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NUMERATOR POLYNOMIAL COEFFICIENT ARRAY FOR THE CONVOLVED FIBONACCI SEQUENCE

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1. INTRODUCTION

In [1], [2], and [3], Hoggatt and Bicknell discuss the numerator polynomial coefficient arrays associated with the row generating functions for the convolution arrays of the Catalan sequence and related sequences. In [4], Hoggatt and Bergum examine the irreducibility of the numerator polynomials associated with the row generating functions for the convolution arrays of the generalized Fibonacci sequence $\left\{H_n\right\}_{n=1}^{\infty}$ defined recursively by

(1.1) $$H_1 = 1,\quad H_2 = P,\quad H_n = H_{n-1} + H_{n-2},\qquad n \geq 3,$$

where the characteristic $P^2 - P - 1$ is a prime. The coefficient array of the numerator polynomials is also examined. The purpose of this paper is to examine the numerator polynomials and coefficient array related to the row generating functions for the convolution array of the Fibonacci sequence. That is, we let $P = 1$.

2. THE FIBONACCI ARRAY

We first note that many of the results of this section could be obtained from [4] by letting $P = 1$.
The convolution array, written in rectangular form, for the Fibonacci sequence is

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Table 1
Convolution Array for the Fibonacci Sequence

1	1	1	1	1	1	1	1	...
1	2	3	4	5	6	7	8	...
2	5	9	14	20	27	35	44	...
3	10	22	40	65	98	140	192	... 6503
5	20	51	105	190	315	490	726	... 6504
8	38	111	256	511	924	1554	2472	...
13	71	233	594	1295	2534	4578	7776	...
21	130	474	1324	3130	6588	12,720	22,968	...
...	...	...	...	...	...	...	...	...

The generating function $C_m(x)$ for the m^{th} column of the convolution array is given by

(2.1) $$C_m(x) = (1 - x - x^2)^{-m}$$

and it is obvious that

(2.2) $$C_m(x) = (x + x^2)C_m(x) + C_{m-1}(x).$$

Hence, if $R_{n,m}$ is the element in the n^{th} row and m^{th} column of the convolution array then the rule of formation for the convolution array is

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(2.3) $$R_{n,m} = R_{n-1,m} + R_{n-2,m} + R_{n,m-1}$$

which is representable pictorially by

$$\begin{array}{cc} & \boxed{w} \\ & \boxed{v} \\ \boxed{u} & \boxed{x} \end{array}$$

where

(2.4) $$x = u + v + w.$$

If $R_m(x)$ is the generating function for the m^{th} row of the convolution array then we see by (2.3) and induction that

(2.5) $$R_1(x) = \frac{1}{1-x}$$

(2.6) $$R_2(x) = \frac{1}{(1-x)^2}$$

and

(2.7) $$R_m(x) = \frac{N_{m-1}(x) + (1-x)N_{m-2}(x)}{(1-x)^m} = \frac{N_m(x)}{(1-x)^m}, \qquad m \geqslant 3$$

with $N_m(x)$ a polynomial of degree

$$\left[\frac{m-1}{2}\right],$$

where $[\ ]$ is the greatest integer function.

The first few numerator polynomials are found to be

$$\begin{aligned} N_1(x) &= 1 \\ N_2(x) &= 1 \\ N_3(x) &= 2 - x \\ N_4(x) &= 3 - 2x \\ N_5(x) &= 5 - 5x + x^2 \\ N_6(x) &= 8 - 10x + 3x^2 \\ N_7(x) &= 13 - 20x + 9x^2 - x^3 \\ N_8(x) &= 21 - 38x + 22x^2 - 4x^3 . \end{aligned}$$

Recording our results by writing the triangle of coefficients for these polynomials, we have

Table 2

Coefficients of Numerator Polynomials $N_m(x)$

1			
1			
2	−1		
3	−2		
5	−5	1	
8	−10	3	
13	−20	9	−1
21	−38	22	−4

Examining Tables 1 and 2, it appears as if there exists a relationship between the rows of Table 2 and the rising diagonals of Table 1. In fact, we shall now show that