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M. Schroder
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Dear Sir,

from Bruno Leclerc (Paris) I heard, that you are planning a new edition of your handbook of integer sequences. I would like to inform you about a sequence which might be of interest to anybody working on data analysis, cluster analysis, classification (especially of qualitative data).

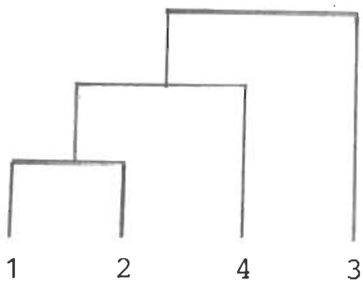
If X is the finite set of objects to be classified, it is known, that there exists a bijection from the set of hierarchies on X (= hierarchical classifications of X) into the set of ultradissimilarity relations on X (cf. the enclosed paper).

A dissimilarity relation on X is a total preorder $\precsim$ on the set $\{\{x,y\}: x,y \in X \text{ and } x \neq y\}$.

An ultradissimilarity relation on X is a dissimilarity relation on X, which satisfies $(\{x,z\} \precsim \{x,y\})$ or $(\{x,z\} \precsim \{y,z\})$ for all $x,y,z \in X$, $x \neq y$, $x \neq z$, $y \neq z$.

For example $\{1,2\} < \{1,4\} \sim \{2,4\} < \{1,3\} \sim \{2,3\} \sim \{3,4\}$ corresponds to the hierarchy

...



Using the Stirling numbers of the second kind $S(\cdot,\cdot)$ it is easy to calculate the number of dissimilarity relations on X, if $|X|$ is given:

$\|X\|$	number of dissimilarity relations on X
3	13
4	4,683
5	102,247,563
6	230,283,190,977,853
n	$\sum_{i=o}^{m-1} (m-i)!\ S(m,m-i)$ where $m = n(n-1)/2$

It should be more interesting to know the number of ultra-dissimilarity relations on X which must be computed recursively (cf. the enclosed paper):

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$\|X\|$	number of ultradissimilarity relations on X
3	4
4	32
5	436
6	9,012
7	262,760
8	10,270,696
9	518,277,560
10	32,795,928,016
n	$\sum_{i=1}^{n-1} N_i(n)$

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$$\sum_{i=1}^{n-1} N_i(n)$$

where $N_k(l) = \sum_{j=k}^{l-1} S(l,j) N_{k-1}(j)$ $\quad \forall\ l = 3,4,\ldots,n \quad \forall\ k = 2,3,\ldots,l-1$

and $N_1(2) = N_1(3) = \ldots = N_1(n) = 1$

Yours sincerely,

M. Schader