Scan

AS676
etc

V E Hoggatt Jr
letter

7 pages
Many seqs

Cnt^d - to be punched ✓

3264
3520
3521 = 5251
3522
3523 = 5252

f91

A HANDBOOK OF INTEGER SEQUENCES: N. J. A. Sloane

Sequences 1130, 1602, 1866, 1981, 2048, 2104 all agree with convolutions of the Catalan sequence 577 for their first ~~nine~~ ten entries. We do not have further terms in the sequences computed. These are all convolutions made an even number of times. 5676

The convolutions for an odd number of times are:

√91 1, 4, 14, 48, 165, 572, 2002, 7072, 25194, 90440, ...
√91 1, 6, 27, 110, 429, 1638, 6188, 23256, 87210, 326876, ...
√91 1, 8, 44, 208, 910, 3808, 15504, 62016, 245157, ...
√91 1, 10, 65, 350, 1700, 7752, 33915, 144210, 600875, ...
1, 12, 90, 544, 2907, 14364, 67298, 303600, 1332045, ...

We call the Catalan sequence itself the zeroth convolution. Then the (k - 1)st convolution of the Catalan sequence is the sequence referred to above which begins with 1, k, ... The mth term of the (k - 1)st convolution of the Catalan sequence is given by

$$\frac{k}{k+m}\binom{2m+k-1}{m}, \quad m = 0, 1, 2, \ldots,$$

where $\binom{n}{k}$ is a binomial coefficient.

References:

V. E. Hoggatt, Jr., and Marjorie Bicknell, "Catalan and Related Sequences Arising from Inverses of Pascal's Triangle Matrices," An unpublished paper. A copy is enclosed for your perusal.

√91

Cont'd (1)

SPECIAL DIAGONAL SUMS OF PASCAL'S TRIANGLE

If Pascal's triangle is written in a left-justified manner, denoting the top row as the zero-th row, then H_n is the sum of the term in the leftmost column in the nth row and the terms obtained starting from this term by moving p rows up and 1 unit to the right.

$$H_n = u(n; p,1) = \sum_{i=0}^{[n/(p+1)]} \binom{n - ip}{i} \quad , n \geq 1, H_0 = 1.$$

Recursively,

$$H_j = j, \quad j = 1, 2, 3, \ldots, p+2, \text{ and } H_n = H_{n-1} + H_{n-p-1}, \; n > p+1 \geq 2.$$

When $p = 1$, we obtain the Fibonacci numbers, and $H_n = F_{n+1}$.

√91 N207 have + many refs

$p = 2$: 1, 2, 3, 4, 6, 9, 13, 19, 28, 41, 60, 88, 129, 189, 277, 406, 595, 872, 1278, 1873, 2745, 4023, 5896, 8641, 12664, 18560, 27201, 39865, 58425, 85626, 125491, 183916, 269542, 395033, 578949 ✓

N188.3 √91 have

$p = 3$: 1, 2, 3, 4, 5, 7, 10, 14, 19, 26, 36, 50, 69, 95, 131, 181, 250, 345, 476, 657, 907, 1252, 1728, 2385, 3292, 4544, 6272, 8657, 11949, 16493, 22765, 31422, 43371, 59864, 82629

N182.7 added √91 3520

$p = 4$: 1, 2, 3, 4, 5, 6, 8, 11, 15, 20, 26, 34, 45, 60, 80, 106, 140, 185, 245, 325, 431, 571, 756, 1001, 1326, 1757, 2328, 3084, 4085, 5411, 7168, 9496, 12580, 16665, 22076

omit

$p = 5$: 1, 2, 3, 4, 5, 6, 7, 9, 12, 16, 21, 27, 34, 43, 55, 71, 92, 119, 153, 196, 251, 322, 414, 533, 686, 882, 1133, 1455, 1869, 2402, 3088, 3970, 5103, 6558, 8427

Dull

$p = 6$: 1, 2, 3, 4, 5, 6, 7, 8, 10, 13, 17, 22, 28, 35, 43, 53, 66, 83, 105, 133, 168, 211, 264, 330, 413, 518, 651, 819, 1030, 1294, 1624, 2037, 2555, 3206, 4025

√91

$p = 7$: 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 14, 18, 23, 29, 36, 44, 53, 64, 78, 96, 119, 148, 184, 228, 281, 345, 423, 519, 638, 786, 970, 1198, 1479, 1824, 2247

References:

Brother Alfred Brousseau, Fibonacci and Related Number Theoretic Tables, The Fibonacci Association, 1972, pp. 117-128. Contains these numbers for $p = 1, 2, \ldots, 11$ and $n = 1, 2, \ldots, 80$, and selected $u(n; p, q)$.

V. C. Harris and Carolyn C. Styles, "A Generalization of Fibonacci Numbers," Fibonacci Quarterly, Vol. 2, No. 4, Dec., 1964, pp. 277-289.

V. E. Hoggatt, Jr., and Marjorie Bicknell, "Diagonal Sums of Generalized Pascal Triangles," Fibonacci Quarterly, Vol. 7, No. 4, Nov., 1969, pp. 341-358.

V. E. Hoggatt, Jr., "A New Angle on Pascal's Triangle," Fibonacci Quarterly, Vol. 6, No. 2, Oct., 1968, pp. 221-234.

91

91

Extend!

