Math Games

Kobon Triangles

Ed Pegg Jr., February 8, 2006

Today's column is dedicated to a problem from *Martin Gardner's Mathematical Games*, 4500 pages of mathemagical goodness published by **maa.org**. The problem resides within *Wheels, Life, and Other Mathematical Amusements*, chapter "Charles Addams' Skier and Other Problems." Considerable progress has recently been made on the problem recently, so I thought I'd recap.

> Kobon Fujimura, a Japanese puzzle expert, recently invented a problem in combinatorial geometry. It is simple to state, but no general solution has yet been found. What is the largest number of nonoverlapping triangles that can be produced by *n* straight line segments! It is not hard to discover by trial and error that for n= 3, 4, 5 and 6 the maximum number of triangles is respectively one, two, five and seven. For seven lines the problem is no longer easy. The reader is asked to search for the maximum number of nonoverlapping triangles that can be produced by seven, eight and nine lines. The problem of finding a formula for the maximum number of triangles as a function of the number of lines appears to be extremely difficult.
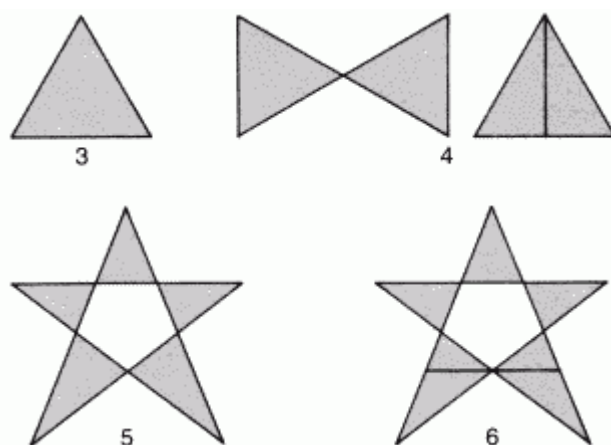

Figure 1. Maximal nonoverlapping triangles with 3-6 lines.

> ANSWER: The maximum number of nonoverlapping triangles that can be produced by seven, eight and nine lines are 11, 15 and 21 respectively. These are thought, although not yet proved, to be maximal solutions."

[The solution is here](), hidden behind a link for the benefit of those wanting to try the problem. Kobon Fujimura went on to write *The Tokyo Puzzles*.

For *n* lines, how many triangles can be made? Saburo Tamura made progress on the Kobon Triangle problem by proving that $\lfloor n(n-2)/3 \rfloor$ was an *upper* bound on the maximal number of nonoverlapping triangles realizable by *n* lines. We can make a table of the solutions known by me, as of a year ago.

	n	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A006066	best known	1	2	5	7	11	15	21	?	?	?	?	?	?	?
A032765	upper bound	1	2	5	8	11	16	21	26	33	40	47	56	65	74

There things remained for decades.

