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Jan. 27, 1987
75-61 177 St.
Flushing, N.Y. 11366

Dear Dr. Sloane-

I am enclosing two series whose extension have never - to my knowledge - been fully solved.

The first is my own - entitled "Unequal Sums" - published in the Journal of Recreational Mathematics, Vol. 15 No. 2, 1983-4. Along with the statement of the problem is a partial solution by an editor of the Journal and partial solutions by myself - Morris Wald - and by Richard J. Hess. I would love to see a complete solution. I can vouch for accuracy up to P=11 on my own account.

The second series was published in the Journal of the London Mathematical Society, Vol. 31, 1956, p. 160-169. I am enclosing only the concluding page. What this is all about is - a stick is divided into segments of varying lengths by appropriately spaced marks - so that the sum of 2, 3, 4 - - - adjacent SEGMENTS will generate all integral lengths FROM 1 up to the value of n (see the TABLE). For unrestricted difference bases the total length of the stick can be anything. For restricted bases the length of the stick must equal the sum of its segments. I will be happy to send you the rest of the article if you like.

Very sincerely -
Morris Wald

MORRIS WALD
75-61 177 ST.
FLUSHING, N.Y. 11366

P.S. I got a real big kick out of the N.Y. Times article. Congratulations!

P.S. I'd welcome an acknowledgment if you use this material.

estimate in (c′) is to replace 3·348 by 3·341, and if (b′) were proved with $\lambda = \frac{5}{3}$, the result would only be to replace the upper estimate by 3·333. These are the corresponding values of $(l+\lambda)^2/(N+1)$ for the basis given above in full, which is the best of any we have constructed for λ in this range. It is likely that more substantial improvement in the upper estimate can be obtained by the direct construction of bases by the present or other methods, as we have found several times during the preparation of this paper. We have, however, exhausted the present construction for $m \leqslant 183$ and for $n \leqslant 36$, so that further improvement by the present method will be hard won.

III. *Difference bases with respect to n for some small values.*

In order to obtain detailed information about the structure of those difference bases which minimise k or l for given n, we have constructed exhaustively the general and restricted difference bases which maximise n for each $k \leqslant 7$ and for each $l \leqslant 8$, and have also constructed some difference bases for greater n including some which are maximal for $k = 8$ and for $l = 9$ but which may not exhaust the difference bases giving these maxima. These bases are given in a table.

The exhaustive construction was effected by means of assessing for indefinite n the different ways of representing $n, n-1, n-2, \ldots$; that is, the difference bases are constructed inwards from their ends. At each stage of the construction, the redundancy ρ is assessed, this being the number of pairs of members of the basis which do not contribute because their difference is either greater than $n+1$ or equal to that between some other pair. We have constructed in this way all general difference bases with $\rho \leqslant 3$ and all restricted difference bases with $\rho \leqslant 5$.

In the table, the *numbers* in each basis are represented by points and the *differences* between them by the numbers in the tables. In our experience the structure of a difference basis is much more perspicuous when it is given in this way, and the similarities between the various bases are more evident. The entries in the column for unrestricted difference bases are limited to those general bases which are not restricted bases—where there are none, the value of n for the restricted bases is given in brackets. There are no such unrestricted difference bases having n as great as for the restricted difference bases with the same number of terms for $k \leqslant 4$, since in these cases the redundancy is zero, and we have failed to construct any with $k = 9$. The entries in the table for $k, l = 10, 11$ have in each case the largest n of any we have constructed, and if any be found with larger, an improvement of the upper estimate in (c) or (c′) will be obtained. In proving that $n = 29$ is maximal for $l = 9$, we have used Brauer's result $l(30) = 10$ in conjunction with our own results for $\rho = 5$ which show that $l(n) \geqslant 10$ for $n \geqslant 31$. It should be noted that $l(n)$ is not necessarily a non-decreasing function of n although $k(n)$ is. The entries for $l \geqslant 9$ were constructed by analogy with those for $l = 8$, while those for $k \geqslant 8$ were constructed from perfect difference sets by selecting a suitable set $b_1, b_2, \ldots$, and adjoining one or more terms $b_1+m, b_2+m, \ldots$. This method is less fruitful for larger values of n. 18 and 24 are the smallest values of n such that $k(n) < l(n)$ and the only ones for which we have proved this so. The basis given for $k = 8$ is that quoted at the end of §I, that given for $l = 10$ is that used for the set (1′) in the example of §II.

