# Some Results of Research in Computational Number Theory


## Dr. Thomas R. Nicely
## (1943-2019)

**http://faculty.lynchburg.edu/~nicely**

**University of Lynchburg Department of Mathematics**


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- Windows Vista may cause memory allocations exceeding 32MB to fail
- Finding DJGPP 2.04
- Other works of the author
- E-mail security alert
- Proprietary marks: Disclaimer
- Links

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**NOTES:** For simplicity, numbers of very large or very small magnitude, appearing in some documents on this site, may be written using the floating-point notation of FORTRAN and C. For example, 56e15 means the same thing as 56000000000000000, 5.6*10^16, 5.6·10^16, 5.6e16, $5.6{\cdot}10^{16}$, $5.6{\times}10^{16}$, etc. However, in some bibliographic references, such a number may be rendered in TeX style, thus: $5.6 \times 10^{16}$. Also, the default on this site is that K and KB equal 1000 bytes; M and MB equal 1000000 bytes; G and GB equal 1000000000 bytes, etc. The FORTRAN/COBOL notation 2**64 (rather than 2^64) is also preferred; see here for a discussion.

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## DESCRIPTION OF RESEARCH

Code written primarily in GNU C, and distributed asynchronously across available personal computers running under extended DOS, Windows, and GNU/Linux, is employed to enumerate primes, prime gaps, prime constellations (twins, triplets, and quadruplets) and their reciprocal sums (to extrapolate estimates for the corresponding Brun's constants). Some related computational results obtained by other researchers are also reported here.

## PAPERS (Unpublished)

- The Baillie-PSW primality test. Includes GNU C source and executable for implementing both the standard and strong versions of the Baillie-PSW and Lucas-Selfridge primality tests, as well as the extra strong Lucas test. Original posting 10 June 2005.
- GNU GMP mpz_probab_prime_p pseudoprimes. Counterexamples for the GNU GMP primality testing function. Original posting December 2004.
- "New evidence for the infinitude of some prime constellations" (20 July 2004).
- "Enumeration to 1.6e15 of the prime quadruplets" (23 August 1999).
- "Enumeration to 1.6e15 of the twin primes and Brun's constant" (16 July 1999).
- "First occurrence of a prime gap of 1000 or greater," Thomas R. Nicely and Bertil Nyman (21 May 1999). The addendum includes listings of the largest prime gaps (deterministic and probabilistic) found to date.

## PAPERS (Published)

- "New prime gaps between 1e15 and 5e16," Bertil Nyman and Thomas R. Nicely, Journal of Integer Sequences 6 (2003), Article 03.3.1, 6 pp. (electronic). MR1997838 (2004e:11143). Published 13 August 2003. Available in various formats (DVI, PS, PDF, LaTeX) at the Journal of Integer Sequences.

- "A new error analysis for Brun's constant," Virginia Journal of Science 52:1 (Spring, 2001) 45-55, MR 1853722 (2003d:11184).
- "New maximal prime gaps and first occurrences," Mathematics of Computation 68:227 (July, 1999) 1311-1315, MR 1627813 (99i:11004).
- "Enumeration to 1e14 of the twin primes and Brun's constant, " Virginia Journal of Science 46:3 (Fall, 1995) 195-204, MR 1401560 (97e:11014).

## TABLES OF PRIME GAPS

- A listing of all first occurrence, maximal, and first known occurrence prime gaps of 1 to 1998, as well as all other prime gaps exceeding 999 which lie below 5e16.
- Tables of first known occurrence prime gaps, of measures:
  - 2000 to 3998.
  - 4000 to 5998.
  - 6000 to 7998.
  - 8000 to 9998.
  - 10000 to 14998.
  - 15000 to 19998.
  - 20000 to 24998.
  - 25000 to 29998.
  - 30000 to 34998.
  - 35000 to 39998.
  - 40000 to 44998.
  - 45000 to 49998.
  - 50000 to 54998.
  - 55000 to 59998.
  - 60000 to 69998.
  - 70000 to 79998.
  - 80000 to 99998.
  - 100000 to 149998.
  - 150000 to 199998.
  - 200000 to 999998.
  - 1000000 to 999999998.
- Note that the above tables display truncated forms of initiating primes which exceed 200 characters in length. However, the zipfile merits.zip (637K) contains a Win/DOS text file specifying the measure G and the merit M=G/ln(p_1) for all presently known first occurrence and first known occurrence prime gaps. This file should be of assistance in determining whether or not some newly discovered gap constitutes a new first known occurrence.
- The complete, untruncated listing of all presently known first occurrence and first known occurrence prime gaps is available as allgaps.dat (9 MB), a Win/DOS text file describing one gap per line, in the standard format. Note that some of the lines are **VERY** long, and will challenge most editors and file utilities (the file is intended primarily as an input file for computer processing).