On 2 October 2005, Toshitaka Suzuki sent *MathWorld* a solution for 65 triangles with 15 lines. This meets the upper bound, so it's unquestionably a valid solution.
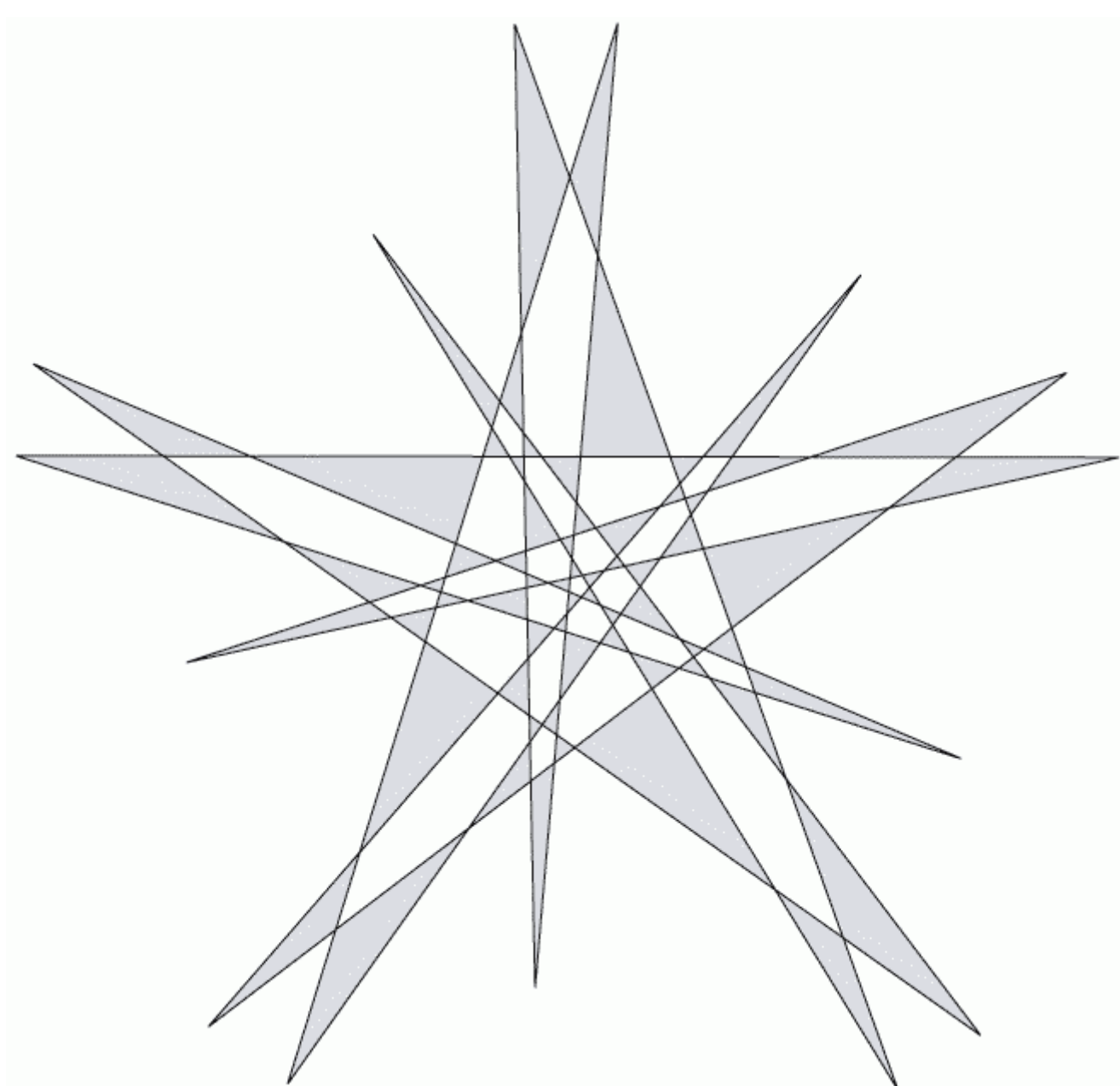
Figure 2. Toshitaka Suzuki's solution for the Kobon Triangle problem with 15 lines and 65 Triangles. (SVG version)

After *MathWorld* listed the above solution, the problem received a resurgence of interest in Japan. I mentioned this on my MathPuzzle site as a tidbit of recreational math news:

> After the discovery of a 15 line Kobon Triangle solution by Toshitaka Suzuki, several recreational math groups in Japan have taken up the problem. Recently, a promising 11-line solution was found, with 32 triangles. 33 triangles is the theoretical maximum.

