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23 September 1992

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Subject: A Handbook of Integer Sequences
On a new Iterated Prime Number Series

Dear Dr. Sloane,

In the above referenced text, the Primes and recurrences occuppy a large portion of the index.

Although iteration and its brother recursion dates back several centuries, in this last decade it has enjoyed equal footing with other numerical methods, I believe I can safety say that this phenomenon is due for the most part to the current interest in "chaos." Being in vogue has brought many other fringe elements into the forefront.

Many processes use iteration as a technique for problem solving. Others are employed for their own sake, as a mathematical amusement. In the former class we have the famous Newton-Raphson method of root extraction (this is not to be confused with the dental proceedure). An example of the later is the famous 3*X +1 or - 2 problem. In the Newton method you have a process which you can observe the speed of convergence, in this case quadratically, to the true answer. In the later the number of iterations it takes for an integer to cascade to one wildly oscillates and today is still an unsolved number theory problem.

The series I wish to present is, I'm sure, of the later, although its progression is much better defined than the Collatz function. The audience is well acquianted with $\pi(x)$, hereinafter denoted as pi(x), which is the arithematic function that counts the number of primes less than or equal to the integer x, with the number one being defined as neither a prime nor a composite number. As an example, the pi(10)=4, they being the primes 2, 3, 5 & 7. There is that noteworthy theorem which states that the Limit of pi(x) as x approaches infinity is the quotient of x divided by the natural logarithm of x. Also by the definition itself, it is obvious that x always exceeds the pi(x). Unlike the hailstone problem though, we know that taking succesive iterations of the pi(x_i) -> x_{i+1} and counting the number of iterations, we always know that this prodeedure will terminate at Zero. Lets call this new function the Q(n) and define it as the number of steps (iterations) we must perform the pi(x) before pi(x)=0. Examples, the Q(10)=4 because pi(10)->4, pi(4)->2, pi(2)->1 and pi(1)->0. We performed the operation pi() four times.

Now let us investigate the growth of Q(n). Since the plot of the pi(x) is relatively smooth, should we not expect the Q(n) to behave similarly? Second, at what values does the Q(n) increment from a lower value to a higher value? The seccesive values of n, the Q(n) will stay the same or increase by just one; it can never go down. The reader is left with his own proof. It is similar to the proof that of pi(x) behaves in the similar manner.

In fact, it can be shown that the first time that Q(n)= whatever, can be generated by the inverse function of pi(x), that is P_i , i being the index. Threfore, we have the following obviously incomplete table:

X	P_i
1	0
2	1
3	2
4	3
5	5
6	11
7	31
8	127
9	709
10	5,381

number = new index

0	1
1	2
2	3
3	5
4	11
5	31
6	127
7	709
8	5,381
9	52,711

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Please enter:

11	52,711	648,391
12	648,391	9,737,333
13	9,737,333	174,440,041
14	174,440,041	3,657,500,101
15	3,657,500,101	88,362,852,307
16	88,362,852,307	2,428,096,940,717
17	2,428,096,940,717	~ 74,900,000,000,000
18	< 74,900,000,000,000	~ 2,560,000,000,000,000

references:

M. Abramowitz and I. Stegun, "Hdbk of Math Fnctns." p870-873
Solomon W. Golomb, E 3385 [1990,427] The Am. Math. Mo. v98n9p858-9, Nov 91.
D. Knuth, "The Art of Computer Programming," v2p365
Hans Riesel, "Prime Numbers and Computer Methods for Factorization"
Stephen Wolfram "Mathmatica," the function Prime[]

Sequentially yours;

Robert G. Wilson

Robert G. Wilson v
Ph.D., ATP/CF&GI

RGWv:hp110+

%N #a(n+1) ~=~ a(n)#-th prime.

%R ~~rgw~~

%O 0,2

%A rgw.