- Detailed instructions for submitting prime gaps are provided. Note that submissions via multimedia (video, audio, images, etc.) and social media (Twitter, Facebook, YouTube, etc.) are not accepted; also, proprietary formats (HTML, Word, Excel, PDF, rich text, etc.) should be avoided (send **plain text files** or **zipfiles**).
- The prime gap listings were last updated 0600 GMT 12 August 2019.

## OTHER TABLES

- A zipfile (111MB) containing complete counts and reciprocal sums of the prime constellations from Nicely's computations (1993-2009). Note the very large size of the zipfile; its data files include more than two million data points from 0 to 2e16.
- A table of π(x), the count of primes, with related functions. Values to 1e16 at intervals of 1e12.
- A table of π(x), the count of primes, with related functions, for 1e16 <= x <= 2e16.
- A table of pi_2(x), the count of twin-prime pairs, with related functions, to 1e16 at intervals of 1e12.
- A table of pi_2(x), the count of twin-prime pairs, with related functions, for 1e16 <= x <= 2e16.
- A table of pi_3a(x), the count of prime triplets (q, q+2, q+6), with related functions, to 1e16 at intervals of 1e12.
- A table of pi_3a(x), the count of prime triplets (q, q+2, q+6), with related functions, for 1e16 <= x <= 2e16.
- A table of pi_3b(x), the count of prime triplets (q, q+4, q+6), with related functions, to 1e16 at intervals of 1e12.
- A table of pi_3b(x), the count of prime triplets (q, q+4, q+6), with related functions, for 1e16 <= x <= 2e16.
- A table of pi_4(x), the count of prime quadruplets (q, q+2, q+6, q+8), with related functions, to 1e16 at intervals of 1e12.
- A table of pi_4(x), the count of prime quadruplets (q, q+2, q+6, q+8), with related functions, for 1e16 <= x <= 2e16.
- Tomás Oliveira e Silva has computed the most extensive tables of π(x) and pi_2(x) of which I am aware, including values of π(x) for x up to 1e23.
- Chris K. Caldwell maintains at his Prime Pages an extensive compilation of values of π(x).
- Xavier Gourdon, Pascal Sebah, and Patrick Demichel have computed the value of π(x) for some extremely large values of x (e.g., 4e22).
- The largest value of x for which π(x) has been computed exactly is x=1e27; specifically,

```
π(10^27) = 1635246042684168044642739 9
```

The computations by David Baugh and Kim Walisch were completed 6 September 2015. Their verifications were completed 9 May 2016. See OEIS A006880 and the Mersenne Forum for details. (Thanks to Rodolfo Ruiz-Huidobro for this information)

## PENTIUM FDIV FLAW

- A personal FAQ regarding the Pentium division flaw. Bibliography attached. Last updated 0900 GMT 19 August 2011.
- Original e-mail message announcing the discovery of the Pentium division flaw, 30 October 1994.
- An account by Richard M. Smith, President of Phar Lap Software, Inc., of the spread of the Pentium flaw announcement across the Internet during the first few days.
- pentbug.zip, a zipfile containing the C source code (pentbug.c) and corresponding DOS executables (pentbug.exe and bug16bit.exe) for a program which will check for the flaw.
- The Pentium division flaw. Thomas R. Nicely. Virginia Scientists Newsletter, Volume 1 (April, 1995), p. 3.
- Untitled article concerning the Pentium division flaw. Thomas R. Nicely. San Francisco *Examiner* (18 December 1994), p. B-5.

