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# Kobon Triangles

### Proof for Tighter Lower Bound

Gilles Clément and I found a tighter bound for the number of Kobon triangles:

**Theorem 1.** *The maximal number of Kobon triangles $K(n)$ for a given number of $n$ straight lines in a plane is upper bounded by*

$$K(n) \leq \left\lfloor \frac{n(n-2)}{3} \right\rfloor - \mathbf{I}_{\{n|(n \mod 6)\in\{0,2\}\}}(n) \qquad (2)$$

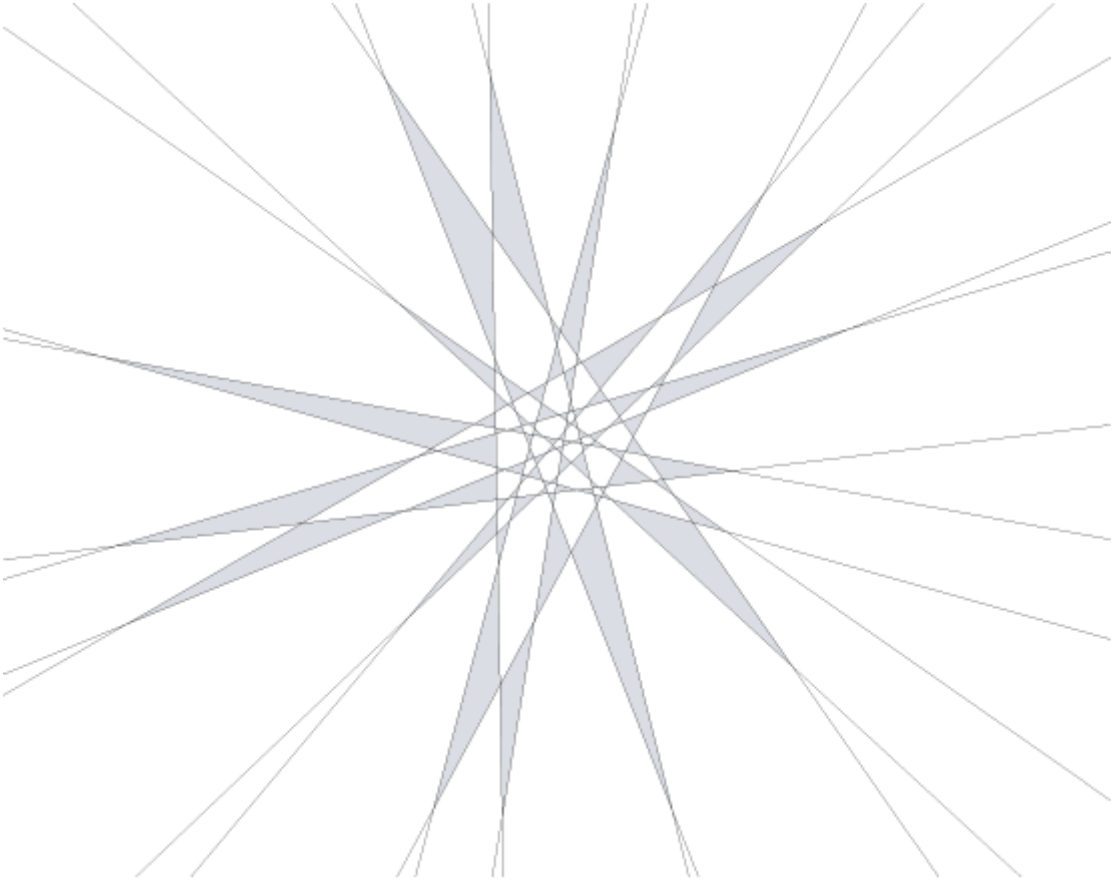
*where $\mathbf{I}_A(x)$ denotes the indicator function. In other words the upper bound known by Saburo Tamura cannot be reached for all $n$ with $n \equiv 0 \mod 6$ and $n \equiv 2 \mod 6$.*

The proof is outlined in the following draft:

- G. Clément and J. Bader. **Tighter Upper Bound for the Number of Kobon Triangles**. Draft Version, 2007. (PDF) (bibtex) (suppl. material)

### Perfect Solution with 17 lines

(Deutsche Version)
In November 2007 I found the following configuration of 85 nonoverlapping triangles constructed using 17 lines, solving the problem of Kobon Fujimura for n = 17



(click to enlarge)

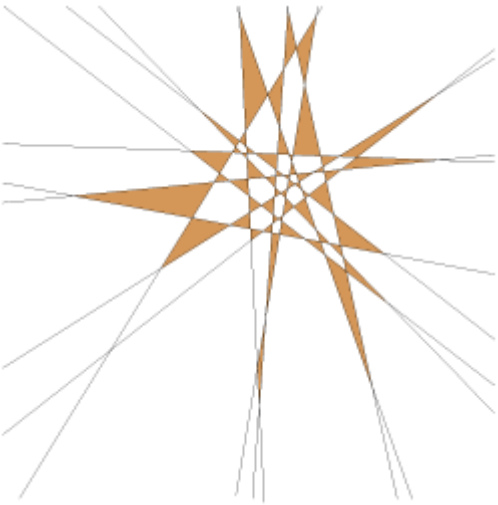
SVG Version, PDF Version (colored to simplify counting)

The placement meets the upper bound proven by Saburo Tamura hence it's the first maximal solution with 17 lines and the 17th term of A006066 is 85.

