(1.3-14) in slightly modified form, from the Poisson moment generating function.

The first seven nonzero moments about the mean are:

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$$M_2 = \lambda, \tag{1.3-16}$$

$$M_3 = \lambda, \tag{1.3-17}$$

$$M_4 = \lambda + 3\lambda^2, \tag{1.3-18}$$

$$M_5 = \lambda + 10\lambda^2, \tag{1.3-19}$$

$$M_6 = \lambda + 25\lambda^2 + 15\lambda^3, \tag{1.3-20}$$

$$M_7 = \lambda + 56\lambda^2 + 105\lambda^3, \tag{1.3-21}$$

$$M_8 = \lambda + 119\lambda^2 + 409\lambda^3 + 105\lambda^4. \tag{1.3-22}$$

These results appear in an anonymous paper (1930) in the first volume of the *Annals of Mathematical Statistics*; probably they were calculated by Carver. The first six are quoted by Kendall and Stuart (1958). Riordan (1937) provides the recursion relationship

$$M_{k+1} = \lambda k M_{k-1} + \lambda\left(\frac{d}{d\lambda}\right)M_k \tag{1.3-23}$$

and the expansion

$$M_k = \sum_{i=0}^{k} \sigma_{i:k}\lambda^i \tag{1.3-24}$$

in terms of the coefficients

$$\sigma_{n:s} = \frac{1}{n!}\sum_{i=0}^{n}(-1)^{n-i}\binom{n}{i}(i-\lambda)^s \tag{1.3-25}$$

$$= \frac{1}{n!}\Delta^n(-\lambda)^s. \tag{1.3-26}$$

These coefficients satisfy the recursion formula

$$\sigma_{n:s+1} = (n-\lambda)\sigma_{n:s} + \sigma_{n-1:s} \tag{1.3-27}$$

and can be expressed in terms of the Stirling numbers of the second kind:

$$\sigma_{n:s} = \sum_{i=0}^{s-n}(-1)^i\binom{s}{i}S_{n:s-i}\lambda^i. \tag{1.3-28}$$

Furthermore, Riordan (1937) expresses the moments in the form

$$M_k = \sum_{i=0}^{[\frac{1}{2}k]} \alpha_{i:k}\lambda^i, \tag{1.3-29}$$

where

$\alpha_{i:k+1} = i\alpha_{i:k} + k\alpha_{i-1:k-1}$