## OTHER WORKS

- Problem Proposal #1109, Mathematics Magazine 53:5 (November, 1980), 300 (with solution), "When will spring next begin on March 21st in the United States?" (Answer: 2103 A.D.)
- "Calculation of the Gregorian Easter cycle," public lecture (October, 1977). The period of Easter in the Gregorian calendar, as presently calculated by the Roman Catholic and Protestant churches, was shown to be 5,700,000 years. The zipfile easter1.zip contains GNU C source code and a DOS/Wintel executable for calculating the dates of Easter Sunday.
- "Special techniques for the solution of a singular integral equation," doctoral dissertation, applied mathematics, University of Virginia, Charlottesville, 1971. Advisor: Gordon E. Latta.
- "Electronic structure of open-shell doublet-state molecules: application to CN," master's thesis, theoretical physics, West Virginia University, Morgantown, 1965. Advisor: Harvey N. Rexroad.
- The PAYDIRT and BOWL BOUND football simulation board games (see below).
- See Downloads for free software.

## PAYDIRT AND BOWL BOUND

The following information is provided in response to numerous inquiries.

For most of the period from 1977 to 1995, I carried out design and development for the football simulation board games Paydirt (pro) and Bowl Bound (college), produced and distributed commercially by Avalon Hill Game Company (Baltimore, Maryland) and Sports Illustrated Enterprises. Commercial support of these games was suspended in April, 1995, and I retired from development in February, 1996. Avalon Hill Game Company was later acquired by Hasbro, Inc., and commercial design, production, and distribution of both games was suspended indefinitely. It appears that Hasbro retains the rights to both games at this time.

Transcripts of these charts may be available from various other parties. I do not authorize, forbid, or restrict sales or distribution by such parties, known or unknown, for profit or not. Since I am not a participant or stakeholder in such operations, I do not accept legal responsibilty or liability for such sales or products.

Please note that I have declared all of my own Paydirt and Bowlbound charts, as well as all related developmental materials, to be in the public domain. However, certain hostile parties dispute my right to do this.

Team charts produced by other parties, whether or not based on my developmental materials, are their own intellectual, legal, and financial property, and are not subject to my declaration of public domain. They are entitled to their own copyrights and authorship notices.

Incidentally, the 1984, 1985, 1986, and 1987 Paydirt team charts (as shipped by Avalon Hill) were *not* my work...despite the fact that my name appears (unauthorized) on many of them.

Please do not contact me regarding the Paydirt or Bowlbound charts or materials. The above exposition contains all that I have to say about the subject.

## NEW LARGEST KNOWN PRIME GAP

Martin Raab has discovered a new first (and largest) known occurrence prime gap of measure G=6582144 following the 216841-digit prime P1=499973#/30030 - 4509212 (where 499973# indicates the product of all primes from 2 through 499973 inclusive). This gap was first reported by Raab on 01 July 2017. The endpoints have passed the strong BPSW test (Nicely, 04 September 2017) for probabilistic primality. All the interior integers have been demonstrated composite (18 August 2017) by ATH on the Mersenne Forum. A test for deterministic certification of primality is at present out of the question. The gap has merit M=13.182884.

## NEW PRIME GAP OF MAXIMUM KNOWN MERIT

The Gapcoin network (Jonnie Frey, developer), a Bitcoin derivative which employs a hashing algorithm to search for prime gaps of high merit, has discovered a new prime gap of maximum known merit, a gap of G=8350 following the 87-digit prime
P1=29370323406802259015872376610441946342570907557481176209858879821789572885867677
The merit M=G/ln(P1) of this gap is M=41.93878373153988, the largest merit of any known prime gap, and the first prime gap to be discovered with a merit exceeding 40. The endpoints of the gap have been certified as primes deterministically, using the Akiyama-Kida-O'Hara UBASIC implementation (1988-1992) of the APRCL2 test, due to Adleman, Pomerance, Rumely, Cohen, H. W. Lenstra, and A. K. Lenstra (1984-1987).

However, Bertil Nyman's maximal gap of 1132, following the prime 1693182318746371 (discovered 24 January 1999), continues to exhibit the greatest known value (0.92063858855742) of the Cramér-Shanks-Granville ratio $G/\ln^2(p\_1)$; this ratio is 0.210642105494715467 for the new Gapcoin gap. The limit superior of this ratio has been conjectured to be unity (or some even larger value); see the discussion in "New prime gaps between 1e15 and 5e16".

Thanks to Dana Jacobsen for alerting me to the discovery of this gap.

