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Eureka Problems Drive
1977

3 pages
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#39 1978

Problems Drive 1977

by J. Mestel

1 Find a positive integer that is multipled by 7 when its last digit is shifted to the front.
Indicate how to construct such an integer larger than any given integer.

2 Dr. Spock was making patterns with his dominoes ($\boxed{i,j}$, $0 \leq i \leq 6$, $0 \leq j \leq 6$) when Captain Kirk sat on his ray gun and fused them into the following array.
Draw in the erased boundaries.

5	4	2	4	1	1	5
4	6	3	2	3	6	3
0	0	1	2	0	6	4
3	4	3	0	1	0	4
1	5	5	5	6	4	2
3	3	4	6	1	0	5
2	6	2	1	5	3	6
5	0	2	1	2	0	6

3 The combination to the secret safe behind the washbasin in the outside toilet consists of seven different integers in descending order, whose sum is 36. A burglar knew the first two numbers, and tried to bribe a porter into telling him the value and position of another.
"You'll have to pay me for two, sir", said the porter, "since no matter which one I gave you, you wouldn't be able to deduce the exact code".
Technically he was correct, but he did not get any money. (The porter is aware of the burglar's knowledge).

4 In the following sums each letter represents a unique digit. Furthermore no two different letters stand for the same digit.
ONE + ONE + ONE ⇒ FOUR, FOUR + ONE = FIVE,
TWO - ONE - ONE = 0
What is NOW FURTIVE?

5 The audience at a Natural Sciences lecture on rag day consists of various numbers of spiders, starfish, tapeworms and relations of Cyclops, a solitary peacock and, of course, Macbeth's corpse. The first lecturer of the day observes as many arms as legs in the audience, and so assumes all is as usual. The second lecturer compares arms and eyes, with a similar result. The third lecturer notes twice as many eyes as heads, but is surprised by how many of them are open.
Describe the constitution of the least possible audience.

Biological Data

	heads	eyes	arms	legs
spider	1	2	0	8
starfish	0	0	5	0
tapeworm	1	0	0	0
Cyclops	1	1	2	2
Macbeth	0	0	2	2
peacock	1	1002	0	2

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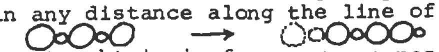
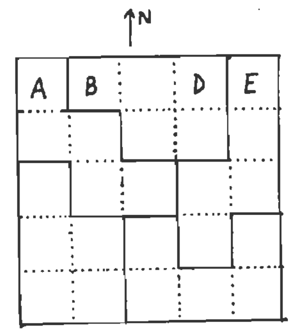
6 Three 2p coins and two 1p coins are arranged thus:

A move consists of translating a 2p coin and a touching 1p coin any distance along the line of their centres.
e.g.: 
Show how to obtain in four moves a position of the form:

7 AND HERE IT IS ... the inevitable what's the next number in the series problem. Explanations of solutions are advisable.
(a) 0, 1, -1, 2, -1, 5, -4, 29, ... on
(b) 6, 11, 37, 135, 2059, ... No
(c) 1, 2, 9, 12, 70, 89, 97, 102, 112, 182, ... No

8 Following Cambridge University's UDI the 25 colleges (unit squares in the diagram) were each occupied by one of five peace-keeping forces, the Apaches, Bootboys, Cowgirls, Deviants and Extras. This was done in such a way that each N-S strip, each E-W strip and every local parish (in black outlines) contained a college occupied by each force.
Given the information in the diagram, colour Cambridge using five colours without reference to any controversial theorems.



9 The hands on my alarm clock are indistinguishable, and there are no numbers around the outside. Accidentally woken up by it one morning, I observed with a snarl that the hands were both pointing at minute divisions, and that they were 9 minutes apart.
Had it not been for my hangover, what could I have deduced?