**Other Configurations**

The following table lists other configurations I found (click on the images to enlarge them):

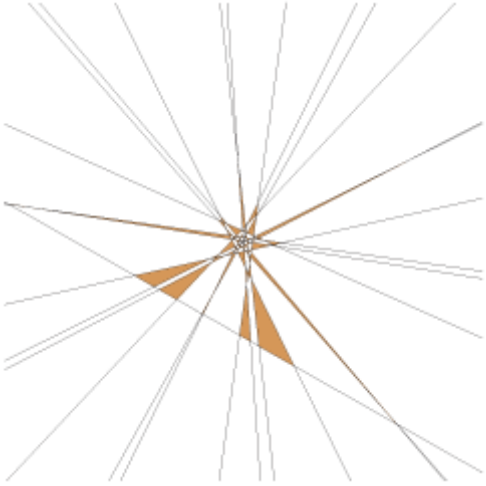
n: 10
triangles: 25
best known: 25
upper bound: 26



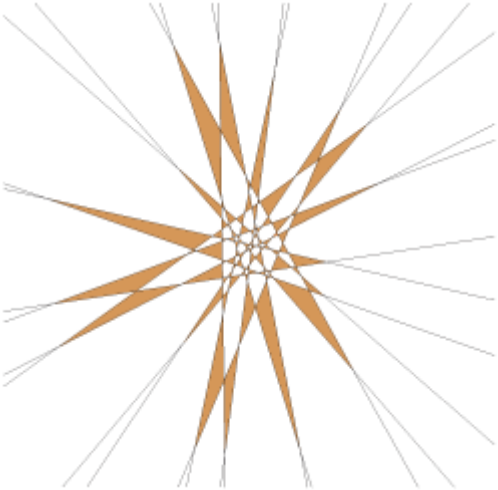
n: 14
triangles: 53
best known: ?
upper bound: 56

comments: The configuration is 5-fold rotational symmetric in contrast to the ones of Serhiy Grabarchuk, Viatcheslav Kabanovitch and S. Honma respectively. pdf version

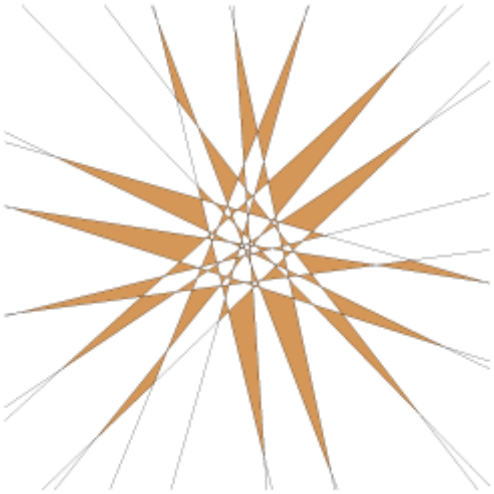
comments: pdf version



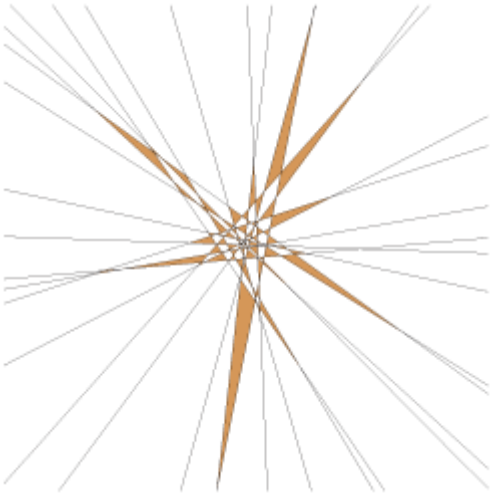
n: 16
triangles: 72
best known: ?
upper bound: 74
comments: Based on the maximal solution for 15 lines found by Toshitaka Suzuki. pdf version



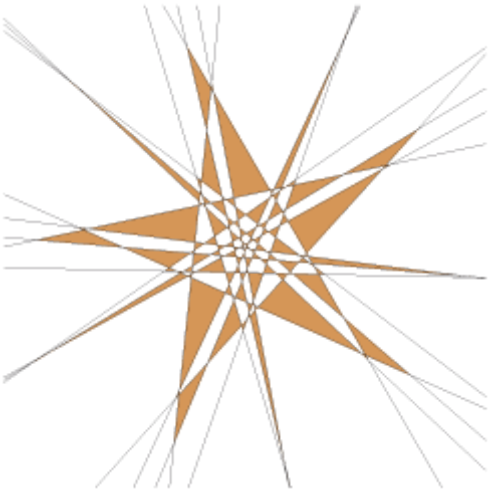
n: 17
triangles: 85
best known: ?
upper bound: 85
comments: Reaches upper bound. pdf version



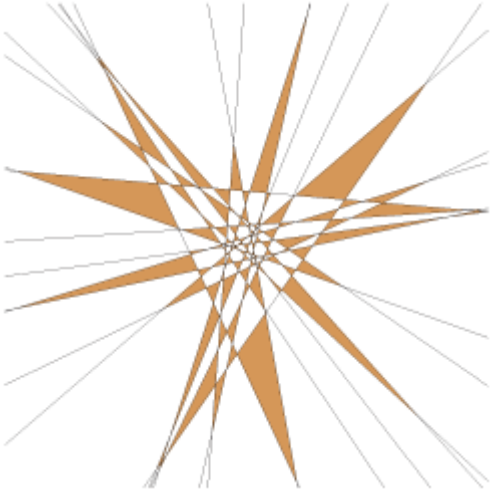
n: 18
triangles: 93
best known: ?
upper bound: 96
comments: 3-fold rotational symmetric. pdf version



n: 19
triangles: 104
best known: ?
upper bound: 107
comments: pdf version



n: 20
triangles: 115
best known: ?
upper bound: 120
comments: 5-fold rotational symmetric. pdf version



n: 21
triangles: 130
best known: ?
upper bound: 133
comments: 3-fold rotational symmetric. pdf version

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