On 08 May 2019, Robert W. Smith discovered a new first known occurrence prime gap of 203890 following the 2485-digit prime 140207*5813#/46410 - 86644. This gap has merit 35.640174363, the greatest merit for any known prime gap exceeding 26892. On 08 July 2019, Smith also discovered a new first known occurrence prime gap of 614640 following the 10004-digit prime 281*23173#/46410 - 267338. This gap has merit 26.6845515588753865, the greatest merit for any known prime gap exceeding 556982. An extended table of previous such gaps, due to Robert W. Smith and axn, is available on the Mersenne PGS forum.

## NEW MAXIMAL PRIME GAPS OF 1530 AND 1550

As a result of the continuing extension of the upper bound of exhaustive scans for prime gaps, the first known occurrence prime gaps of 1530 and 1550, following respectively the primes 17678654157568189057 and 18361375334787046697, and discovered respectively 19 April 2014 and 13 July 2014 by the late Dr. Bertil Nyman, have now been confirmed (13 August 2018) as first occurrence prime gaps and maximal prime gaps. Nyman's maximal prime gap of 1550 is the largest maximal prime gap presently known.

The merit of Nyman's new maximal prime gap G=1550 is M=34.9439.

## E-MAIL SECURITY ALERT

My current e-mail address is always available elsewhere on this site.

If you receive an e-mail claiming to be from my address (or some slight variation of my address), which is threatening, abusive, solicitous, commercially oriented, questionable in nature, or otherwise suspicious, treat it as a fraudulent act of vandalism on the part of some third party; ignore its contents and delete it! **I DID NOT SEND IT!**

Be aware that malicious parties and spammers frequently spoof legitimate e-mail addresses, including my own, using forged headers. My own e-mails will always have distinctive identification headers, aside from those inserted by the mail provider. On the rare occasions when I send attachments with e-mails, it will be with the prior permission of the recipient, or there will be a clear explanation within the message of the contents of the attachment. Furthermore, I never include active links, embedded images, JavaScript, VBScript, or Active-X controls in e-mail (although the e-mail providers, such as Hotmail, might add such features without my permission, just as they append commercial footers without warning).

If possible, send your e-mail messages as **plain text**; avoid HTML and rich text, especially in e-mails containing data to be processed. Attachments and large data files should be sent as zipfiles (this protects the contents from corruption by the mailers). Please **DO NOT** send embedded images (jpg, gif, bmp, etc.) in your messages, as these constitute a security hole for viruses and worms, and create a serious bottleneck in e-mail processing. If such images are deemed critical, send them in separate zipped attachments.

I have provided detailed instructions for submitting lists of prime gaps.

Make sure that your subject line is to the point---otherwise, your message might be deleted, unread, as likely spam. Also, if you are seeking information or advice, please send, on your own behalf, a clear and concise explanation of the question or problem. Ordinarily, I will not reply to carbon copies, inquiries by a third person on behalf of others, or unsolicited transcripts of conversations, dialogues, or group discussions to which I was not party.

If your zipfiles or other attachments are extremely large (over 10MB), I do not advise sending them via e-mail. For such extremely large files, provide instead a pointer to a website from which I can download the file.

## DOWNLOADS

- **NOTE**: These applications are distributed as freeware, copyright © 2019 Dr. Thomas R. Nicely, released into the public domain by the author, who disclaims any legal liability arising from their use. All are 32-bit console (terminal, non-GUI, tty, command-line, shell) applications, optimized for a window size of 80x25 or greater. Unless otherwise stated, any source code provided is in GNU C (4.5.2 or later), including the GMP 5.0.2 and MPFR 3.0.1 libraries (earlier versions of these packages may or may not require modifications of the source codes). Primary development and testing are carried out in the 32-bit Windows x86 environment, on 32-bit standalone machines with administrative privileges, using the MinGW/MSYS compilers and development environment (version 28 January 2011). Any executables provided are native to 32-bit Windows (98SE and later). However, efforts are being made to maintain portability to other compilers and platforms, including GNU/Linux (SUSE 10.x as root user), Cygwin, Digital Mars, DJGPP, and Borland (version 5.51). Compatibility with these compilers and platforms is in some cases limited by their lack of support for GNU extensions, C99, C0x, GMP, MPFR, the long double data type, various glibc functions, the conio functions of DOS/DJGPP/Borland, and by non-standard interfaces to 64-bit integers. Support for compilers and platforms other than MinGW/MSYS/Windows is at an early beta stage, and will be extended in breadth and depth as time and resources allow. Support for MSVC, Cygwin, and 64-bit compilers is only at alpha level.