DIAGONAL SUMS OF PASCAL'S TRIANGLE

If Pascal's Triangle is written in a left-justified manner, denoting the top row as the zero-th row, then u(n;, p, q) is the sum of the term in the left-most column in the n-th row and the terms obtained ~~starting~~ from this term by moving p rows up and q units to the right:

$$u(n;\ p,\ q) = \sum_{i=0}^{[n/(p+q)]} \binom{n-ip}{iq}, \qquad n \geq 1, \quad u(0;\ p,q) = 1$$

$$\sum_{n=0}^{\infty} u(n;\ p,q)x^n = \frac{(1-x)^{q-1}}{(1-x)^q - x^{p+q}}$$

Same references as "Special Diagonal Sums of Pascal's Triangle."

$A(N) = A(N-1) + A(N-2) + A(N-4)$ N397.7 = ~~3521~~

Entd f91

p = 1, q = 2: 1, 1, 1, 2, 4, 7, 12, 21, 37, 65, 114, 200, 351, 616, 1081,
1897, 3329, 5842, 10252, 17991 5251

Entd

p = 1, q = 3: 1, 1, 1, 1, 2, 5, 11, 21, 37, 64, 113, 205, 377, 693, 1266,
2301, 4175, 7581, 13785, 25088 N541.7 = 3522

new 91

~~p = 1, q = 4: 1, 1, 1, 1, 1, 2, 6, 16, 36, 71, 127, 212, 340, 541,~~
~~881, 1497, 2653, 4825, 8829~~

p = 1, q = 4: 1, 1, 1, 1, 1, 2, 6 , 16, 36, 71, 128, 220, 376, 661, 1211,
2290, 4382, 8347, 15706, 29191 5676 f91

new 91

p = 1, q = 5: 1, 1, 1, 1, 1, 1, 2, 7, 22, 57, 127, 253, 464, 804, 1354,
2289, 4005, 7372, 14198, 28033

p = 1, q = 6: 1 , 1, 1, 1, 1, 1, 1, 2, 8, 29, 85, 211, 463, 925, 1718,
3017, 5097, 8464, 14197, 24753

Entd

p = 2, q = 2: 1, 1, 1, 1, 2, 4, 7, 11, 17, 27, 44, 72, 117, 189, 305,
493, 798, 1292, 2091, 3383 ⇒ N 392.1 = ~~3523~~
5252

f91

Extend

f91

INTEGERS NOT REPRESENTABLE BY TRUNCATED FIBONACCI SEQUENCE

2, 3, 5, 8, 13, 21, ...

1, 4, 6, 9, 12, 14, 17, 19, 22, 25, 27, 30, 33, 35, 38, 40, 43, 46, 48, 51, 53, 56, 59, 61, 64, 67, 69, 72, 74, 77, 80, 82, 85, 88, 90, 93, 95, 98, 101, 103, 106, 108, 111, 114, 116, 119, 122, 124, 127, 129, 132, 135, 137, 140, 142, 145, 148, 150, 153, 156

3264

Brother Alfred Brousseau, Fibonacci and Related Number Theoretic Tables, Fibonacci Association, 1972

CHARACTERISTIC NUMBERS OF FIBONACCI SEQUENCES

The characteristic number of a Fibonacci sequence is the absolute value of $T_n^2 - T_{n-1}T_{n+1}$ where $T_{n+1} = T_n + T_{n-1}$, $T_1 \geq 1$, $T_2 > T_1$, and $(T_1, T_2) = 1$.

(Same reference as above.) Arranged in order of increasing value, possible characteristic numbers are:

1, 5, 11, 19, 29, 31, 41, 55, 59, 61, 71, 79, 89, 95, 101, 109, 121, 131, 139, 145, 149, 151, 155, 179, 181, 191, 199, 205, 209, 211, 229, 239, 241, 251, 269, 271, 281, 295, 305, 311, 319, 331, 341, 349, 355, 359, 361, 379, 389, 395, 401, 409, 419, 421, 431, 439, 445, 449, 451, 461, 479, 491, 499

Omit

5.91

FIBONACCI NUMBERS WHICH ARE PRIMES

Brother Alfred Brousseau, Fibonacci and Related Number Theoretic Tables,

(1), 2, 3, 5, 13, 89, 233, 1597, 28657, 514229, 433494437, 2971215073

LUCAS NUMBERS WHICH ARE PRIMES

(1), 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451

SUBSCRIPTS OF FIBONACCI PRIMES:

(1), (2), 3, 4, 5, 7, 11, 13, 17, 23, 29, 43, 47, 83, 131, 137, 359, 431, 433, 449, 509, 569, 571

SUBSCRIPTS OF LUCAS PRIMES:

(1), 2, 4, 5, 7, 8, 11, 13, 16, 17, 19, 31, 37, 41, 47, 53, 61, 71, 79, 113, 313.

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SEQUENCES ARISING FROM INVERSES OF PASCAL'S TRIANGLE MATRICES

(V. E. Hoggatt, Jr., and Marjorie Bicknell, "Catalan and Related Sequences Arising from Inverses of Pascal's Triangle Matrices," unpublished paper.)

The sequence S_k which begins 1, 1, k + 1, ..., is expressible by the formula

$$\frac{1}{kn+1}\binom{(k+1)n}{n} = \frac{1}{n}\binom{(k+1)n}{n-1}, \quad n = 0, 1, 2, \ldots$$

The sequences S_k are given below. S_0 is all ones, and S_1 is the Catalan numbers.

k = 2: 1, 1, 3, 12, 55, 273, 1428, 7752, 43263, 246675, 1430715

k = 3: 1, 1, 4, 22, 140, 969, 7084, 53820, 420732, 3362260, 54687776

k = 4: 1, 1, 5, 35, 285, 2530, 23751, 231880, 2330445, 23950355, 250543370

k = 5: 1, 1, 6, 51, 506, 5481, 62832, 749398, 9203634, 113147580, 1478314266

k = 6: 1, 1, 7, 70, 819, 10472, 141778, 1997688, 86969025

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