153 155 132

$1 \leq n^5 \leq 3$ $5 \leq m^5 \leq 4$

$n' = 1 \Rightarrow m' = 5$ $n' = 5 \Rightarrow m' = 3$

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Irredundant weighted binary codes

Marc Davio
1975

Consider a vector $\underline{\underline{w}} = [w_0, w_1, \ldots, w_{n-1}]$ the components of which are positive integers arranged by increasing values. Denote furthermore by W_i the quantity:

$$W_i = \sum_{j=0}^{i} w_i \qquad (i = 0, 1, \ldots, n-1)$$

and put

$$w_{-1} = W_{-1} = 0$$

Then it is easy to prove the following:

Theorem 1. The weight vector $\underline{\underline{w}}$ of an irredundant weighted binary code with positive weights satisfies the following inequalities

what is this?

$$w_{i-1} \leq w_i \leq W_{i-1} + 1. \quad (i = 0, \ldots, n-1). \quad (1)$$

Problem Compute the number $P(n)$ of vectors of length n satisfying (1).* This provides an upper bound to the number of irredundant weighted binary codes of length n.

$w_{-1} = W_{-1} = 0$

* and $W_0 = 1$? $w_0 = W_0 = 1$

Then we have: For w_1: $1 \leq w_1 \leq 2$ $\therefore w_1 = 1$ or 2

$w_1 = 1 \Rightarrow W_1 = 2$		$w_1 = 2 \Rightarrow W_1 = 3$	
For w_2: $1 \leq w_2 \leq 3$		$2 \leq w_2 \leq 4$	
w	W	w	W
111	123	122	135
112	124	123	136
113	125	124	137

A recurrent construction is suggested by ~~theorem 1~~ eqn (1).
It is illustrated hereunder for $n = 4$

- 1 →
  - 11 →
    - 111 → 1111, 1112, 1113, 1114
    - 112 → 1122, 1123, 1124, 1125
    - 113 → 1133, 1134, 1135, 1136
  - 12 →
    - 122 → 1222, 1223, 1224, 1225, 1226
    - 123 → 1233, 1234, 1235, 1236, 1237
    - 124 → 1244, 1245, 1246, 1247, 1248

<u>Rule</u>: last term runs from penult term to (old row sum) + 1

eg old row sum = 4
penult = 2
∴ 1122 to 1125

It is clear that the number of vectors of length $(n+k)$ generated by the vector $\underline{\underline{w}}$ of length n depends upon w (Compare 11 and 12 for $k = 2$).

Put now for sake of simplicity $x = W_{n-1}$ and $y = W_{n-2}$.
Then, it is possible to show that the vector $\underline{\underline{w}}$ generates

(i) $(y+2)$ vectors of length $(n+1)$

(ii) $(x+2)(y+2)$ vectors of length $(n+2)$

(iii) $\frac{1}{2}(x+2)(y+2)(4x-y+5)$ vectors of length $(n+3)$

(iv) $\frac{1}{12}(x+2)(y+2)(84x^2 - \overset{45}{\cancel{37}}xy + 8y^2 + 189x - \overset{43}{\cancel{38}}y + 111)$
vectors of length $(n+4)$

Hence, since the only vector of length 1 is [1] $(x=1, y=0)$ one immediately obtains

n	1	2	3	4	5	6
$P(n)$	1	2	6	27	192	2280

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Similarly, the vectors of length 2 [1, 1] and [1, 2] (with, respectively $x=2, y=1$ and $x=3, y=1$) generate
and vectors of length 6, i.e

$P(6) = \qquad + \qquad = 2280$ (?)