  Compilation of the sources will typically require a command line such as

  ```
  gcc xxx.c trn.c conio3.c -std=gnu99 -lm -lmpfr -lgmp -oxxx.exe
  ```

  where xxx.c is the name of the main source file; the exact command line parameters will depend upon your operating environment and the specific code being compiled. The support library trn.c (and its header trn.h), and the GMP library (4.3.1), will be needed for the great majority of the codes; MPFR (2.4.1) will be required for some applications; while the support library conio3.c (and its header conio3.h) will only be required if the code calls conio console functions (such as gotoxy, wherex, etc.) and is being compiled outside of DJGPP and Borland C. No makefiles are required. The extensions of the gnu99 standard (including most of C99) are used throughout these codes.

  If you are *not* linking with GMP or MPFR, you will need to append the qualifiers `-D__NOGMP__` and/or `-D__NOMPFR__` on the command line.

Note that, in general, if you wish to recompile the codes or examine the source code of the support routines, you will need to **download trn.zip separately in order to obtain the files trn.c and trn.h** (a few packages still include older, dedicated versions of these support files).

- trn.zip, a zipfile (61K) containing the latest revisions of the source code (trn.c) and header file (trn.h) for the support routines called by many of the downloadable applications listed below (some of the applications include their own support files, or are self-contained). Multiple platforms. Last updated 0010 GMT 05 October 2018.
- pix4.zip, a zipfile (58K) containing the source code and a Wintel executable for calculating π(x) using the Legendre-Meissel-Lehmer algorithm. Command-line syntax: pix4 [LB] UB. Run time for x near 1e9 is less than one second; x may be as large as 1e19, but execution time balloons to several hours near 1e15 or 1e16, and memory requirements also increase. The LML algorithm is written as a function (sllLML) in the module lml.c, which may be called from your own code by recompilation and linking. For recompilation, you will also need to download the library files trn.c and trn.h in trn.zip, and include the command-line qualifier -std=gnu99. Not compatible with 64-bit compilers or operating systems; does not require or use GMP or MPFR. Last updated 0545 GMT 20 July 2011.
- conio3.zip, a zipfile (9K) containing the latest revisions of the source code (conio3.c) and header file (conio3.h) for a library of functions which emulate some of the conio functions (gotoxy, wherex, etc.) native to DJGPP and Borland C in DOS console environments. Needed only if the main code calls such functions and is being compiled outside of DJGPP and Borland C. Portions of this code, notably the Win32 sections, were adapted from the package devpak CONIO 2.0 (CONIO2), written and released to the public domain by Hongli Lai, tkorrovi, Andrew Westcott, and Michal Molhanec, and targeted at the Win32 MinGW/Dev-C++ platform. The original CONIO 2.0 is available here; thanks to David Hoke for this pointer, and for his own adaptation of CONIO 2.0. Multiple platforms (but does not support Unicode/wchar_t). Last updated 0400 GMT 04 April 2016.
- bpsw1.zip, a zipfile (229K) containing the source code and executable for an application which illustrates the standard and strong versions of the Baillie-PSW primality test, as well as the standard and strong Lucas-Selfridge tests and the extra strong Lucas test. Requires trn.zip and GMP, but does not require MPFR (recompile with -D__NOMPFR__). The actual code implementing most of these tests is contained in the support module trn.c. Last updated 0200 GMT 28 March 2013.
- cglp4.zip, a zipfile (243K) containing the source code and executable (MinGW/Win32) for an application which checks prime gaps for validity, using the strong Baillie-PSW primality test. Requires GMP, trn.zip, and possibly conio3.zip. Multiple platforms. Last updated 0010 GMT 05 October 2018.
- easter1.zip, a zipfile (39K) containing source code and an executable for calculating the date of Easter Sunday for specified years. Support is provided for both the Western Church (Catholic/Protestant) and Eastern Orthodox algorithms, and for both the Gregorian and Julian (Old Style) calendars. No warranty expressed or implied; this code has not been endorsed or approved by any religious institution, organization, or authority. Last updated 0950 GMT 23 April 2010.
- factor1.zip, a zipfile (190K) containing source files (GNU C with GMP) and an executable for a code which illustrates some algorithms used for factoring integers, including small prime

