that satisfy linear recurrences

the [input](#) file generates the [output](#) file.

- If you want to see the same output, but in TeX

the [input](#) file generates the [output](#) file.

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