J fgt

## UNIVERSITY OF CHICAGO

Department of Education 5835 S. Kimbark Avenue Chicago, IL 60637 A7018

October 26, 1991

Dr. N.J.A. Sloane Room 2C-376 AT & T Bell Laboratories Murray Hill, NJ 07974

Dear Dr. Sloane:

I am responding to the request for "new sequences, extensions, and corrections" for a second edition of your remarkable volume *A Handbook of Integer Sequences*. I have your Supplement I but I do not know if there are others. I have put all references to sequence numbers in boldface.

New sequences: The sequence numbers are the N-numbers,  $\frac{1}{344.5}$  1, 2, 3, 12, 20, 60, 210, 840, 2520, 2520, ... If we have the sequence numbers are the N-numbers,  $\frac{1}{344.5}$  1, 2, 4, 8, 14, 22, ... If we have the sequence numbers are the N-numbers,  $\frac{1}{344.5}$  1, 2, 4, 8, 14, 22, ... If we have the sequence numbers  $\frac{1}{344.5}$  1, 2, 4, 8, 14, 22, ... If we have  $\frac{1}{344.5}$  1, 2, 4, 8, 14, 22, ... If we have  $\frac{1}{344.5}$  1, 2, 4, 8, 14, 22, ... If we have  $\frac{1}{344.5}$  1, 2, 4, 8, 14, 22, ... If we have  $\frac{1}{344.5}$  1, 2, 4, 8, 14, 22, ... If we have  $\frac{1}{344.5}$  1, 2, 4, 8, 14, 22, ... If we have  $\frac{1}{344.5}$  1, 2, 4, 8, 14, 22, ... If we have  $\frac{1}{344.5}$  1, 2, 4, 8, 14, 22, ... If we have  $\frac{1}{344.5}$  1, 2, 4, 8, 14, 22, ... If we have  $\frac{1}{344.5}$  1, 2, 4, 8, 14, 22, ... If we have  $\frac{1}{344.5}$  1, 2, 4, 8, 14, 22, ... If we have  $\frac{1}{344.5}$  1, 2, 4, 8, 14, 22, ... If we have  $\frac{1}{344.5}$  1, 2, 4, 8, 14, 22, ... If we have  $\frac{1}{344.5}$  1, 2, 4, 8, 14, 22, ... If we have  $\frac{1}{344.5}$  1, 2, 4, 8, 14, 22, ... If we have  $\frac{1}{344.5}$  1, 2, 4, 8, 14, 22, ... If we have  $\frac{1}{344.5}$  1, 2, 4, 8, 14, 22, ... If we have  $\frac{1}{344.5}$  1, 2, 4, 8, 14, 22, ... If we have  $\frac{1}{344.5}$  1, 2, 4, 8, 14, 22, ... If we have  $\frac{1}{344.5}$  1, 3, 3, 3, 4, 22, 64, 196, ... If we have  $\frac{1}{344.5}$  1, 3, 3, 3, 6, 630, ... If we have  $\frac{1}{344.5}$  1, 3, 3, 3, 630, ... If we have  $\frac{1}{344.5}$  1, 3, 3, 3, 630, ... If we have  $\frac{1}{344.5}$  1, 3, 30, 630, ... If we have  $\frac{1}{344.5}$  1, 3, 30, 630, ... If we have  $\frac{1}{344.5}$  1, 3, 30, 630, ... If we have  $\frac{1}{344.5}$  1, 3, 30, 630, ... If we have  $\frac{1}{344.5}$  1, 3, 30, 630, ... If we have  $\frac{1}{344.5}$  1, 3, 30, 630, ... If we have  $\frac{1}{344.5}$  1, 3, 30, 630, ... If we have  $\frac{1}{344.5}$  1, 3, 30, 630, ... If we have  $\frac{1}{344.5}$  1, 3, 30, 630, ... If we have  $\frac{1}{344.5}$  1, 3, 30, 630, ... If we have  $\frac{1}{344.5}$  1, 3, 30, 630, ... If we have  $\frac{1}{344.5}$  1, 3, 30, 630, ... If we have  $\frac{1}{344.5}$  1, 3, 30, 630, ... If we have  $\frac{1}{344.5}$  1, 3

 $(2n-1)!/2^{n-1}$ 

**1866.5** 1, 7, 36, 165, ...

(2n+3)C(n-1)

Extensions:
In general, I found it useful to have explicit or recursive formulas for sequences when they are known. In my own copy, I have such formulas by the following sequences (these are, as you know, a very small percentage of the sequences for which such formulas are possible):

 $103 \quad a_{n+1} = a_n + a_{n-1} - 1$ 

297  $a_{n+1} = \text{Sum } (a_k \text{ to } a_n), \text{ where } k = n - [\frac{n+3}{2}].$ 

585  $a_{n+1} = Sum(0 \text{ to } k)C(n,k)a_k$ . 755 is quite similar to this one.

1793 (2n)!/2n

New references

1760 Two Year College Mathematics Journal Nov 83, p. 407.

Errors:

560 9th term should be 4271.

9th term is 4466 in an article in the Mathematics Teacher from May 1985.

966 formula needs greatest integer function

1375 Easier to put in 3\*\*N than to have the three dots.

## Comments:

It is nice to know when sequences are related in a simple way. Here are some examples:

391 is 1022 (triangular numbers) plus one. 522 is 1022 less one.

**468** has an interesting pattern:  $1^{\frac{1}{2}}$ , 2,  $2^{2}$ ,  $2 \cdot 5$ ,  $5^{2}$ ,  $5 \cdot 14$ ,  $14^{2}$ ,  $14 \cdot 41$ ,  $41^{2}$ . This would seem easy to continue, as the numbers 1, 2, 5, 14, 41, are the sums of powers of 3. **1130** is differences of consecutive terms of **577** (Catalan numbers).

A421 is C(2n,n-1).

1574 and 1714 are related by sums and differences.

1630 is every second term of 552.

1705 is every second triangular number, and the differences of consecutive terms of 1839

1709 is the differences of consecutive terms of 1845; or one could say 1845 is the sum of the first n terms of 1709.

1719, 1847, and 1911 are similarly related by sums and differences.

1725 and 1851 are related by sums and differences.

(I have others of these, but it seems a computer search could be done to be exhaustive in the given list.)

1746 could be divided by 6.

It seems that 1835 could be divided by 7.

1997 is the square of corresponding terms of 1217.

**2291** is the square of **1760**.

I hope all this is useful to you.

Sincerely,

Zalman Usiskin

Talgan Usuka

Professor of Education