generation, trial divisors, Brent's variation of Pollard's rho method, Pollard's (p-1) method, and a partial implementation of the ECM method. Expression parsing capability is included to allow input in formula form, such as factor1 "2**150 + 1" (command line arguments may require enclosure in double quotes under operating systems such as Windows XP). No claim is made that this code is "state of the art" or "research caliber"; it is most certainly no threat to current encryption schemes. It may eventually be improved by incorporating additional factoring algorithms. Last updated 0900 GMT 08 July 2011.

- lirz.zip, a zipfile (252K) containing source, documentation, data files, and an executable for the purpose of computing the number-theoretic functions Li (logarithmic integral); HL2, HL3, and HL4 (Hardy-Littlewood integral approximations); and R(x), Riemann's prime number function/formula. Routines are included for GNU C with ultraprecision (GCC 4.1.2, GMP 4.2.1, MPFR 2.2.1), GNU C with long double precision, UBASIC 8.8f (ultraprecision), and Mathematica 2.1 (ultraprecision). Last updated 0530 GMT 03 October 2008.
- pentbug.zip, a zipfile (55K) containing the C source code (pentbug.c) and executables (pentbug.exe and bug16bit.exe) for an application which will check for the Pentium FDIV flaw. The executable bug16bit.exe is a 16-bit executable intended for use in standalone DOS, such as systems prior to Windows 95 (which might appear on machines containing flawed processors). Last updated 0700 GMT 20 August 2011.
- pi2.zip, a zipfile (148K) containing the C source codes (pi2e.c and pi2f.c) and executables (pi2e.exe and pi2f.exe) for programs illustrating some practical techniques for generating the twin primes and tabulating their properties. The pi2f code takes advantage of the sieve of Eratosthenes; the pi2e code uses the simple square-root test for primality. The pi2f code is faster in most cases, but either one can enumerate all the twin primes below 1e6 in less than one second on a 600 MHz Celeron; pi2f can enumerate all those below 1e8 in under 15 seconds. Last updated 2100 GMT 22 November 2004.
- pix.zip, a zipfile (176K) containing the C source codes and executables for enumerating the primes and π(x). Three algorithms are illustrated, using the GMP mpz_probab_prime_p and Baillie-PSW tests, trial divisors to the square root, and the sieve of Eratosthenes over byte arrays. Last updated 0530 GMT 15 July 2009.
- td2k.zip, a zipfile (20K) containing the source code (td2k.ub) and documentation (td2k.txt) for a UBASIC application designed for discovering new first known occurrence prime gaps. This is a fully operational research production code. If you download and use it, I encourage you to notify me of any new first known occurrence prime gaps you discover; I will then post them (with proper attribution and credit) in my lists. NOTE: The input and data files of td2k are incompatible with those of the previous version, td2j. Runs begun with td2j should be completed with td2j, or re-started from scratch with td2k. Last updated 0225 GMT 29 April 2005.
- UBASIC (725K), a freeware GW-BASIC-like interpreted programming environment developed by Professor of Mathematics Yûji Kida of Rikkyo University, Japan. UBASIC features easily accessible ultraprecision integer and floating point arithmetic (hundreds of digits), as well as numerous additional intrinsic functions of specific interest in computational number theory. No computational number theorist should be without UBASIC! Also very effective for classroom instructional use. The zipfile provided here contains Version 8.8f (08

October 2000), the last stable version of which I am aware. See also http://www.rkmath.rikkyo.ac.jp/~kida/ubasic.htm.

- **WARNING**: Be aware that, due to the peculiar command-line parsing algorithm incorporated in recent versions of Microsoft Windows, mathematical expressions in command lines should, to avoid misinterpretation, be specified within double quotes; e.g.,

  mycode "2**150 + 1"

  This syntax is also valid under DOS and older versions of Windows, but the double quotes were optional in those operating environments. Depending on the programming language, it may also be necessary (within the source code) to strip off the double quotes and/or concatenate command-line arguments. Finally, replacing the exponentiation operator "^" (a particularly troublesome token for Windows) with "**" (as in FORTRAN/COBOL) may be helpful, if the application permits.

