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David Singmaster
letter

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Neil Sloane
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Dear Neil Sloane,

I've just worked out some sequences that may interest you.

It is well known that the number of ways of packing a $2 \times n$ board with dominoes is F_{n+1} , where the F_n are the Fibonacci numbers with $F_0 = 0$. I believe the following are well known, though I don't have ~~xxxx~~ any references. The number of ways of packing a $3 \times 2n$ rectangle with dominoes satisfies $f(n) = 4f(n-1) - f(n-2)$ with $f(0) = 1$, $f(1) = 3$, $f(2) = 11$. This ^{is} sequence ~~1160~~ in the Handbook, but you do not give a reference to its occurrence in this context (though I haven't checked the Euler reference, but it seems unlikely that he did this). The number of ways of packing a $4 \times n$ rectangle with dominoes satisfies $f(n) = f(n-1) + 5f(n-2) + f(n-3) - f(n-4)$, with $f(0) = 1$, $f(1) = 1$, $f(2) = 5$, $f(3) = 11$. I do not find this in the Handbook. The terms are: 1, 1, 5, 11, 36, 95, 281, 781, 2245, 6336, 18061, 51205, 145601, 413351, 1174500, 3335651, 9475901, 26915305, ...

I don't see that you refer to the following sources which give other evaluations of the number of packings.

M. E. Fisher, Statistical mechanics of dimers on a plane lattice, Phys. Rev. 124 (1961) 1664-1672.

P. W. Kasteleyn, The statistics of dimers on a lattice, Physica 27 (1961) 1209-1225.

I haven't seen these yet, but they are referred to in E. W. Montroll's chapter "Lattice Statistics" in Applied Combinatorial Mathematics and elsewhere.

Some other odd sequences.

Let $e(n)$ be the smallest integer e with exactly n divisors. Then $e(1) = 1$ and the sequence is: 1, 2, 4, 6, 16, 12, 64, 24, 36, 48, 1024, 60, 4096, 192, 144, 120, 65536, 180, 262144, 240, 576, 3072, 4194304, 360, 1296, 12288, 900,

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 960, 268435436, 720, 1073741824, 840, 9216, 196608, 5184, 1260, ...

I can't remember where I got this from. Possibly by looking in Glaisher's Number-Divisor Tables.

Let $a_1 = 1$, $b_1 = 2$, $a_{n+1} = a_n + b_n$, $b_{n+1} = \text{least integer} > b_n$ and which is not an a_i . The sequences go:

a_n 1 3 7 12 18 26 35 45 56 69 83 98 114 131 151 172 194 217
 b_n 2 4 5 6 8 9 10 11 13 14 15 16 17 19 20 21 22 23

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A friend gave this sequence to me, but he didn't say where it came from. It is very close to your sequence 1042 and 355. It seems like it should be related to Beatty and/or Wythoff, but I haven't really tried to find such a relation.

At one time I wrote down the following sequences.

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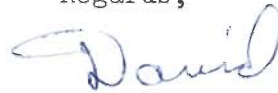
Smallest prime factor of n : 2, 3, 2, 5, 2, 7, 2, 3, 2, 11, 2, 13, 2, 3, 2, ...

These would be the first factors which cancel the corresponding number in carrying out the Sieve of Erasthenes.

Largest prime factor of n : 2, 3, 2, 5, 3, 7, 2, 3, 5, 11, 3, 13, 7, 5, 2, 17, ...

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Regards,



David Singmaster

PS. It was a pleasure to see you at Bell in January.