10 The C.I.A. have developed a new technique to destroy their secret book-keeping. Unfortunately for the free world, it merely obscures the digits, and does not remove decimal points from a calculation.
As an aspiring Commy, can you fill in the blurred digits in the exact long division below?

```
          @@@@.@@@@
@@@) @@@@@@
     @@@
      @@@
      @@@
       @@@
       @@@
        @@@
        @@@
         @@@@
         @@@@
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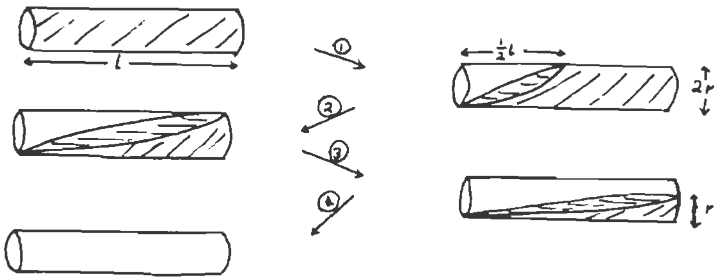
11 Four impoverished students have a solitary, full, cylindrical glass of beer. They share it out according to the following measurements:



What proportion does each get of the total volume?

MAGDALENE

Ramsey Problems in Euclidean Geometry

by Dr. B. Bollobás

Most of you are probably familiar with Ramsey's theorem, proved by the Cambridge logician F. P. Ramsey [2] in 1930: any colouring of the pairs of the natural numbers with two colours contains a monochromatic infinite set, i.e. an infinite set of natural numbers, all of whose pairs have the same colour. For over twenty years this result seemed to be no more than a curiosity. In the fifties, however, it became increasingly clear (due mostly to the efforts of Professor Paul Erdős) that there are many similar results and problems under rather different conditions.

In the early sixties a branch of set theory was born, called the <u>partition calculus</u>, which aims at answering the question: for which cardinals n, m, r and c is the following statement true?

"Let X be a set with $|X|=m$ and X(r) the set of subsets of X with cardinality r. If we colour X(r) with c colours, then we can find $Y \subseteq X$ with $|Y|=m$ and all elements of Y(r) having the same colour".

In a variant of this problem we restrict our attention to colourings which are 'regular' in some sense. Thus if we colour $2^{\mathbb{N}} = P(\mathbb{N})$, the set of all subsets of $\mathbb{N}$, with two colours, then there need not be an infinite set $M \subseteq \mathbb{N}$, all of whose infinite subsets have the same colour; but if one of the colour classes is open (in the product topology on $2^{\mathbb{N}}$, the product of countably many 2-point discrete spaces), then there has to be such an M. (This result and some extensions of it are very useful in analysis.)

The nature of the problem changes again if we colour an algebraic or geometric object and look for a monochromatic set with a given structure. A result of this kind was proved by van der Waerden [3] three years before Ramsey's theorem: given n there exists an N such that if $\{1,2,\ldots,N\}$ is coloured with two colours then at least one colour class contains an arithmetic progression of length n. Recently several deep results were proved in this vein, including extensions of van der Waerden's theorem to commutative semigroups.

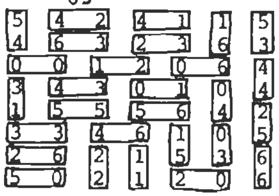
The aim of this article is to draw attention to some Ramsey type problems discussed in [1], which, though not very deep, are rather amusing, have not been investigated too much and can be tackled by first year undergraduates with a fair chance of success. (See [1] for many more results and problems.)

Let L be a <u>finite</u> set of points in $\mathbb{R}^m$, m-dimensional

Solutions to Problems Drive

1 $a\dfrac{10^{68n} - 1}{69}$, a = 7, 8 or 9; n any positive integer.

2

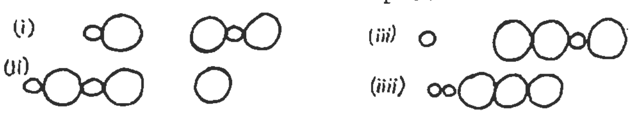


3 9, 8, 7, 5, 4, 2, 1

4 439 1806725

5 166 spiders, 266 starfish, 2 tapeworms, 2553 cyclops 1 peacock and Macbeth's corpse.

6



7 (a) $u_n = u_{n-2}^2 - u_{n-1}$, so next term is -13.

(b) if p_n is the nth prime, $u_n = 2^{p_n} + p_n$, so next term is 8205.

(c) $u_n - 1$ is the nth natural number with rotational symmetry, so next term is 610.

8

A	B	C	D	E
D	C	A	E	B
C	E	B	A	D
B	D	E	C	A
E	A	D	B	C

9 The time was 7.48 a.m. (4.12 can never be in the morning).

10 $625\overline{)631938}$

11 $\frac{1}{4}, \frac{1}{4}, \frac{1}{2} - \frac{2}{3\pi}, \frac{2}{3\pi}$.