After that, I started learning about various 10-line solutions that had 25 triangle (26 is the upper bound). Branko Grünbaum found the first such solution in 1967.
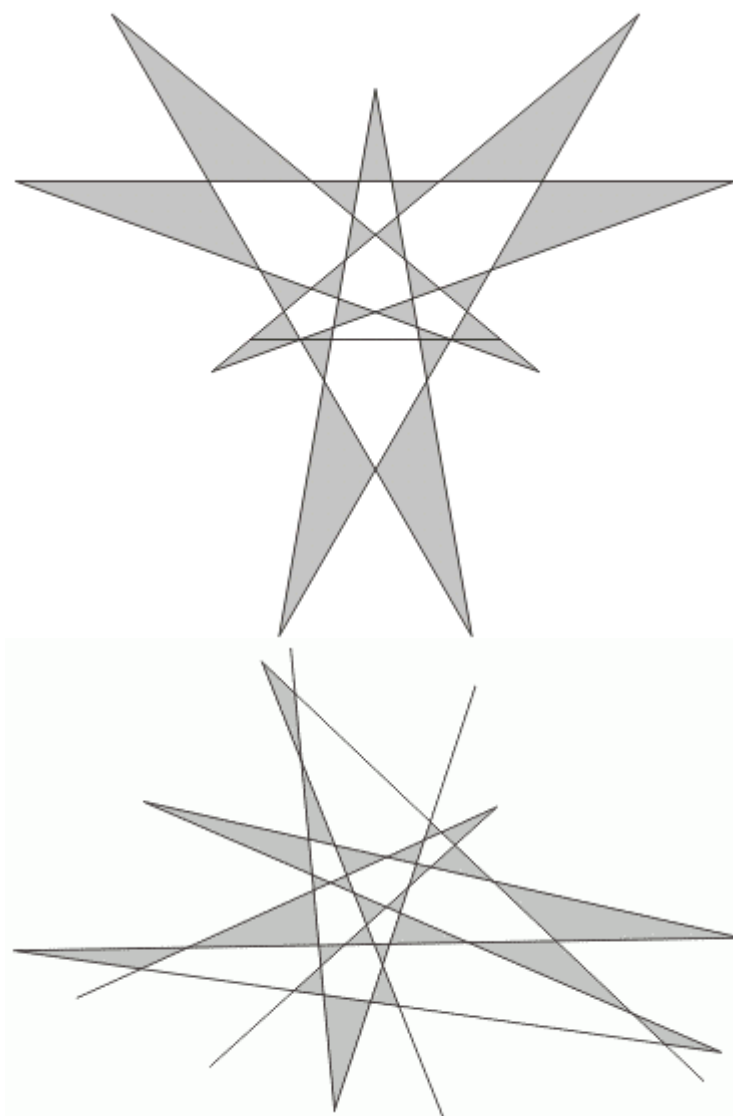

Figure 3. Two solutions for 25 nonoverlapping triangles with 10 lines. Solutions by Serhiy Grabarchuk and Viatcheslav Kabanovitch.

So, I figured that might be it for awhile. I was wrong. Serhiy Grabarchuk, a well-known puzzle creator and solver currently working on the *Age of Puzzles* site, let me know I'd missed a lot! As an aside, let me point at Serhiy's site, and mention his related arco-triangles problem. Now, Serhiy let me know the following:

I was through your latest update, and one of your topics is especially interesting to me. That's the Kobon Triangles theme. I thought you will be interested to know that in 1996 and 1999 there were several solutions which evolve this theme with results for 10-13 lines. These results were published in the Russian puzzle bulletin "Sharada" ("Charade") of the Russian puzzle club "Diogen" ("Diogenes"). I attach the respective GIFs of these solutions, and the respective pages of the bulletin.

Specifically, he sent me page 1 and page 2 of the June 1999 issue of Charade-6(72), as well as page 2 of the May 1996 Charade. In addition to the solutions above with 10 lines, Viatcheslav Kabanovitch found excellent solutions with 11, 12, and 13 lines.
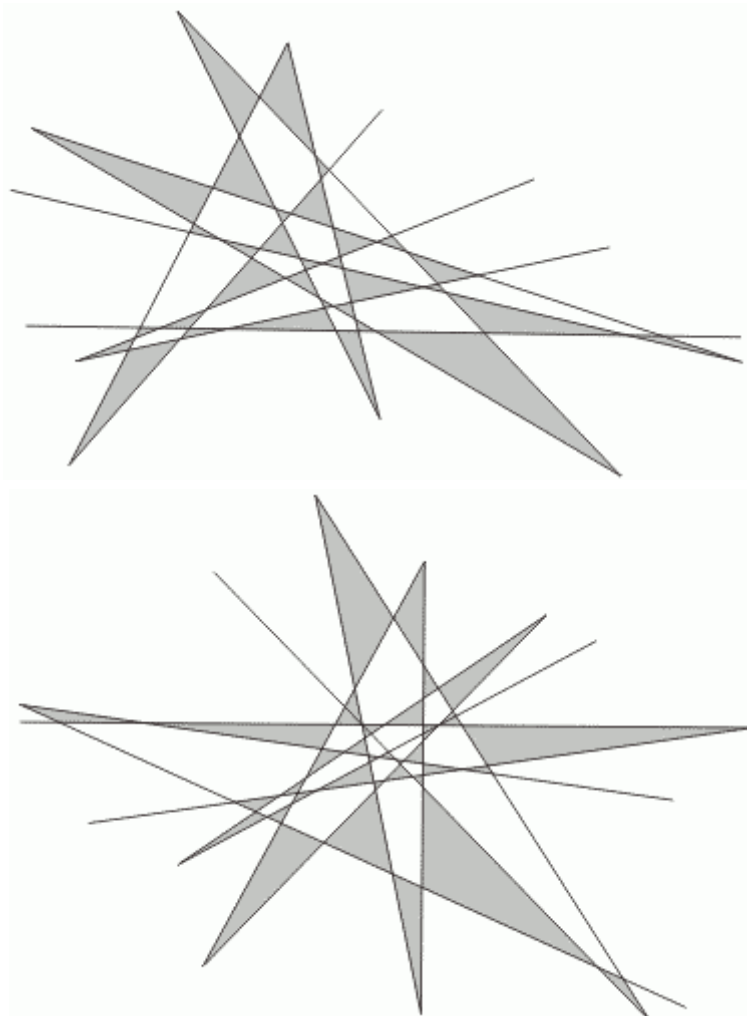

