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N. J. A. Sloane May 29, 1991

Dear Sir,

I am currently a graduate student at the University of Georgia, and I recently caught your e-mail message requesting integer sequences.

Phillip E. Parker and I would like to submit the enclosed sequences for your consideration.

Also enclosed are two articles we have written concerning these sequences. The first, The Lorentzian Modular Group and Nonlinear Lattices, is to appear (if it has not already) in _Geometry, Topology and Number Theory_, ed. G.M. Rassias, World Scientific. The other article, The Lorentzian Modular Group and Nonlinear Lattices II, has not yet been accepted for publication anywhere.

Now for some explanation concerning the sequences. We consider two groups

$$SO_1^{2+}(\mathbb{Z}) = \left\langle \begin{bmatrix} 3 & 2 & 2 \\ 2 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \right\rangle$$

$$O_1^{2+}(\mathbb{Z}) = \left\langle \begin{bmatrix} 3 & 2 & 2 \\ 2 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right\rangle$$

and their action on the sets

$$\{(z,x,y) \in \mathbb{Z}^3 \mid z^2 - k = x^2 + y^2,\ z > 0\},\quad k \in \mathbb{Z},\ k > 0$$

$$\{(z,x,y) \in \mathbb{Z}^3 \mid z^2 - k = x^2 + y^2\},\quad k \in \mathbb{Z},\ k < 0.$$

For each $k \in \mathbb{Z}$, $k \neq 0$, we have proved that both groups

$SO_1^{2+}(\mathbb{Z})$ and $O_1^{2+}(\mathbb{Z})$ generate a finite number of orbits as groups acting on these sets.

For $k>0$ we can characterize a set of unique representations from each orbit, and thus can count the exact number of orbits for a given k.

For $k<0$ we have found no way to characterize a set of representatives from each orbit, but have computationally searched and obtained a list, of some amount of certainty, of the number of orbits for a given k.

Note that for $k=0$, there are an infinite number of orbits for either group.

In summary, the tables for $k>0$ should be exact, but we have not proved that those for $k<0$ are exact.

I hope that you can make use of these.

Thank you
Glenn J. Fox

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k	5793 # $O_1^{24}(\mathbb{Z})$ orbits ~~maps~~	5794 # $SO_1^{24}(\mathbb{Z})$ orbits ~~maps~~
1	1	1
2	1	1
3	1	1
4	2	2
5	1	1
6	1	1
7	2	2
8	3	3
9	2	2
10	1	1
11	2	3
12	3	3
13	1	1
14	2	2
15	3	4
16	4	4
17	2	2
18	2	2
19	2	3
20	4	5
21	2	2
22	1	1
23	4	6
24	5	5
25	2	2
26	2	3
27	3	4
28	4	4
29	2	3
30	2	2
31	4	6
32	6	7
33	2	2
34	2	2
35	4	6
36	6	7
37	1	1
38	2	3
39	5	8
40	5	5

Omit

k	# $O_1^{2+}(\mathbb{Z})$ orbits ~~reps~~	# $SO_1^{2+}(\mathbb{Z})$ orbits ~~reps~~
	5795	5796
-1	2	2
[illegible]	1	1
-3	1	1
-4	4	4
-5	1	1
-6	1	1
-7	2	2
-8	3	3
-9	5	6
-10	1	1
-11	1	1
-12	3	3
-13	1	1
-14	2	2
-15	2	2
-16	8	8
-17	2	2
-18	2	2
[illegible]	1	1
-20	3	3
-21	2	2
-22	1	1
-23	2	2
-24	5	5
-25	7	10
-26	1	1
-27	2	2
-28	4	4
-29	1	1
-30	2	2
-31	2	2
-32	5	5
-33	3	4
-34	3	4
-35	2	2
[illegible]	11	12
-37	2	3
-38	1	1
-39	2	2
-40	5	5