## LINKS

Following are some websites of relevance to mathematics in general, and number theory in particular. Note that these pages may open in a new browser window.

**DISCLAIMER**: No endorsement of, or by these sites is expressed or implied, and Thomas R. Nicely accepts no responsibility or liability in consequence of their access or content. Furthermore, no endorsement, expressed or implied, is granted to other sites which link to this site (with or without my authorization), and no responsibility or liability is accepted for the access, content, accuracy, or integrity of any external site.

- Complete counts and reciprocal sums of the prime constellations from Nicely's computations (1993-2009). NOTE: These data files are quite large (over 60MB each, even for the zipped versions), including more than two million data points from 0 to 2e16.
- The GNU project ("GNU's Not UNIX"), launched in 1984 to develop and provide as free software (under the terms of the GNU GPL, Lesser GPL, and FDL licenses) a complete UNIX-like operating system, including utilities, applications, and development tools. Linux is one kernel for the GNU operating system. Supported by the Free Software Foundation.
- The GMP (GNU MP) multiple precision software package. Excellent for ultraprecision integer arithmetic; incomplete support for floating-point arithmetic and DOS/Windows platforms. Version 4.2.1 or later recommended.
- MPFR, a C library for multiple-precision floating-point computations with correct rounding, reliable precision control, and compatibility with the ANSI/IEEE 754-1985 standard. MPFR is based on (and assumes pre-installation of) the GMP multiple-precision library. It is open-source software, distributed under the terms of the GNU Lesser GPL license. MPFR is supported and maintained by French teams at INRIA, LORIA, and LIP. It provides many features unavailable with the GMP mpf_t data type and libraries, notably a large collection of transcendental functions. Version 2.2.1 or later recommended.
- The Prime Gap Searches project at the Mersenne Forum, coordinated by Robert W. Smith. Computer codes in C and Perl, developed by Robert Gerbicz, Dana Jacobsen, Antonio P. Key,

*et al*. A more extensive list of contributors is available here. Project initiated April 2017.

- MPC, a portable library written in C for arbitrary precision arithmetic on complex numbers providing correct rounding. Ultimately, it should implement a multiprecision equivalent of the C99 standard. MPC builds upon the GNU MP and the GNU MPFR libraries. Written and maintained at INRIA by Andreas Enge, Philippe Théveny, and Paul Zimmermann. Distributed under the GNU LGPL as free software.
- MinGW, minimalist GNU for Windows. MinGW is a collection of freely available and freely distributable Windows specific header files and import libraries, combined with GNU toolsets that allow one to produce native Windows programs which do not rely on any third-party C runtime DLLs. MinGW is distributed in conjunction with MSYS, a Minimal SYStem (shell) providing POSIX/Bourne configure, make, and libtool services within 32-bit Windows. MinGW and MSYS together provide a scalable development environment for GCC applications within 32-bit Windows, with support for GMP and MPFR. The executables require no third-party DLLs, but are specific to the Win32 platform, and rely on the presence (and share some of the shortcomings) of certain Microsoft system DLLs (e.g., MSVCRT.DLL). The deficiencies of MinGW with regard to long doubles, 64-bit integers, and conio are partially remedied by the functions incorporated in the trn and conio3 libraries. Further comments are provided.
- The On-Line Encyclopedia of Integer Sequences (OEIS), founded in 1964 by Neil J. A. Sloane at AT & T Labs (Bell Labs).
- DJ Delorie's DJGPP port of the GNU GCC compilers and utilities (including GMP) to the DOS/Windows platform.
- Home of the C standard. The current standard (C11) for Programming Language C is ISO/IEC 9899:2011, published 08 December 2011. The latest publically available version of the C11 standard is the draft document WG14 N1570, dated 12 April 2011, which refers to the standard as ISO/IEC 9899:201x. In addition, a Technical Corrigendum 1 (ISO/IEC 9899:2011/Cor. 1:2012) was published in 2012.
- Home of the C++ standard. The current standard for Programming Language C++ is ISO/IEC 14882:2017(E) (5th edition), also known as "C++17", published December 2017. It appears that this standard is only available offline, from member bodies of ISO or IEC; but see also the ISO standardization page. Development of the C++20 standard is underway.
- Tomás Oliveira e Silva's projects in computational number theory.
- The home page of Professor Donald E. Knuth of Stanford University.
- Jens Kruse Andersen's site featuring The Top-20 Prime Gaps, the successor to a compilation maintained prior to February 2004 by Paul Leyland.
- The Prime Pages, Chris K. Caldwell, University of Tennessee at Martin. Includes an elementary introduction to prime numbers and number theory.
- The Number Theory Web, maintained by Keith Matthews, University of Queensland, Brisbane, Australia.
- MathWorld, a Wolfram Web resource, maintained by Eric W. Weisstein.
- Mathematical constants and computations. Ultraprecision mathematical constants; very fast and very compact algorithms and codes for the evaluation of certain classical mathematical constants; evaluation of $\pi(x)$ for extremely large x ( > 1e20). Site maintained by Xavier