TABLE.

k	n	Unrestricted difference bases.	l	n	Restricted difference bases.
1	(0)		1	0	.
2	(1)		2	1	. 1 .
3	(3)		3	3	. 1 . 2 .
4	(6)		4	6	. 1 . 3 . 2 .
5	9	. 3 . 1 . 5 . 2 .	5	9	. 1 . 3 . 3 . 2 .
		. 4 . 1 . 2 . 6 .			. 1 . 1 . 4 . 3 .
6	13	. 4 . 1 . 1 . 7 . 3 .	6	13	. 1 . 1 . 4 . 4 . 3 .
		. 6 . 1 . 2 . 2 . 8 .			. 1 . 5 . 3 . 2 . 2 .
		. 1 . 3 . 6 . 2 . 5 .			. 1 . 3 . 1 . 6 . 2 .
		. 1 . 7 . 3 . 2 . 4 .			
7	18	. 6 . 3 . 1 . 7 . 5 . 2 .	7	17	. 1 . 1 . 4 . 4 . 4 . 3 .
		. 8 . 1 . 3 . 6 . 5 . 2 .			. 1 . 1 . 1 . 5 . 5 . 4 .
		. 14 . 1 . 3 . 6 . 2 . 5 .			. 1 . 1 . 6 . 4 . 2 . 3 .
		. 13 . 1 . 2 . 5 . 4 . 6 .			. 1 . 1 . 6 . 4 . 3 . 2 .
					. 1 . 3 . 6 . 2 . 3 . 2 .
					. 1 . 7 . 3 . 2 . 2 . 2 .
8	24	. 8 . 10 . 1 . 3 . 2 . 7 . 8 .	8	23	. 1 . 1 . 9 . 4 . 3 . 3 . 2 .
					. 1 . 3 . 6 . 6 . 2 . 3 . 2 .
9	(29)		9	29	. 1 . 1 . 12 . 4 . 3 . 3 . 3 . 2 .
					. 1 . 3 . 6 . 6 . 6 . 2 . 3 . 2 .
					. 1 . 2 . 3 . 7 . 7 . 4 . 4 . 1 .
10	37	. 16 . 1 . 11 . 8 . 6 . 4 . 3 . 2 . 22 .	10	36	. 1 . 2 . 3 . 7 . 7 . 7 . 4 . 4 . 1 .
		. 7 . 15 . 5 . 1 . 3 . 8 . 2 . 16 . 7 .			
11	45	. 18 . 1 . 3 . 9 . 11 . 6 . 8 . 2 . 5 . 28 .	11	43	. 1 . 2 . 3 . 7 . 7 . 7 . 7 . 4 . 4 . 1 .

King's College,
Cambridge.

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EMBEDDINGS IN SEMIGROUPS WITH ONE-SIDED DIVISION

P. M. COHN†.

1. By a *semigroup* we shall understand a set with a single-valued binary operation (denoted by juxtaposition: ab) defined on it, which is associative: $a(bc) = (ab)c$.

† Received 15 March, 1955; read 24 March, 1955.

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For what values of n is this possible such that no two knights arrive at the same place?

Solution by E. C. Buissant des Amorie, Amstelveen, Netherlands

It is only possible for odd n. Say $n = 2m + 1$. There are m pairs with sum n: $[1 + (n - 1)]$, $[2 + (n - 2)]$, ... Going clockwise, you can place knights with their numbers $1, 2, \ldots, m, n, (n - m), (n - m - 1), \ldots$.

For even n it is impossible. Proof:

Let the knight-numbers be: $1\ 2\ 3\ \ldots\ 2m$ Sum $= m(2m + 1)$

and the permutation-numbers be: . . . . Sum $= m(2m + 1)$.

Adding modulo $2m$ gives the impossibility:

$$m(2m + 1) + m(2m + 1) \equiv m(2m + 1) (\mathrm{mod}\ 2m).$$

***1192. Unequal Sums** by Morris Wald, Flushing, NY (*JRM, 15:*2, p. 143)

Let S be a set of positive integers less than or equal to n. As a function of n, how many members may S contain such that no two disjoint subsets of S have the same sum?

Partial Solution by Friend H. Kierstead, Jr., Cuyahoga Falls, OH

Let p be the maximum number of members of S such that all of the sums are unequal. We will determine upper and lower bounds on p for all positive integer values of n.