Figure 4. Solutions by Viatcheslav Kabanovitch for the Kobon Triangle problem on 11 and 12 lines.

Viatcheslav Kabanovitch also found the below solution with 13 lines. Amazingly, it meets the upper bound of 47 triangles.
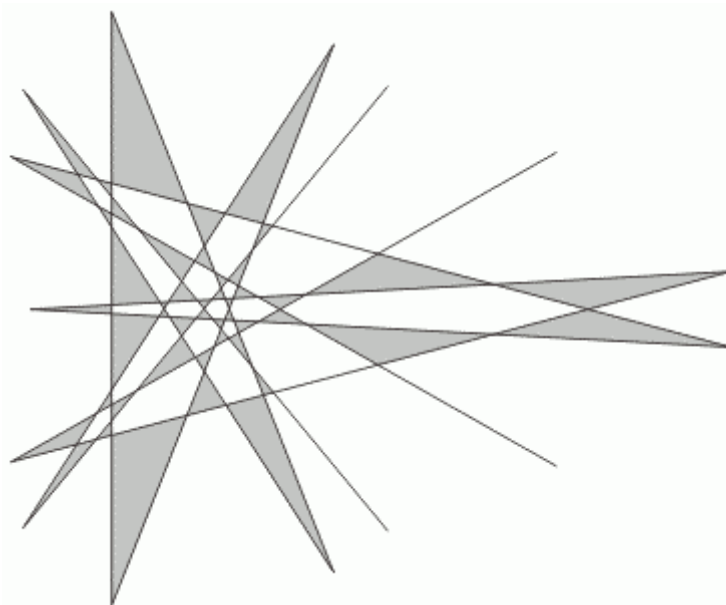
Figure 5. Solution by Viatcheslav Kabanovitch for the Kobon Triangle problem on 13 lines.

Okay, so that's a lot of progress. Here is an updated table:

	n	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A006066	best known	1	2	5	7	11	15	21	25	32	38	47	?	65	?
A032765	upper bound	1	2	5	8	11	16	21	26	33	40	47	56	65	74

Based on the above information, we can state that the problem is definitely solved for 13 (Viatcheslav Kabanovitch, 1999) and 15 lines (Toshitaka Suzuki, 2005). Solutions exist that come within 1 of the maximum for 8, 10, and 11. The best known solution for 12 lines is 2 away from the upper bound. Nothing is known for 14, 16, and beyond.

Anybody else? More results would be greatly appreciated. I am reminded of the words of Tutte upon first seeing Blanusa Snarks. "I saw Blanusa's paper soon after it appeared. Alas, I did not understand the language, but the diagram made all clear!" Here, although the source material was Japanese and Russian, everything was crystal clear.

References:

David Eppstein, "Kabon Triangles."
http://www.ics.uci.edu/~eppstein/junkyard/triangulation.html.

Martin Gardner, *Wheels, Life, and Other Mathematical Amusements.* New York: W. H. Freeman, pp. 170-171 and 178, 1983.

Branko Grünbaum, *Convex Polytopes, 2nd ed.* New York: Springer-Verlag, p. 400, 2003.

S. Honma, "Kobon Triangles." http://www004.upp.so-net.ne.jp/s_honma/triangle/triangle2.htm.

Viatcheslav Kabanovitch, "Kobon Triangle Solutions." *Sharada* (Charade, by the Russian puzzle club Diogen), p. 1-2, June 1999.

N. J. A. Sloane, Sequences [A006066](A006066)/M1334 and [A032765](A032765) in "The On-Line Encyclopedia of Integer Sequences."

Eric W. Weisstein. "Kobon Triangle." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/KobonTriangle.html

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Math Games archives.

Comments are welcome. Please send comments to Ed Pegg Jr. at ed@mathpuzzle.com.

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