Gourdon and Pascal Sebah. Sebah also plans to post at this site periodically updated results of his own enumeration of the twin primes and the associated estimates of Brun's constant.

- The Mathematics WWW Virtual Library of Florida State University.
- The Penn State index of Mathematics Websites around the world.
- The American Mathematical Society (AMS).
- The Mathematical Association of America (MAA).
- The Society for Industrial and Applied Mathematics (SIAM).
- The Society of Actuaries (SOA).
- The Association for Computing Machinery (ACM).
- PARI-GP, a software package for computer-aided number theory, including the ultraprecision libpari C libraries and the gp programmable interactive calculator. Targeted at UNIX platforms, with some DOS/Wintel support. Site maintained by Henri Cohen and Karim Belabas.
- GIMPS, the Great Internet Mersenne Prime Search, a group enterprise using distributed computing across the Internet to search for new primes of the form 2^p - 1, where p is prime. The Prime95 code employed uses George Woltman's gwnum library, highly optimized for x86 processors, as well as code by Richard Crandall.
- PrimeForm/GW, highly optimized x86 software by Yves Gallot and George Woltman, designed to perform compositeness tests, probabilistic primality tests, and (limited) deterministic primality tests. This software incorporates Woltman's gwnum library, the core of the Prime95 code used by GIMPS.
- TtH, Ian Hutchinson's TeX to HTML translator.
- UPX, "the Ultimate Packer for eXecutables". UPX is a free, portable, extendable, high-performance executable packer for several different executable formats. It achieves an excellent compression ratio and offers very fast decompression. Executables suffer little or no memory overhead or other drawbacks for most of the formats supported, because of in-place decompression. UPX is copyrighted software distributed under the terms of the GNU General Public License, with special exceptions granting free usage for commercial programs as stated in the UPX License Agreement. Maintained and copyrighted by Markus F. X. J. Oberhumer, László Molnar, and John F. Reiser (all rights reserved).
- DOSBox, an emulator that recreates an MS-DOS compatible environment (complete with sound, input, graphics and even basic networking). This environment is accurate enough to run many classic MS-DOS games completely unmodified. DOSBox has been ported to many different platforms, including Windows, BeOS, Linux, and Mac OS X. I can personally testify that DOSBox allows me to run Derive XM 3.01, Scrabble Deluxe 1.0 (29 April 1991), and Chess88 (version 2.0, 16 March 1984) in full-screen mode under Vista SP1. DOSBox is free of charge and open-source, published under the GNU GPL license. Copyright DOSBox Team.
- Spybot - Search & Destroy, a freeware application designed to detect and remove spyware of different kinds from your computer. Spybot provides a free software alternative to costly proprietary anti-spyware programs. Also, Spybot is a passive (manual or on-demand) anti-spyware application, and thus avoids the python-like grip of some commercial anti-spyware packages, whose on-access real-time scanners can seriously impact the performance and interface of a system. Frequent signature updates are made available. Copyright Safer Networking Ltd., County Wicklow, Ireland.

- ClamWin, a free anti-virus application for various platforms. ClamWin provides a free software alternative to costly proprietary anti-virus programs. Also, ClamWin is a passive (manual or on-demand) anti-virus application, and thus avoids the python-like grip of commercial anti-virus packages, whose on-access real-time scanners can seriously impact the performance and interface of a system. Based on the Clam AntiVirus engine, ClamWin is an open source code released under the terms of the GNU General Public License. Daily virus signature updates are provided. Copyright ClamWin Pty Ltd.

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