Let $n = 2^{p_i - 1}$. Then the set $(1, 2, 4, \ldots, 2^{p_i - 1})$ contains p_i members and generates uniquely all of the positive integers from 1 to $2^{p_i} - 1$. Therefore, $p \geqslant p_i$ and p_i is a valid lower bound for $n = 2^{p_i - 1}$, and in fact for all values of n from $2^{p_i - 1}$ to $2^{p_i} - 1$.

The pigeonhole principle furnishes an upper bound for p, say p_u. The largest sum that can be generated by p_u integers less than or equal to n is $\frac{1}{2}p_u(2n - p + 1)$, and this must be larger than the number of sums generated by all subsets of S, which is $2^{p_u} - 1$. Therefore,

$$\tfrac{1}{2}p_u(2n - p_u + 1) \geqslant 2^{p_u} - 1\ ,$$

whence,

$$n \geqslant [\ 2^{p_u} + \tfrac{1}{2}p_u(p_u - 1) - 1\ ]/p_u.$$

From this relation we can readily find all values of n for which p_u is an upper bound on p.

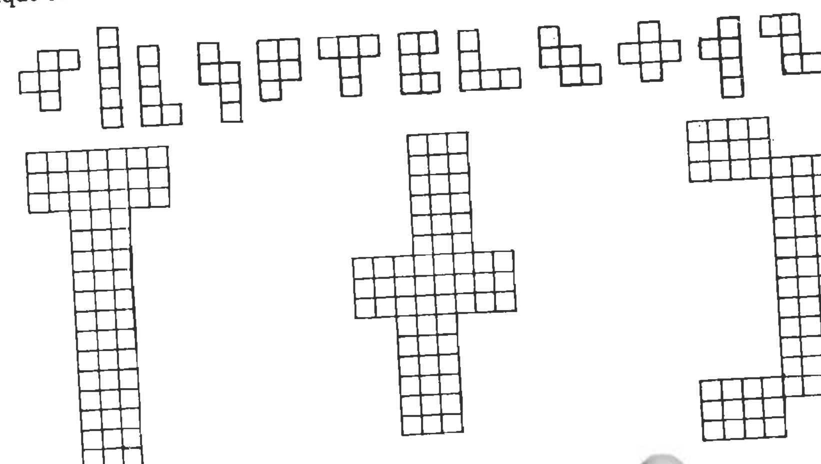
We now have all of the information to construct the accompanying table. The first column lists a value of p_i or p_u; the second column shows the range of values of n for which the p in the first column is a lower bound; the third column shows the range of n for which the p is an upper bound; the fourth column shows the true range of n as hand-calculated by Mr. Wald to $p = 9$ and to $p = 16$ by Richard I. Hess of Palos Verdes, CA; the fifth column shows a few example subsets by Mr. Hess. It may be seen that for a given value of n, Wald's value for p never differs by more than 1 [illegible] the lower bound. Also, the upper and lower bounds

are identical up to $n = 5$, differ by no more than 1 up to $n = 12$, by 2 up to 106, and by 3 up to 4013. new 5318

p	*Lower*	*Upper*	*Wald & Hess*	*Examples of Subsets*
1	1	1	1	
2	2-3	2-3	2-3	
3	4-7	4-5	4-6	
4	8-15	6-8	7-12	7, 6, 5, 3
5	16-31	9-12	13-23	13, 12, 11, 9, 6
6	32-63	13-21	24-43	24, 23, 22, 20, 17, 11
7	64-127	22-35	44-83	
8	128-255	36-60	84-160	
9	256-511	61-106	161-308	
10	512-1023	107-191	309-593	309, 308, 305, 302, 296, 285, 265, 225, 148
11	1024-2047	192-346	594-1163	
12	2048-4095	347-636	1164-2283	
13	4096-8191	637-1176	2284-4483	
14	8192-16383	1177-2191	4484-8806	
15	16384-32767	2192-4013	8807-17304	
16	32768-65535	4014-7718	17305-34300	

1193. Pentomino* Packing II, by Yoshio Ohno, Tokyo, Japan (*JRM, 15:*2, p. 143)

Fill each of the large figures below with the set of 12 pentominoes. Each has a unique solution.



* *Pentomino* is a registered trademark of Solomon W. Golomb.

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February 12, 1987

Mr. Morris Wald
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Flushing, N.Y. 11366

Dear Mr. Wald:

Thank you for your kind letter of January 27. I was aware of Leech's sequence, but I had not seen the other before. Yes, I am collecting material for the second edition. Enclosed is a paper related to Leech's.

Yours sincerely,

N. J. A. Sloane

NJAS:yc

Enc.
As above